Resource Allocation Schemes for Target Localization in Distributed Multiple Radar Architectures

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***Abstract*—In this paper, two resource allocation schemes for multiple radar systems are proposed. The first approach fully utilizes all available infrastructure in the localization process, i.e., all transmit and receive radars, while minimizing the total transmit energy. The power allocation among the transmit radars is optimized such that a predefined estimation mean-square error (MSE) objective is met, while keeping the transmitted power at each station within an acceptable range. The second scheme minimizes the number of transmit and receive radars employed in the estimation process by effectively choosing a subset of radars such that the required MSE performance threshold is attained. In the latter, the transmit antennas are assumed to fully utilize the admissible power range. The Cramer-Rao bound (CRB), which is known to be asymptotically tight to the maximum likelihood estimator (MLE) MSE at high signal-to-noise ratio (SNR), is used as an optimization metric for the estimation MSE. Subset selection is implemented through a heuristic algorithm, offering reduced computational cost compared with an exhaustive search.**

***Index Terms*—MIMO radar, Multistatic radar, CRB, convex optimization, power allocation, target localization.**

I. Introduction

In recent years, radar architectures employing multiple, widely distributed stations have been introduced, such as multiple-input multiple-output (MIMO) radar systems with widely spread antennas [1] and multistatic radar systems [2]. These systems have been shown to offer significant advantages over traditional single antenna radars, referred to as monostatic, or systems with one transmitter and one receiver which are widely separated, often referred to as bistatic. MIMO radar systems with widely distributed antennas offer enhanced target localization capabilities by exploiting increased spatial spread [1]. A study of localization estimation mean square error (MSE) performance based on the Cramer-Rao bound (CRB) [3] is presented in [4], demonstrating performance improvement proportional to the product of the number of transmit and receive antennas. An analysis of the localization performance for multistatic radar systems is provided in [2], where the MSE is shown to be a function of the geometric spread. Localization MSE in MIMO radar systems with non-coherent processing is inversely proportional to the signal effective bandwidth, the signal-to-noise ratio (SNR), and the product of the number of transmit and receive antennas [4].

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Localization performance improvement can be achieved with an increase of either the number of participating radars or the transmission power. In practice, most systems have a predetermined performance goal including, but not limited to, target localization accuracy and maximum total radiated energy. Consequently, full system utilization may result in inefficient use of system resources, such as the number of operating radars, transmitted power, and communication load between the radars and a central fusion center.

The notion of resource-aware design is of critical importance when it comes to radar applications that include mobile deployment of stations or systems operating over prolonged time periods, in which the cost of operation becomes significant. For surveillance radars that are mounted on vehicles and thus have limited energy resources, or for anti-missile defense radar systems, powered off-grid by diesel generators, power aware design is beneficial in extending the ability of such systems to operate before refueling. Furthermore, power management is an essential part of military operations in hostile environments, where low-probability-of-intercept (LPI) operation may be required. In these scenarios, minimizing the power that is required to perform the task is important. Another aspect of the problem is the use of the available infrastructure. For a given multiple radar system, multiple mission assignments may be accomplished by minimizing the number of transmit and received stations engaged in a specific estimation task.

In this paper, two resource allocation schemes are proposed. The first approach optimizes power allocation among all radars in the system, while the latter offers a more effective utilization of the existing radar stations, where the most advantageous antenna subset is selected to accomplish the task requirements. The choice of an optimal subset is normally implemented through exhaustive examination of all possibilities. Here, we propose a heuristic algorithm for the selection of this subset and provide a performance comparison to an exhaustive search alternative.

The paper is organized as follows: The system model is introduced in Section II. Resource allocation schemes are proposed in Section III, in which the CRB is derived first in Subsection III-A, followed by the development of a power allocation scheme that optimizes power allocation for a given MSE threshold in Subsection III-B. A subset selection algorithm, minimizing the number of transmit and receive radars

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active in the localization process, is given in Subsection III-C. Numerical analysis is presented in Section IV. Finally, Section V concludes the paper.

