A HYBRID SS-TOA WIRELESS GEOLOCATION BASED ON PATH ATTENUATION UNDER IMPERFECT PATH LOSS EXPONENT

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ABSTRACT

We consider the wireless geolocation using the time of arrival (ToA) of radio signals in a cellular setting. The main concern in this paper involves the effects of the error knowledge of the path loss exponent (PLE). We derive the asymptotic error performance of the maximum likelihood (ML) estimator under the imperfect PLE. We point out that a previous method provides inaccurate performance prediction and then present a new method based on the Taylor series expansion. Numerical examples illustrate that the Taylor analysis captures the bias and the error variance of the ML estimator under the imperfect PLE better than the conventional method. Simulation results also illustrate that in the threshold region, the ML estimator outperforms the MC estimator even in the presence of the PLE error. However, in the asymptotic region the MC estimator and the ML estimator with the perfect PLE outperform the ML estimator under the imperfect PLE.

Keywords: Time-of-arrival estimation, maximum likelihood estimator, path loss exponent.

1. INTRODUCTION

One of the requirements in wireless communications is the knowledge of the mobile location. For the localization based on distance estimation where the distance is estimated from received signal strength (RSS), path loss exponent (PLE) appears as a key parameter. In many wireless networks, the value of the PLE is assumed to be known *a priori*. However, this assumption is often too ideal for realistic environment, because the PLE may change according to surrounding variation and thus may need to be estimated. Since the accuracy of the PLE is crucial for the geolocation, the imperfect PLE plays a crucial role in the performance analysis of the wireless systems. Despite the uncertain knowledge of the radio propagation caused by, e.g., the estimation of the PLE and the possible fluctuation of the PLE, the performance of the wireless localization systems has to be determined as accurately as possible.

1.1 Literature Review

Several related works are devoted to the problem of the unknown PLE in the wireless location. In [1], the PLEs are assumed to be different and random with uniform and normal distributions, and based on the RSS the consideration of the different PLEs for each link increases the localization accuracy compared to the identical PLE assumption. In [2] the PLE is calibrated from the measurements, whereas in [3] and [4] several algorithms for the PLE estimation are proposed. In [4], the algorithms for the estimation of the path loss between any sensor and any arbitrary point inside the network are designed using the path loss measurements among sensors. In [5], a handover algorithm is presented using the least squares estimate of path loss parameters for each link from mobile station to base station (BS). Furthermore, the sensitivity of the maximum likelihood (ML) estimator under model error is investigated for the application of direction-of-arrival estimation (see, e.g., [6, 7]).

In the previous works, no attention was paid to the imperfect PLE in the ToA estimation with path attenuation yet. As far as robustness is concerned, the study of the ML estimator for the ToA estimation under an imperfect PLE is deemed meaningful since a system designer will be able based on such a study to decide whether the path loss model under the imperfect PLE is useful or not.

1.2 Purpose and Problem Statement

The objectives of this work are twofold: i) to investigate the performance of the ML estimator in the ToA estimation under the imperfect PLE, and ii) to extend the hybrid RSS-ToA geolocation approach to the case that the transmitted signal is the second-derivative Gaussian monocycle pulse. In this work, we evaluate the ToA estimation performance based on the Taylor series expansion. The performance of the ML estimator under the imperfect PLE is investigated and compared to the ML estimator with the perfect PLE and to the maximum correlation (MC) estimator, which does not require the knowledge of the PLE.

2. TRANSCEIVER MODEL

The received signal at the b-th BS is given by

$$r_b(t) = a_b s(t-\tau_b) + n_b(t), \tag{1}$$

where a_b is the path gain at the b-th BS, $s(t-\tau_b)$ is the transmitted signal delayed by τ_b, which is the time delay of the propagation to the b-th BS, and $n_b(t)$ is the additive noise at the b-th BS, which is assumed as a circularly-symmetric complex-valued zero-mean white Gaussian process with the double-sided power spectral density σ_{n}^2 (Joule). Based on the path attenuation, the loss gain a_b can be written as (see, e.g., [8])

$$a_b = \sqrt{\kappa}\left(\frac{d_0}{c\tau_b}\right)^{\frac{1}{2}\gamma_b}, \tag{2}$$

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where d_0 is the close-in distance in the far field region, γ_b is the PLE at the b-th BS, and κ is the unitless constant depending on antenna characteristics and average channel attenuation given by

$$\kappa = \frac{c^2}{16\pi^2 d_0^2 f_0^2}, \tag{3}$$

with f_0 being the central frequency and c being the speed of the light. The estimated values of the ToA from the MC and ML estimators are given by (see [9])

$$\hat{\tau}_{\mathrm{MC},b} = \arg\max_{\tau_b} \rho(\tau_b), \tag{4a}$$

$$\hat{\tau}_{\mathrm{ML},b} = \arg\min_{\tau_b} a_b^2 E_\mathrm{s} - 2a_b\rho(\tau_b), \tag{4b}$$

where $\rho(\tau_b) = \int_0^{T_\mathrm{o}} \Re(r_b(t)s^*(t-\tau_b))\,\mathrm{d}t$ is the correlation between the transmitted and received signals and $E_\mathrm{s} = \int_0^{T_0} |s(t)|^2 \mathrm{d}t$ is the transmitted signal energy with T_o, $\Re(\cdot)$, and $(\cdot)^*$ being the observation period, the real part, and the conjugate, respectively.

3. PERFORMANCE ANALYSIS METHODS

We assume for simplicity that the PLEs are the same for all BSs. The PLE is assumed to be subject to an additive error, i.e.,

$$\gamma = \gamma_0 + \delta_\gamma, \tag{5}$$

where γ_0 is the true value of the PLE and δ_γ is the additive error. To study the ML estimator under the imperfect PLE, the objective function of the ML estimator can be expressed as (see [10, eq. (5.2.1)])

$$f_\mathrm{ML}(\tau_b|\gamma) = a_b^2(\tau_b|\gamma)E_\mathrm{s} - 2a_b(\tau_b|\gamma)\rho(\tau_b). \tag{6}$$

For notation brevity, we introduce $a_{b,0}$ as the loss gain for the true values of the ToA and the PLE, and $\tilde{a}_{b,0}$ as the loss gain for the true value of the ToA and the imperfect PLE. Moreover, the first and the second derivatives of $f_\mathrm{ML}(\tau_b|\gamma)$ with respect to τ_b and γ for the true value of the ToA $\tau_b = \tau_{b,0}$ can be derived as (see [10, Appendix A. 6])

$$\begin{aligned}&\frac{\partial}{\partial\tau_b} f_\mathrm{ML}(\tau_b|\gamma)\bigg|_{\tau_b=\tau_{b,0}} \\ &= -\frac{1}{\tau_{b,0}}\gamma\left(E_\mathrm{s}\tilde{a}_{b,0} - E_\mathrm{s}a_{b,0} - \rho_{\mathrm{ns},0}\right)\tilde{a}_{b,0} - 2\tilde{a}_{b,0}\dot{\rho}_{\mathrm{ns},0},\end{aligned} \tag{7}$$

$$\begin{aligned}&\frac{\partial^2}{\partial\tau_b^2} f_\mathrm{ML}(\tau_b|\gamma)\bigg|_{\tau_b=\tau_{b,0}} \\ &= \frac{1}{\tau_{b,0}^2}\gamma(1+\gamma)E_\mathrm{s}\tilde{a}_{b,0}^2 - \frac{1}{\tau_{b,0}^2}\gamma\left(1+\frac{1}{2}\gamma\right)\tilde{a}_{b,0}\rho(\tau_{b,0}) \\ &\quad + \frac{1}{\tau_{b,0}}2\gamma\tilde{a}_{b,0}\dot{\rho}_{\mathrm{ns},0} - 2\tilde{a}_{b,0}\left(-4\pi^2\bar{\beta}^2E_\mathrm{s}a_{b,0} + \ddot{\rho}_{\mathrm{ns},0}\right),\end{aligned} \tag{8}$$

and

$$\begin{aligned}\frac{\partial^2}{\partial\gamma\partial\tau_b} f_\mathrm{ML}(\tau_b|\gamma)\bigg|_{\tau_b=\tau_{b,0}} &= \frac{1}{\tau_{b,0}}\tilde{a}_{b,0}(a_{b,0}E_\mathrm{s} + \rho_{\mathrm{ns},0} - E_\mathrm{s}\tilde{a}_{b,0}) \\ &- \frac{1}{\tau_{b,0}}\gamma\ln\left(\frac{d_0}{c\tau_{b,0}}\right)\tilde{a}_{b,0}\left(E_\mathrm{s}\tilde{a}_{b,0} - \frac{1}{2}a_{b,0}E_\mathrm{s} - \frac{1}{2}\rho_{\mathrm{ns},0}\right) \\ &- \ln\left(\frac{d_0}{c\tau_{b,0}}\right)\tilde{a}_{b,0}\dot{\rho}_{\mathrm{ns},0},\end{aligned} \tag{9}$$