II. System Model

We consider a distributed multiple radar system with M transmit and N receive radars, forming an $M\times N$ distributed multiple radar system. An extended target, with a center of mass located at position (x,y), is assumed. The variation in the location of the targets' center of mass, as viewed by the set of radars, is assumed to be small with respect to the system resolution capabilities. The system is tracking the target's location and has available estimates for unknown parameters, such as the target radar cross section (RCS), from previous cycles. The search cell is confined to $(x_c\pm kc/\beta, y_c\pm kc/\beta)$. The transmit and receive radars are located in a two dimensional plane. The M transmit radars are arbitrarily located at coordinates $(x_{m_{Tx}}, y_{m_{Tx}})$, $m=1,\ldots,M$, and the N receiver radars are arbitrarily located at coordinates $(x_{n_{Rx}}, y_{n_{Rx}})$, $n=1,\ldots,N$. A set of orthogonal waveforms is transmitted, with a lowpass equivalent $s_m(t)$, where $\int_{\mathcal{T}_m}|s_m(t)|^2dt=1$, and $\mathcal{T}_m$ is the duration of the m-th transmitted signal. The waveform effective bandwidth is denoted by β_m and defined in [5]. The waveforms' transmitted powers $p_{m_{tx}}$ are constrained by maximal values $\mathbf{p}_{tx_{\max}}=[p_{1_{tx\max}}, p_{2_{tx\max}},...,p_{M_{tx\max}}]^T$.

Let $\tau_{m,n}(x,y)$ denote the propagation time of a signal transmitted by radar m, reflected by the target, and received by radar n:

$$\tau_{m,n}(x,y)=\frac{R_{m_{Tx}}+R_{n_{Rx}}}{c}, \tag{1}$$

where $R_{m_{Tx}}$ is the range from transmitter m to the target and $R_{n_{Rx}}$ is the range from receiver n to the target, i.e.,

$$R_{m_{Tx}}=\sqrt{(x_{m_{Tx}}-x)^2+(y_{m_{Tx}}-y)^2} \tag{2}$$

and

$$R_{n_{Rx}}=\sqrt{(x_{n_{Rx}}-x)^2+(y_{n_{Rx}}-y)^2},$$

where c is the speed of light. The baseband representation for the signal transmitted from radar m received at radar n is

$$r_{m,n}(t)=\sqrt{\alpha_{m,n}(x,y)\,p_{m_{tx}}}\,h_{m,n}s_m(t-\tau_{m,n})+w_{m,n}(t). \tag{3}$$

The term $\alpha_{m,n}(x,y)=\frac{1}{R^2_{m_{Tx}}R^2_{n_{Rx}}}$ represents the variation in the signal strength due to path loss effects. The target RCS $h_{m,n}$ is modeled as deterministic, complex, and is assumed to be unknown. The term $w_{m,n}(t)$ represents circularly symmetric, zero-mean, complex Gaussian noise, spatially and temporally white with autocorrelation function $\sigma_w^2\delta(\tau)$.

We define a vector of unknown parameters:

$$\mathbf{u}=\left[x,y,\mathbf{h}^T\right]^T, \tag{4}$$

where $\mathbf{h}=[h_{1,1},h_{1,2},...,h_{M,N}]^T$. The following vector notation is defined for later use: $\boldsymbol{\beta}=[\beta_1,\beta_2,...,\beta_M]^T$, $\mathbf{p}_{tx}=[p_{1_{tx}},p_{2_{tx}},...,p_{M_{tx}}]^T$, and $\boldsymbol{\tau}=[\tau_1,\tau_2,...,\tau_{MN}]^T$.

III. Resource Allocation Schemes

For a predetermined threshold for localization MSE, denoted by $\eta_{x,y_{\max}}$, system resource utilization may be optimized by minimizing the total power radiation needed to achieve this goal. Another option is for the system to select a subset of radars that will perform the localization mission. The given system parameters include the transmit radar locations set $S_{Tx}=\{(x_{1_{tx}},y_{1_{tx}}), (x_{2_{tx}},y_{2_{tx}}),...,(x_{3_{Tx}},y_{3_{Tx}})\}$, receive radar locations set $S_{Rx}=\{(x_{1_{Rx}},y_{1_{Rx}}),(x_{2_{Rx}},y_{2_{Rx}}),...,(x_{3_{Rx}},y_{3_{Rx}})\}$, targets' RCS, $\mathbf{h}$, propagation path loss, $\boldsymbol{\alpha}$, and noise variance, σ_w^2. The controllable design parameters are the transmit power at each radar, $p_{m_{tx}}$, and the signal effective bandwidth $\boldsymbol{\beta}$. In general, power radiation is constrained by a maximum value $p_{m_{tx\max}}$, determined by the operational design, and a minimal value $p_{m_{tx\min}}$, chosen such that it may still be classified as operating in the high SNR region. As the CRB is known to be asymptotically tight to the maximum likelihood estimator (MLE) MSE at high SNR [6], it is used here to represent the localization MSE as a function of the power allocation. Next, the CRB expression is derived, to support the proposed resource allocation schemes.

A. The CRB

Given a vector of unknown parameters $\mathbf{u}$, its unbiased estimate $\hat{\mathbf{u}}$ satisfies the following inequality [3]:

$$E_{\mathbf{u}}\left\{(\hat{\mathbf{u}}-\mathbf{u})(\hat{\mathbf{u}}-\mathbf{u})^T\right\}\geq\mathbf{J}^{-1}(\mathbf{u}), \tag{5}$$

where $\mathbf{J}(\mathbf{u})$ is the Fisher Information matrix (FIM) given by:

$$\mathbf{J}(\mathbf{u})=E_{\mathbf{u}}\left\{\frac{\partial}{\partial\mathbf{u}}\log f(\mathbf{r}|\mathbf{u})\left(\frac{\partial}{\partial\mathbf{u}}\log f(\mathbf{r}|\mathbf{u})\right)^T\right\}, \tag{6}$$

where $f(\mathbf{r}|\mathbf{u})$ is the conditional, joint probability density function (pdf) of the observation $\mathbf{r}=[r_{1,1},r_{1,2},...,r_{M,N}]$. Given the received signal in (3), the conditional pdf $f(\mathbf{r}|\mathbf{u})$ is of the following form:

$$f(\mathbf{r}|\mathbf{u}) = \frac{1}{(\pi\sigma_w^2)^{\frac{MN}{2}}}\exp\left\{-\frac{1}{\sigma_w^2}\sum_{\ell=1}^{N}\sum_{k=1}^{M}\int_T\left|r_{m,n}(t)-\sqrt{\alpha_{m,n}(x,y)\,p_{m_{Tx}}}\,h_{m,n}s_m(t-\tau_{m,n})\right|^2dt\right\}. \tag{7}$$

The FIM, $\mathbf{J}(\mathbf{u})$, is derived in Appendix A (see (18), (23), and (24)). The CRB matrix, $\mathbf{C}_{x,y}$, is defined as the 2×2 upper right block sub-matrix of the inverse of the FIM, $\mathbf{J}^{-1}(\mathbf{u})$, resulting in the following matrix:

$$\mathbf{C}_{x,y}(\mathbf{u})=\left\{\sum_{m=1}^{M}p_{m_{tx}}\kappa_m\begin{bmatrix}a_m & c_m\\ c_m & b_m\end{bmatrix}\right\}^{-1}, \tag{8}$$

where the elements a_m, b_m, and c_m, are defined as

$$a_m = \sum_{n=1}^{N}\alpha_{m,n}|h_{m,n}|^2\left(\frac{x_{m_{Tx}}-x}{R_{m_{Tx}}}+\frac{x_{n_{Rx}}-x}{R_{n_{Rx}}}\right)^2 \tag{9}$$

$$b_m = \sum_{n=1}^{N}\alpha_{m,n}|h_{m,n}|^2\left(\frac{y_{m_{Tx}}-y}{R_{m_{Tx}}}+\frac{y_{n_{Rx}}-y}{R_{n_{Rx}}}\right)^2 \tag{10}$$

and

$$c_m = \sum_{n=1}^{N} \alpha_{m,n} |h_{m,n}|^2 \left(\frac{x_{m_{Tx}} - x}{R_{m_{Tx}}} + \frac{x_{n_{Rx}} - x}{R_{n_{Rx}}} \right) \times \left(\frac{y_{m_{Tx}} - y}{R_{m_{Tx}}} + \frac{y_{n_{Rx}} - y}{R_{n_{Rx}}} \right), \quad (11)$$

where $\kappa_m = \frac{8\pi^2\beta_m^2}{\sigma_w^2 c^2}$. The trace of the matrix $\mathbf{C}_{x,y}$ represent the lower bound on the sum of the MSEs for the target location estimation, i.e., $\operatorname{tr}(\mathbf{C}_{x,y}) \le \sigma_x^2 + \sigma_y^2$, where σ_x^2 and σ_y^2 are the target's x and y location estimation MSE, respectively. Following some additional matrix manipulations, the trace of the CRB matrix $\mathbf{C}_{x,y}$ can be expressed as

$$\operatorname{tr}(\mathbf{C}_{x,y}) = \frac{\mathbf{p}_{tx}^T (\mathbf{a} + \mathbf{b})}{\mathbf{p}_{tx}^T \left(\mathbf{a}\mathbf{b}^T - \mathbf{c}\mathbf{c}^T \right) \mathbf{p}_{tx}}, \quad (12)$$

where $\mathbf{a} = \kappa_m [a_1, a_2, ..., a_M]^T$, $\mathbf{b} = \kappa_m [b_1, b_2, ..., b_M]^T$, and $\mathbf{c} = \kappa_m [c_1, c_2, ..., c_M]^T$ are defined by the elements in (9), (10), and (11). For the case of equal power allocation, $\mathbf{p}_{tx_{EQ}} = p_{tx_{EQ}} [1, 1, ..., 1]^T$, the trace of the CRB is