where $\rho_{\mathrm{ns},0} = \int_0^{T_\mathrm{o}} \Re(n(t)s^*(t-\tau_b))\mathrm{d}t\Big|_{\tau_b=\tau_{b,0}}$ is the correlation between the noise and the transmitted signal at the true value of the ToA, and $\dot{\rho}_{\mathrm{ns},0} = \frac{\partial}{\partial\tau_b}\int_0^{T_\mathrm{o}} \Re(n(t)s^*(t-\tau_b))\mathrm{d}t\Big|_{\tau_b=\tau_{b,0}}$ and $\ddot{\rho}_{\mathrm{ns},0} = \frac{\partial^2}{\partial\tau_b^2}\int_0^{T_\mathrm{o}} \Re(n(t)s^*(t-\tau_b))\mathrm{d}t\Big|_{\tau_b=\tau_{b,0}}$ are the first and the second derivatives of the correlation between the noise and the transmitted signal at the true value of the ToA, respectively. To derive the error performance, we should also use the results $\mathrm{E}_{n_b(t)}\{\dot{\rho}_{\mathrm{ns},0}\} = 0$, $\mathrm{E}_{n_b(t)}\left\{\rho_{\mathrm{ns},0}^2\right\} = \frac{1}{2}E_\mathrm{s}\sigma_\mathrm{n}^2$, $\mathrm{E}_{n_b(t)}\left\{\dot{\rho}_{\mathrm{ns},0}^2\right\} = 2\pi^2E_\mathrm{s}\bar{\beta}^2\sigma_\mathrm{n}^2$, $\mathrm{E}_{n_b(t)}\{\rho_{\mathrm{ns},0}\dot{\rho}_{\mathrm{ns},0}\} = 0$, where $\mathrm{E}_{n_b(t)}\{\cdot\}$ is the expectation with respect to the noise $n_b(t)$, and $\bar{\beta}$ is the effective bandwidth of the transmitted signal.

3.1 Friedlander Method

The theoretical expression of the error between the estimated and the true ToAs is given by (see [7, eq. 20])

$$\hat{\tau}_{b,\mathrm{ML}}(\gamma) - \tau_{b,0} = -\frac{(\gamma-\gamma_0)\mathrm{E}_{n_b(t)}\left\{\frac{\partial^2}{\partial\gamma\partial\tau_b}f_\mathrm{ML}(\tau_b|\gamma)\Big|_{\tau_b=\tau_{b,0}}\right\}}{\mathrm{E}_{n_b(t)}\left\{\frac{\partial^2}{\partial\tau_b\partial\tau_b}f_\mathrm{ML}(\tau_b|\gamma)\Big|_{\tau_b=\tau_{b,0}}\right\}}. \tag{10}$$

Substituting the expectation of (8) and (9) with respect to the noise $n_b(t)$ into (10), we obtain the error between the estimated and the true values of the ToA as follows (see [10, eq. (5.2.2)])

$$\begin{aligned}&\hat{\tau}_{b,\mathrm{ML}}(\gamma) - \tau_{b,0} = - \\ &\frac{1}{\frac{1}{\tau_{b,0}^2}\gamma(1+\gamma)E_\mathrm{s}\tilde{a}_{b,0} - \frac{1}{\tau_{b,0}^2}\gamma\left(1+\frac{1}{2}\gamma\right)E_\mathrm{s}a_{b,0} + 8\pi^2\bar{\beta}^2E_\mathrm{s}a_{b,0}} \\ &(\gamma-\gamma_0)\left[\frac{1}{\tau_{b,0}}(E_\mathrm{s}a_{b,0} + \rho_{\mathrm{ns},0} - E_\mathrm{s}\tilde{a}_{b,0}) - \ln\left(\frac{d_0}{c\tau_{b,0}}\right)\dot{\rho}_{\mathrm{ns},0}\right. \\ &\left. - \frac{1}{\tau_{b,0}}\gamma\ln\left(\frac{d_0}{c\tau_{b,0}}\right)\left(E_\mathrm{s}\tilde{a}_{b,0} - \frac{1}{2}E_\mathrm{s}a_{b,0} - \frac{1}{2}\rho_{\mathrm{ns},0}\right)\right].\end{aligned} \tag{11}$$