$$\operatorname{tr}(\mathbf{C}_{x,y}) = \frac{1}{p_{uni/M}} \frac{\mathbf{1}^T (\mathbf{a} + \mathbf{b})}{\mathbf{1}^T \left(\mathbf{a}\mathbf{b}^T - \mathbf{c}\mathbf{c}^T \right) \mathbf{1}}, \quad (13)$$

where $p_{uni} = p_{tx_{EQ}} * M$. The expression for the CRB as given in (12), offers a metric that may be used to represent the MLE MSE in the power allocation schemes provided next.

B. Power Allocation: Minimize the Power Budget

In this power allocation scheme, the total radiating power, $\sum_{m=1}^{M} p_{m_{tx}}$, is minimized to meet a given localization accuracy threshold, η_{thd}. This can be formulated into an optimization problem of the form

$$\begin{aligned} \underset{\mathbf{p}_{tx}}{\text{minimize}} \quad & \mathbf{1}^T \mathbf{p}_{tx}, \\ s.t. \quad & \operatorname{tr}(\mathbf{C}_{x,y}(\tilde{\mathbf{u}})) \le \eta_{\max}, \\ & p_{m_{tx}} \le p_{m_{tx\max}}, \quad \forall m = 1, 2, ..., M, \\ & p_{m_{tx}} \ge p_{m_{tx\min}}, \quad \forall m = 1, 2, ..., M, \end{aligned} \quad (14)$$

where $\mathbf{C}_{x,y}(\tilde{\mathbf{u}})$ is the 2×2 CRB matrix given in (8) and $\tilde{\mathbf{u}} = \left[\tilde{x}, \tilde{y}, \tilde{\mathbf{h}}^T \right]^T$ is a vector of preliminary estimates of the target location and RCS, obtained in previous cycles. The search cell center coordinates, (x_c, y_c), may also be used instead of an estimated target location$(\tilde{x}, \tilde{y})$. The optimization problem in (14) may be rewritten as

$$\begin{aligned} \underset{\mathbf{p}_{tx}}{\text{minimize}} \quad & \mathbf{1}^T \mathbf{p}_{tx}, \\ s.t. \quad & \mathbf{p}_{tx}^T \left[\left(\tilde{\mathbf{a}}\tilde{\mathbf{b}}^T - \tilde{\mathbf{c}}\tilde{\mathbf{c}}^T \right) \mathbf{p}_{tx} - \frac{1}{\eta_{thx}} \left(\tilde{\mathbf{a}} + \tilde{\mathbf{b}} \right) \right] \ge 0, \\ & p_{m_{tx}} \le p_{m_{tx\max}}, \forall m, \\ & p_{m_{tx}} \ge p_{m_{tx\min}}, \forall m, \end{aligned} \quad (15)$$

where $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$, and $\tilde{\mathbf{c}}$ are calculated for the estimated vector $\tilde{\mathbf{u}}$. The optimization problem in (15) is nonconvex [7]. To solve it we first relax the constraints by exchanging the first inequality by an equality, i.e., $\mathbf{p}_{tx}^T \left[\eta_{x,y} \left(\tilde{\mathbf{a}}\tilde{\mathbf{b}}^T - \tilde{\mathbf{c}}\tilde{\mathbf{c}}^T \right) \mathbf{p}_{tx} - \left(\tilde{\mathbf{a}} + \tilde{\mathbf{b}} \right) \right] = 0$, and since $p_{m_{tx}} \neq 0$, $\forall m = 1, 2, ..., M$, the latter constraint is replaced by $\left(\mathbf{a}\mathbf{b}^T - \mathbf{c}\mathbf{c}^T \right) \mathbf{p}_{tx} - \frac{1}{\eta_{x,y}} (\mathbf{a} + \mathbf{b}) = 0$. Minimizing the power vector results in maximizing the MSE, bringing it as close as possible to the threshold point η_{thx}. The following relaxed convex optimization problem may be composed:

$$\begin{aligned} \underset{\mathbf{p}_{tx}}{\text{minimize}} \quad & \mathbf{1}^T \mathbf{p}_{tx}, \\ s.t. \quad & \left(\tilde{\mathbf{a}}\tilde{\mathbf{b}}^T - \tilde{\mathbf{c}}\tilde{\mathbf{c}}^T \right) \mathbf{p}_{tx} - \frac{1}{\eta_{\max}} \left(\tilde{\mathbf{a}} + \tilde{\mathbf{b}} \right) \ge 0, \\ & p_{m_{tx}} \le p_{m_{tx\max}}, \forall m, \\ & p_{m_{tx}} \ge p_{m_{tx\min}}, \forall m. \end{aligned} \quad (16)$$

The relaxed convex optimization problem formulated in (16) can be solved using available convex optimization tools, such as CVX [8]. The optimal solution to (16), $\mathbf{p}_{tx_{PW}}^*$, is then used as the starting point for a local optimization, applied to the original nonconvex problem in (15). An appropriate search algorithm is proposed in Table 1. The local optimum obtained in this process, $\mathbf{p}_{tx_{PWu}}$, is then compared with the uniform power distribution budget, for which $p_{tx_{EQ}} = \frac{1}{\eta_{thd}} \frac{\mathbf{1}^T(\mathbf{a}+\mathbf{b})}{\mathbf{1}^T\left(\mathbf{a}\mathbf{b}^T - \mathbf{c}\mathbf{c}^T\right)\mathbf{1}}$. If needed, the locally optimum search is updated and repeated.

Table 1: Optimization algorithm for (15)	
1. Init:	$\mathbf{p}_{tx_o} = \mathbf{p}_{tx_{PW}}^*$ $\eta(\mathbf{p}_{tx}) = \frac{\mathbf{p}_{tx}^T(\tilde{\mathbf{a}}+\tilde{\mathbf{b}})}{\mathbf{p}_{tx}^T\left(\tilde{\mathbf{a}}\tilde{\mathbf{b}}^T - \tilde{\mathbf{c}}\tilde{\mathbf{c}}^T\right)\mathbf{p}_{tx}}$ Iteration step $\triangle p_0$ Stop conditions ε
2. Repeat,	$\mathbf{p}_{tx_k} = \arg\max \left\{ \eta_{hd} - \eta(\mathbf{p}_{tx_k}) \right\}$ s.t. $p_{m_{tx\min}} \le p_{m_{tx_{k-1}}}$ $\triangle p_k = \triangle p_{k-1} \frac{\mathbf{1}^T \mathbf{p}_{tx_k}}{\mathbf{1}^T \mathbf{p}_{tx_{k-1}}}$
3. while	$\left(\eta_{\max} - \eta\left(\mathbf{p}_{tx_{k-1}}\right) \right) \ge \varepsilon$
4.	$\mathbf{p}_{tx_{PWu}} = \mathbf{p}_{tx_{k-1}}$
5. end.	

C. Subset Selection: Minimize the Number of Operational Radars

The power allocation scheme introduced previously adapts the transmitted energy to the system characteristics, such as physical location of transmit and receive antennas with respect to the target, reflectivity, and propagation path losses. For a given scenario, some transmit/receive antenna pairs are contributing more to the localization performance than others, i.e., transmit/receive pairs that have lower path propagation losses and better views of the target are advantageous over ones with higher path losses and/or low reflectivity viewing angles. Thus, a given localization accuracy threshold, η_{thd}, may be obtained by using a smaller subset of the available transmit and receive antennas. An optimal set of transmit and receive antennas may be chosen such that $\operatorname{tr}\left(\mathbf{C}_{x,y}(\hat{x}, \hat{y})|_{S_{\min}} \right) \le \eta_{thd}$, where $S_{\min} = \{ \mathbf{x}_{i_{Tx}} \in S_{Tx}, \mathbf{x}_{j_{Rx}} \in S_{Tx} \,|i = 1, .., L_M, \; j = 1, .., L_N \}$, $1 \le L_M \le M$, $1 \le L_N \le N$, is a minimal set of transmit and receive antennas that delivers the required performance goal η_{thd}. The choice of the minimal subset of radars may be implemented through exhaustive examination of all possibilities. Such a search has a complexity of $\sim O\left(2^{M+N}\right)$. In Table 2, a heuristic algorithm is proposed for the selection of this subset, offering a reduced complexity of $\sim O\left(2MN\right)$. In the proposed algorithm, a subset is initially generated by selecting a transmitter and a receiver that are closest to the target. Following, a receiver that minimizes the trace of the CRB matrix is added to the subset. Transmitters and receivers are added sequentially until the trace of the CRB matrix value is lower than the threshold $\eta_{x,y}$. A minimum of four antennas is required for localization, i.e., the cardinality of $S_{\min}$, $|S_{\min}|$, is set to be at least four. The transmitters in the subset are assumed to use their maximum available power $p_{m_{tx}\max}$.