3.1.1 ML Estimator Bias by the Friedlander Analysis Method

Taking the expectation of (11), we obtain the bias of the ML estimation under the imperfect PLE as follows (see [10, eq. (5.2.5)])

$$\begin{aligned}&\mathrm{E}_{n_b(t)}\left\{\hat{\tau}_{b,\mathrm{ML}}(\gamma) - \tau_{b,0}\right\} = - \\ &\frac{(\gamma-\gamma_0)(a_{b,0}-\tilde{a}_{b,0}) - \gamma(\gamma-\gamma_0)\ln\left(\frac{d_0}{c\tau_{b,0}}\right)\left(\tilde{a}_{b,0} - \frac{1}{2}a_{b,0}\right)}{8\pi^2\bar{\beta}^2\tau_{b,0}^2a_{b,0} - \left(\gamma(a_{b,0}-\tilde{a}_{b,0}) + \gamma^2\left(\frac{1}{2}a_{b,0} - \tilde{a}_{b,0}\right)\right)}\tau_{b,0}.\end{aligned} \tag{12}$$

3.1.2 ML Estimator Error Variance by the Friedlander Analysis Method

Taking the expectation of the square of (11), we obtain the error variance of (11) as follows (see [10, eq. (5.2.14)])

$$\begin{aligned}
&\mathrm{E}_{n_b(t)}\left\{(\hat{\tau}_{b,\mathrm{ML}}(\gamma)-\tau_{b,0})^2\right\} \\
&= \frac{\tau_{b,0}^2(\gamma-\gamma_0)^2}{\left(\frac{\tilde{a}_{b,0}}{a_{b,0}}\gamma(1+\gamma)-\gamma\left(1+\frac{1}{2}\gamma\right)+8\pi^2\bar{\beta}^2\tau_{b,0}^2\right)^2} \\
&\left[\left(1-\frac{\tilde{a}_{b,0}}{a_{b,0}}+\gamma\ln\left(\frac{d_0}{c\tau_{b,0}}\right)\left(\frac{1}{2}-\frac{\tilde{a}_{b,0}}{a_{b,0}}\right)\right)^2\right. \\
&+\frac{1}{2}\frac{1}{a_{b,0}^2}\frac{\sigma_\mathrm{n}^2}{E_\mathrm{s}}\left(1+\frac{1}{2}\gamma\ln\left(\frac{d_0}{c\tau_{b,0}}\right)\right)^2 \\
&\left.+\frac{1}{a_{b,0}^2}\frac{\sigma_\mathrm{n}^2}{E_\mathrm{s}}2\pi^2\bar{\beta}^2\tau_{b,0}^2\ln^2\left(\frac{d_0}{c\tau_{b,0}}\right)\right].
\end{aligned} \tag{13}$$

It is worthwhile to observe that when $\gamma=\gamma_0$, i.e., for the perfect PLE, the error variance in (13) becomes zero. This result is inconsistent with the intuition, since the error variance should reduce to (see [9])

$$\mathrm{E}_{n_b(t)}\left\{\left(\hat{\tau}_{b,\mathrm{ML}}(\gamma_0)-\tau_{b,0}\right)^2\right\}=\frac{1}{\mathrm{SNR}a_{b,0}^2\left(8\pi^2\bar{\beta}^2+\frac{\gamma_b^2}{2\tau_{b,0}^2}\right)}, \tag{14}$$

where $\mathrm{SNR}=\frac{E_\mathrm{s}}{\sigma_\mathrm{n}^2}$ is the transmitted signal-to-noise ratio (SNR). Therefore, the error variance of the ML estimator calculated by the Friedlander method, i.e., (13), cannot well predict the error performance. The major reason of the inaccurate prediction by the Friedlander method is that the Taylor expansion in [7, eq. (19)] has missed the derivative term $\frac{\partial}{\partial\tau_b}f_\mathrm{ML}(\tau_b|\gamma)$ at the true values of γ_0 and $\tau_{b,0}$. Next we correct the expansion based on the Taylor series.

3.2 Theoretical Error Performance Based on the First-Order Taylor Series

By using the first-order Taylor series of the $\frac{\partial}{\partial\tau_b}f_\mathrm{ML}(\tau_b|\gamma)$ in two variables around the true values $\tau_{b,0}$ and γ_0, the theoretical expression of the error between the estimated and the true ToAs can be written as

$$\begin{aligned}
&\hat{\tau}_{b,\mathrm{ML}}(\gamma)-\tau_{b,0}=- \\
&\frac{\left.\frac{\partial}{\partial\tau_b}f_\mathrm{ML}(\tau_b|\gamma)\right|_{\tau_b=\tau_{b,0}}+(\gamma-\gamma_0)\mathrm{E}_{n_b(t)}\left\{\left.\frac{\partial^2}{\partial\gamma\partial\tau_b}f_\mathrm{ML}(\tau_b|\gamma)\right|_{\tau_b=\tau_{b,0}}\right\}}{\mathrm{E}_{n_b(t)}\left\{\left.\frac{\partial^2}{\partial\tau_b\partial\tau_b}f_\mathrm{ML}(\tau_b|\gamma)\right|_{\tau_b=\tau_{b,0}}\right\}}.
\end{aligned} \tag{15}$$

Substituting (7) and the expectation of (8) and (9) with respect to the noise $n_b(t)$ into (15), we obtain the error between the estimated and the true values of the ToA (see [10, eq. (5.3.5)])

$$\begin{aligned}
&\hat{\tau}_{b,\mathrm{ML}}(\gamma)-\tau_{b,0}= \\
&\left[\frac{1}{\tau_{b,0}}\gamma\left(E_\mathrm{s}\tilde{a}_{b,0}+E_\mathrm{s}a_{b,0}+\rho_{\mathrm{ns},0}\right)-\frac{1}{\tau_{b,0}}(\gamma-\gamma_0)E_\mathrm{s}(a_{b,0}-\tilde{a}_{b,0})\right. \\
&\left.+2\dot{\rho}_{\mathrm{ns},0}+\frac{1}{\tau_{b,0}}\gamma(\gamma-\gamma_0)\ln\left(\frac{d_0}{c\tau_{b,0}}\right)E_\mathrm{s}\left(\tilde{a}_{b,0}-\frac{1}{2}a_{b,0}\right)\right] \\
&\left(\frac{1}{\frac{1}{\tau_{b,0}^2}\gamma(1+\gamma)E_\mathrm{s}\tilde{a}_{b,0}-\frac{1}{\tau_{b,0}^2}\gamma\left(1+\frac{1}{2}\gamma\right)E_\mathrm{s}a_{b,0}+8\pi^2\bar{\beta}^2E_\mathrm{s}a_{b,0}}\right).
\end{aligned} \tag{16}$$

3.2.1 ML Estimator Bias by the First-Order Taylor Series

Taking the expectation of (16), we obtain the bias of the ML estimation under the imperfect PLE as follows (see [10, eq. (5.3.6)])

$$\begin{aligned}
&\mathrm{E}_{n_b(t)}\left\{\hat{\tau}_{b,\mathrm{ML}}(\gamma)-\tau_{b,0}\right\}=- \\
&\frac{(2\gamma-\gamma_0)\left(a_{b,0}-\tilde{a}_{b,0}\right)+\gamma(\gamma-\gamma_0)\ln\left(\frac{d_0}{c\tau_{b,0}}\right)\left(\frac{1}{2}a_{b,0}-\tilde{a}_{b,0}\right)}{8\pi^2\bar{\beta}^2\tau_{b,0}^2a_{b,0}-\left(\gamma(a_{b,0}-\tilde{a}_{b,0})+\gamma^2\left(\frac{1}{2}a_{b,0}-\tilde{a}_{b,0}\right)\right)}\tau_{b,0}.
\end{aligned} \tag{17}$$

3.2.2 ML Estimator Error Variance by the First-Order Taylor Series

Using the error expression derived from the Taylor expansion in (16), we obtain the error variance of the ML estimation under the imperfect PLE as follows (see [10, eq. (5.3.16)])

$$\begin{aligned}
&\mathrm{E}_{n_b(t)}\left\{(\hat{\tau}_{b,\mathrm{ML}}(\gamma)-\tau_{b,0})^2\right\}=\left(\left((2\gamma-\gamma_0)\left(1-\frac{\tilde{a}_{b,0}}{a_{b,0}}\right)+\gamma(\gamma-\gamma_0)\right.\right. \\
&\left.\ln\left(\frac{d_0}{c\tau_{b,0}}\right)\left(\frac{1}{2}-\frac{\tilde{a}_{b,0}}{a_{b,0}}\right)\right)^2+\frac{1}{a_{b,0}^2}\frac{\sigma_\mathrm{n}^2}{E_\mathrm{s}}\left.\left(\frac{1}{2}\gamma^2+8\pi^2\bar{\beta}^2\tau_{b,0}^2\right)\right) \\
&\left(\frac{\tilde{a}_{b,0}}{a_{b,0}}\gamma(1+\gamma)-\gamma\left(1+\frac{1}{2}\gamma\right)+8\pi^2\bar{\beta}^2\tau_{b,0}^2\right)^{-2}\tau_{b,0}^2.
\end{aligned} \tag{18}$$

For the perfect PLE, the error variance in (18) reduces to (14), and therefore, (18) gives a consistent RMSE. The reason why the method based on the Taylor series expansion is able to better characterize the error performance than the Friedlander's method is that in the Taylor series expansion method, the term $\frac{\partial}{\partial\tau_b}f_\mathrm{ML}(\tau_b|\gamma)$ is taken into account. This additional term fulfils the expansion of the first-order Taylor series in [7, eq. (19)], thus leading to higher prediction accuracy.