Once a subset is chosen, the transmitted power may be further optimized by using the power allocation scheme proposed previously with the subset. For large numbers of radars, significant complexity reduction is achievable through the use of the proposed algorithm.

IV. Numerical analysis

To evaluate the performance of the proposed algorithms, numerical analysis of some specific cases is presented in this section.

The spatially diverse multiple propagation paths between the transmit and receive radars have different error characteristics, reliant on the specific path loss, target reflectivity, effective bandwidth, and transmitted power (as seen from (8)). The power allocation methods, proposed in the previous section, dilute the error variation through adequate distribution of the transmit power. In Figure 1 a 5×5 multiple radar system ($M = N = 5$) is illustrated; it has four different radars spreads, accounting for different error characteristics. These are exploited for a numerical analysis of the proposed power allocation algorithms.

Case 1 to Case 4 in Figure 1 are equivalent to the case of different path loss on the MN transmit/receive paths, denoted by $\alpha_{m,n} \neq \alpha_{m',n'}; \forall m, m', n, n'$. The reflectivity of the target on paths originating from transmitter 1 and 5 are set to be relatively low, $h_{1,n} \leq 0.1$ and $h_{5,n} \leq 0.4; \forall n$. Table 3 summarizes the power allocation optimization, $p_{PWtotal}$, for a given localization MSE threshold, which in this case it is set to $\eta_{thd} = 3\text{m}^2$. The power values are normalized to the noise variance, σ_w^2. The total transmit power for uniform allocation, p_{uni}, is calculated with (13), where $\text{tr}\left(\mathbf{C}_{x,y}\right) = \eta_{thd}$. The total power utilized to generate a localization MSE of η_{thd} or less, $p_{PWtotal}$, is minimized using the allocation algorithm in Subsection III-B. It is observed that uniform power allocation is not necessarily the best allocation, where in Case 1 and Case 2, the total power allocated in the optimization process, $p_{PWtotal}$, is about half of that used in the uniform allocation (p_{uni}). The power efficiency of the optimized allocation compared with the uniform one is dependent on the radar spread and the path loss. For Case 4, distributed and unified power allocations have the same power budget, yet, there are more than one possible power distribution that meet performance. This supports integration of additional decision criteria in determining the final power distribution, based on the transmitters' individual power resource status.

Table 2: Subset choice - heuristic algorithm

1.	init: Choose $\mathbf{x}_{i_{Tx}} \in S_{Tx}$ s.t. $\min \left\|\mathbf{x} - \mathbf{x}_{i_{Tx}}\right\|^2$ Choose $\mathbf{x}_{j_{Rx}} \in S_{Rx}$ s.t. $\min \left\|\mathbf{x} - \mathbf{x}_{i_{Rx}}\right\|^2$ Select subsets: $S_{\min} = \{\mathbf{x}_{i_{Tx}}, \mathbf{x}_{j_{Rx}}\}$, Update: $S'_{Tx} = S_{Tx} \backslash \mathbf{x}_{i_{Tx}}$, $S'_{Rx} = S_{Rx} \backslash \mathbf{x}_{j_{Rx}}$, Set: $count = 1$

2.	while $\left(\text{tr}\left(\mathbf{C}_{x,y}\left(\hat{x}, \hat{y}\right)\|_{S_{\min}}\right) > \eta_{x,y}\right)$
	&& $(\|S_{\min}\| < 4)$
	if $count$ =even:
	if $S'_{Tx} \neq null$ than choose $\mathbf{x}_{i_{Tx}} \in S'_{Tx}$ s.t.
	$\min \left\|\text{tr}\left(\mathbf{C}_{x,y}\left(\hat{x}, \hat{y}\right)\|_{S_{\min} \cup \mathbf{x}_{i_{Tx}}}\right) - \eta_{thd}\right\|^2$
	Update: $S_{\min} = S_{\min} \cup \{\mathbf{x}_{i_{Tx}}\}$, $S'_{Tx} = S'_{Tx} \backslash \{\mathbf{x}_{i_{Tx}}\}$
	Set: $count = count + 1$
	if $count$ =odd:
	if $S'_{Rx} \neq null$ than choose $\mathbf{x}_{i_{Rx}} \in S'_{Rx}$ s.t.
	$\min \left\|\text{tr}\left(\mathbf{C}_{x,y}\left(\hat{x}, \hat{y}\right)\|_{S_{\min} \cup \mathbf{x}_{i_{Rx}}}\right) - \eta_{thd}\right\|^2$
	Update: $S_{\min} = S_{\min} \cup \{\mathbf{x}_{j_{Rx}}\}$, $S'_{Rx} = S'_{Rx} \backslash \{\mathbf{x}_{j_{Rx}}\}$
	Set: $count = count + 1$
	if $S'_{Tx} \neq null$ and $S'_{Rx} \neq null$ than go to (4)