4. NUMERICAL EXAMPLES

In classical impulse radio ultrawideband systems, one of the most considered waveforms is the second-derivative Gaussian pulse, which can be expressed as (see, e.g., [11]) $p(t)=\left(1-4\pi\left(\frac{t}{\tau_\mathrm{p}}\right)^2\right)\mathrm{e}^{-2\pi\left(\frac{t}{\tau_\mathrm{p}}\right)^2}$ where $t>0$, τ_p is the pulse-shaping factor chosen to adjust the pulse width T_p. Let us consider the transmitted signal as $s(t)=p\left(t-\frac{1}{2}T_\mathrm{p}\right);t>0$, whose effective bandwidth and effective absolute central frequency are given by (see [10, eq. (3.2.2) and eq. (3.3.5)])

$$\bar{\beta} = \frac{1}{2\pi}\sqrt{\frac{\int_{-\infty}^{\infty}\omega^2|S(\omega)|^2\mathrm{d}\omega}{\int_{-\infty}^{\infty}|S(\omega)|^2\mathrm{d}\omega}} = \frac{1}{\tau_\mathrm{p}}\sqrt{\frac{5}{2\pi}}, \tag{19a}$$

$$\bar{f}_\mathrm{abs} = \frac{1}{2\pi}\frac{\int_{-\infty}^{\infty}|\omega||S(\omega)|^2\mathrm{d}\omega}{\int_{-\infty}^{\infty}|S(\omega)|^2\mathrm{d}\omega} = \frac{1}{\tau_\mathrm{p}}\frac{16}{3\pi}, \tag{19b}$$

where ω is the angular frequency, and $S(\omega)$ is the Fourier transform of $s(t)$. Note that $\bar{f}_\mathrm{abs}$ is considered as an approximation of the central frequency in (3). Substituting the above parameters, i.e., (19a) and (19b), into the expressions of the biases and the variances given by the Friedlander and Taylor expansion methods, we obtain the theoretical error performance of the ML estimator under the imperfect PLE for the second derivative of the Gaussian pulse as the transmitted signal.

Fig. 1 shows the bias and the RMSE of the position estimate as a function of the received SNR defined by $\mathrm{SNR}_\mathrm{Rx} = \frac{a_b^2 E_\mathrm{s}}{\sigma_\mathrm{n}^2}$. In the upper sub-figure, the bias expressions in (12) and (17) depend on neither the signal energy E_s nor the noise variance σ_n^2. As the SNR is varied, the bias is invariant to the SNR. For SNR = 20 or 25 dB, the simulation has a zigzag tendency, because the derived bias is accurate for only asymptotic region, i.e., for large SNR. In the lower sub-figure, the ML and MC estimates approach their asymptotic error variances when the SNR is large. In the threshold region from 0 to 10 dB, i.e., in the transition region where the error falls quickly, and for a small PLE error, the ML estimator presents lower RMSE than the MC estimator even in the presence of the imperfect PLE. However, in the asymptotic region, especially for the SNR larger than 35 dB, the MC estimator and the ML estimator with the perfect PLE outperform the ML estimator under the imperfect PLE. This is because although the RMSE decreases with the increase in the received SNR, the bias of the ML estimator under the imperfect PLE shown in the upper sub-figure does not decrease with the increase in the received SNR. Therefore, the bias dominates the RMSE at high SNR. Although the RMSE on the order of 0.01 mm for the received SNR to be from 35 to 50 dB seems to be impractical. The figure is used just to indicate the merits of the approach proposed in this paper.

Moreover, the bias and the RMSE of the position estimate as a function oft the PLE error $\delta_\gamma = \gamma - \gamma_0$ are shown in Fig. 2. It can be seen from both sub-figures that, the Taylor expansion analysis presents higher accuracy in the performance prediction than the Friedlander analysis method. The plot of the theoretical ML estimator under the imperfect PLE actually changes with the PLE error. A zoomed version of this RMSE figure is available in [10, Fig. 5.17]. This unzoomed version is chosen, because the curve of "ML Analysis: Friedlander" still appears in the figure. The disagreement between the bias analysis and the simulation results mainly results from the two-dimentional Taylor series truncation.

5. CONCLUSION

We have derived the asymptotic error performance of the ML estimator under the imperfect PLE. It is pointed out that a previous method provides inaccurate performance prediction and a new method based on the Taylor series expansion, which can better capture the error performance of the ML estimator than the conventional method, is presented. Moreover, one can observe from the simulation results that in the presence of the PLE error, the ML estimator outperforms the MC estimator for the small PLE error in the threshold region. However, in the asymptotic region the MC estimator and the ML estimator with the perfect PLE outperform the ML estimator under the imperfect PLE.

REFERENCES

[1] J. Shirahama and T. Ohtsuki, "RSS-based localization in environments with different path loss exponent for each link," in *Proc. IEEE Veh. Technol. Conf.*, Marina Bay, Singapore, May. 2008, pp. 1509-1512.

[2] G. Mao, B. D. Anderson, and B. Fidan, "Path loss exponent estimation for wireless sensor network localization," *Computer Networks*, vol. 51, no. 10, pp. 2467-2483, Jul. 2007.

[3] S. Srinivasa and M. Haenggi, "Path loss exponent estimation in large wireless networks," in *Proc. IEEE Inform. Theo. Appl. Works. 2009 (ITA 2009)*, San Diego, CA, Feb. 2009, pp. 124-129.

[4] X. Zhao, L. Razoumov, and L. J. Greenstein, "Path loss estimation algorithms and results for RF sensor networks," *Proc. IEEE Veh. Technol. Conf.*, vol. 7, Los Angeles, LA, Sep. 2004, pp. 4593-4596.

[5] N. Benvenuto and F. Santucci, "A least squares path-loss estimation approach to handover algorithms," *IEEE Trans. Veh. Technol.*, vol. 48, no. 2, pp. 437-447, Mar. 1999.

[6] C. D. Richmond, "On the threshold region mean-squared error performance of maximum-likelihood direction-of arrival estimation in the presence of signal model mismatch," in *Proc. IEEE Works. Sens. Array Multi. Proces. 2006*, Waltham, MA, Jul. 2006, pp. 268-272.

[7] B. Friedlander, "Sensitivity analysis of the maximum likelihood direction-finding algorithm," *IEEE Trans. Aerosp. Electron. Syst.,* vol. 26, no. 6, pp. 953-968, Nov. 1990.

[8] V. Erceg, L. J. Greenstein, S. Y. Tjandra, S. R. Parkoff, A. Gupta, B. Kulic, A. A. Julius, and R. Bianchi, "An empirically based path loss model for wireless channels in suburban environments," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 7, pp. 1516-1526, Jul. 1999.

[9] B. T. Sieskul, F. Zheng, and T. Kaiser, "Time-of-arrival estimation in path attenuation," in *Proc. IEEE Int. Works. Signal Proces. Adv. Wirel. Commun. 2009 (SPAWC 2009)*, Perugia, Italy, Jun. 2009, pp. 573-577.

[10] M. I. Valera Martínez, "A hybrid SS-ToA wireless geolocation based on path attenuation: Robustness investigation under imperfect path loss exponent," Master's thesis, Leibniz Universität Hannover & Universidad Politécnica de Cartagena, Hannover, Germany & Cartagena, Spain, Aug. 2009. [Online]. Available: http://www.ikt.uni-hannover.de/3.html

[11] M. Z. Win and R. A. Scholtz, "Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple access communications," *IEEE Trans. Commun.*, vol. 48, no. 4, pp. 679-691, Apr. 2000.