3.	go to (2)
4.	end

Applying the subset selection algorithm to the four cases in Figure 1 results in the following selections: $S_{\min}$ (Case 1) $= \{\mathbf{x}_{2_{Tx}}, \mathbf{x}_{4_{Tx}}, \mathbf{x}_{1_{Rx}}, \mathbf{x}_{4_{Rx}}\}$, $S_{\min}$ (Case 2) $= \{\mathbf{x}_{2_{Tx}}, \mathbf{x}_{3_{Tx}}, \mathbf{x}_{4_{Tx}}, \mathbf{x}_{1_{Rx}}, \mathbf{x}_{4_{Rx}}\}$, $S_{\min}$ (Case 3) $= \{\mathbf{x}_{2_{Tx}}, \mathbf{x}_{4_{Tx}}, \mathbf{x}_{2_{Rx}}, \mathbf{x}_{4_{Rx}}\}$, and $S_{\min}$ (Case 4) $= \{\mathbf{x}_{2_{Tx}}, \mathbf{x}_{4_{Tx}}, \mathbf{x}_{1_{Rx}}, \mathbf{x}_{4_{Rx}}\}$. The estimation performance goal may be achieved by using four to five antennas out of the available ten. An exhaustive search would have given the same number of radars for all cases, though for Case 2 it would select a configuration with two transmitters and three receivers and for Case 3, a subset of one transmitter and three receivers. In the first of these cases, the selection of the first transmitter and receiver in the proposed algorithm prevents the choice given by the exhaustive search. In the optimization process, transmit/receive pairs with better locations with respect to the target and lower path losses are selected. The selection based on the proposed algorithm is nearly the same as the optimal result achieved by exhaustive search. This suggests that significant complexity reduction can be achieved with low penalty via the proposed algorithm.

V. Conclusions

To support resource-aware design for target localization in distributed multiple radar systems, two resource allocation schemes have been developed. One minimizes the total radiating power to accomplish a predetermined localization

MSE threshold, while the other minimizes the number of radars employed to achieve this threshold. The transmitters' powers are constrained to specific ranges that follow system design criteria such as minimal SNR or values set by antenna parameters. A closed-form expression for the CRB has been used to represent the localization MSE. The power allocation nonconvex optimization problem has been solved by first relaxing the original constraints and then using its solution to find a local optimum for the original nonconvex problem. It has been shown that uniform power allocation is not necessarily the best choice, and significant power savings can be obtained through proper distribution of power, based on the radars' geometric spread with respect to the target location and the target RCS. The same accuracy performance may be obtained by using only a fraction of the available radars, supporting efficient infrastructure operation. An efficient subset selection algorithm has been proposed, providing reduced complexity with little or no penalty compared with an exhaustive search.

Table 3: Minimize power: Different path loss and RCS.

	case 1	case 2	case 3	case 4
$p_{PWtotal}$	166	160	75	165
p_{uni}	238	322	75	165
$\mathbf{p}_{tx_{PWu}}$	$\begin{bmatrix} 68 \\ 1 \\ 1 \\ 21 \\ 75 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \\ 60 \\ 100 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 100 \\ 1 \\ 62 \\ 1 \end{bmatrix}$

APPENDIX A
DERIVATION OF THE J(U) MATRIX

The FIM for the unknown parameter vector $\mathbf{u} = [x, y, \mathbf{h}]$ is derived in this appendix. The conditional pdf in (7) is used as follows:

$$\mathbf{J}(\mathbf{u}) = E_{\mathbf{r}|\mathbf{u}}\left[\nabla_{\mathbf{u}} \ln f(\mathbf{r}|\mathbf{u}) \left(\nabla_{\mathbf{u}} \ln f(\mathbf{r}|\mathbf{u})\right)^H\right]. \quad (17)$$

As the conditional pdf in (7) is given as a function of the time delays, $\tau_{m,n}$, and not the target location, (x, y), we first compute the FIM with respect to the vector $\boldsymbol{\gamma} = [\boldsymbol{\tau}, \mathbf{h}]$, $\mathbf{J}(\boldsymbol{\gamma})$, and use the chain rule to evaluate $\mathbf{J}(\mathbf{u})$, as follows [6]:

$$\mathbf{J}(\mathbf{u}) = \mathbf{Q}\mathbf{J}(\gamma)\mathbf{Q}^T, \quad (18)$$