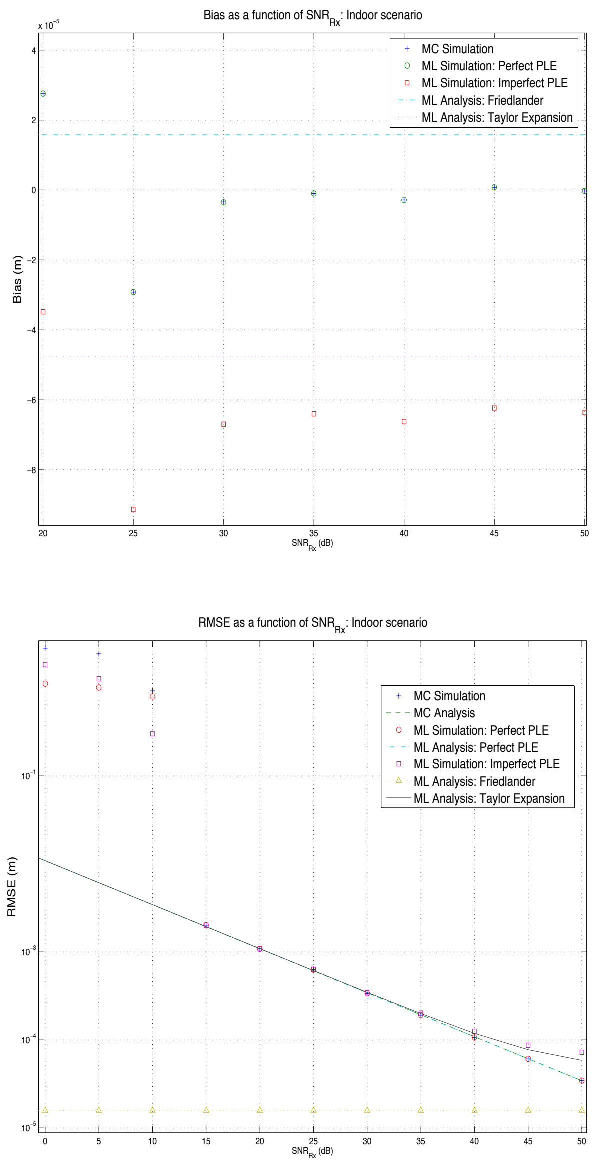

Figure 1: Bias and RMSE of the position estimate as a function of the received SNR $\mathrm{SNR_{Rx}}$ for the imperfect PLE, $\gamma_0 = 2$, $\delta_\gamma = 0.5$, $d_b = 3$ m, $\bar{\beta} = 3.1007 \times 10^9$ Hz, sampling time $= 0.01$ ps, and $N_\mathrm{R} = 1{,}000$ independent runs. A zoomed version of the RMSE in the lower figure is available in [10, Fig. 5.13].

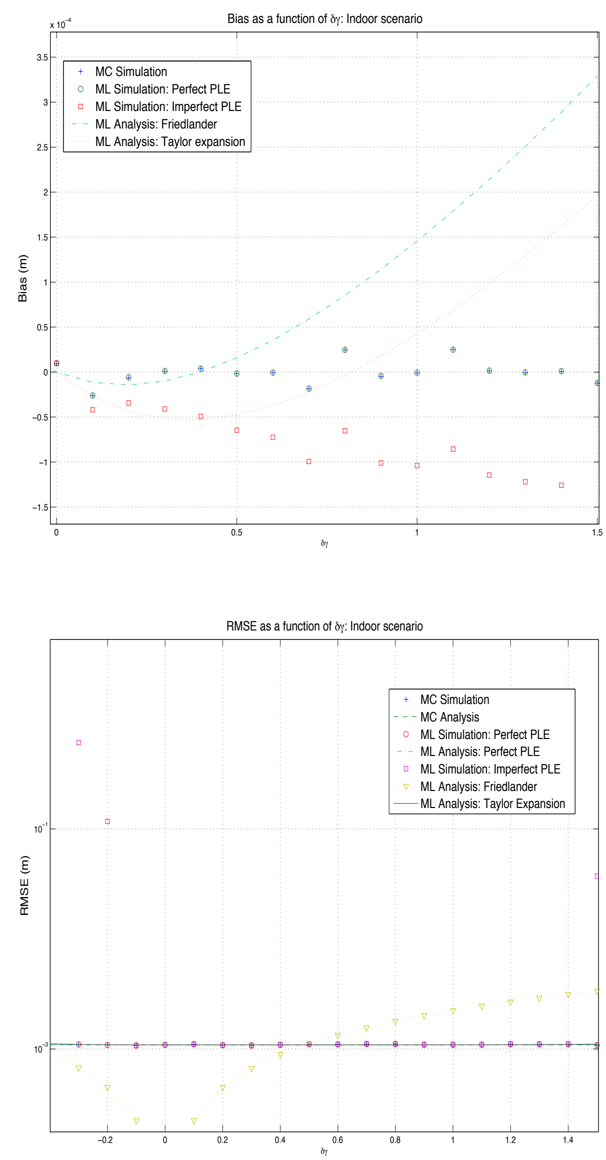

Figure 2: Bias and RMSE of the position estimate as a function of the PLE error δ_γ for the imperfect PLE, $\gamma_0 = 2$, $\mathrm{SNR_{Rx}} = 20$ dB, $d_b = 3$ m, $\bar{\beta} = 3.1007 \times 10^9$ Hz, sampling time $= 0.01$ ps, and $N_\mathrm{R} = 5{,}000$ independent runs. A zoomed version of the RMSE in the lower figure is available in [10, Fig. 5.17].