where the Jacobian matrix $\mathbf{Q}$ is

$$\mathbf{Q} = \frac{\partial \gamma}{\partial \mathbf{u}}. \quad (19)$$

To calculate $\mathbf{J}(\boldsymbol{\gamma})$, we first derive the second order derivative of $f(\mathbf{r}|\boldsymbol{\gamma})$ given in (3) with respect to the elements of vector $\boldsymbol{\tau}$ as

$$\frac{\partial^2 [\ln f(\mathbf{r}|\boldsymbol{\gamma})]}{\partial \tau_{m,n} \partial \tau_{z,q}} = \begin{cases} 4\pi^2 \beta_m^2 \alpha_{m,n} p_m |h_{m,n}|^2 & m = z \text{ and } n = q \\ 0 & o.w. \end{cases} \quad (20)$$

The derivative of $f(\mathbf{r}|\boldsymbol{\gamma})$ with respect to the elements of $\boldsymbol{\tau}$ and $\mathbf{h}$, $\frac{\partial^2 [\ln f(\mathbf{r}|\boldsymbol{\gamma})]}{\partial \tau_{m,n} \partial h_{z,q}}$, are

$$\frac{\partial^2 [\ln f(\mathbf{r}|\boldsymbol{\gamma})]}{\partial \tau_{m,n} \partial h_{z,q}} = \frac{\partial^2 [\ln f(\mathbf{r}|\boldsymbol{\gamma})]}{\partial h_{m,n} \partial \tau_{z,q}} = 0 \ ; \quad \forall\, m, n, z, q \ . \quad (21)$$

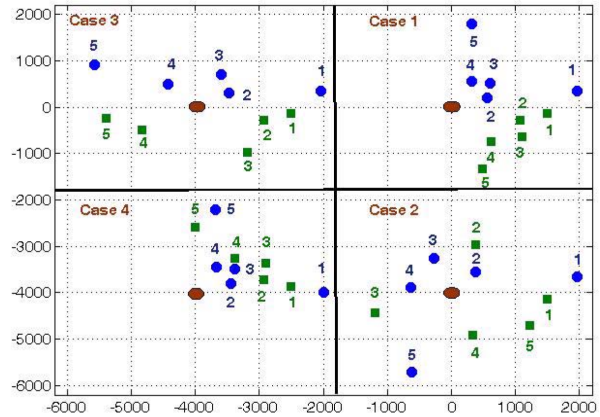

Fig. 1. MIMO radar with 5×5 elements with different geometric layouts.

Finally, the second order derivative of $f(\mathbf{r}|\boldsymbol{\gamma})$ with respect to the elements of $\mathbf{h}$, $\frac{\partial^2 [\ln f(\mathbf{r}|\boldsymbol{\gamma})]}{\partial h_{m,n} \partial h_{z,q}}$, are

$$\frac{\partial^2 [\ln f(\mathbf{r}|\boldsymbol{\gamma})]}{\partial h_{m,n} \partial h_{z,q}} = \begin{cases} \alpha_{m,n} p_m & m = z \text{ and } n = q \\ 0 & o.w. \end{cases} . \quad (22)$$

Combining the expressions in (20), (21), and (22), provides the FIM for vector $\boldsymbol{\gamma}$:

$$\mathbf{J}(\boldsymbol{\gamma}) = diag\left(4\pi^2 \beta_m^2 \alpha_{m,n} p_m |h_{m,n}|^2, \alpha_{m,n} p_m\right), \quad (23)$$

where $diag(\circ)$ is a diagonal matrix with diagonal entries listed in $(\circ)$. The Jacobian matrix $\mathbf{Q}$ is

$$\mathbf{Q} = \begin{bmatrix} \frac{x_{1_{Tx}} - x}{R_{1_{Tx}}} + \frac{x_{1_{Rx}} - x}{R_{1_{Rx}}} & \cdots & \frac{x_{M_{Tx}} - x}{R_{M_{Tx}}} + \frac{x_{N_{Rx}} - x}{R_{N_{Rx}}} & 0 \\ \frac{y_{1_{Tx}} - y}{R_{1_{Tx}}} + \frac{y_{1_{Rx}} - y}{R_{1_{Rx}}} & \cdots & \frac{y_{M_{Tx}} - y}{R_{M_{Tx}}} + \frac{y_{N_{Rx}} - y}{R_{N_{Rx}}} & 0 \\ 0 & \cdots & 0 & \mathbf{I} \end{bmatrix}. \quad (24)$$

The FIM $\mathbf{J}(\mathbf{u})$ is composed by applying the chain rule in (18), combining the FIM $\mathbf{J}(\boldsymbol{\gamma})$, given in (23), and the Jacobian matrix $\mathbf{Q}$, given in (24).

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