A SIGNAL–ADAPTIVE DISCRETE EVOLUTIONARY TRANSFORM

Sultan Aldirmaz[1], *Lutfiye Durak-Ata*[2], *Aydin Akan*[3], *and Luis F. Chaparro*[4]

[1,2]Dept. of Electronics and Commun. Eng., Yildiz Technical Univ., Yildiz, Besiktas, 34349, Istanbul Turkey.
Phone: +90 212 383 24 98, +90 212 383 24 86 [1]aldirmaz@yildiz.edu.tr, [2]lutfiye@ieee.org
[3]Dept. of Electrical and Electronics Eng., Istanbul Univ., Avcilar, 34320, Istanbul, Turkey, akan@istanbul.edu.tr
[4]Dept. of Electrical and Computer Eng., University of Pittsburgh, Pittsburgh, Pennsylvania 15261, USA, chaparro@ee.pitt.edu

ABSTRACT

The discrete evolutionary transform is applied to signals in a blind-way, i.e., without using any parameters to characterize the signal. For this reason, it is not optimal and needs an improvement by using some information about the signal. In this paper, we propose an improvement for the discrete evolutionary transform and redefine the generalized time-bandwidth product optimal shorttime Fourier transform as a special case of the discrete evolutionary transform. The optimized kernel function of the shorttime Fourier transform is determined according to the instantaneous frequency of linear FM signals-type signals. Even in case of quadratic FM signals, the resulting localization of the time-frequency representations improves remarkably. The performance of this adaptive discrete evolutionary transform is presented on signals with time-varying instantaneous frequencies.

1. INTRODUCTION

In nature, most of the signals vary both in time and frequency simultaneously and their characterization in both domains is an important issue for various applications such as processing speech, acoustic and biomedical signals, modeling and estimating the impulse responses of wireless communication channels, analyzing sonar, radar and seismic signals. To acquire more information about the signal, we need to reveal its joint time-frequency behavior besides individual time-domain or frequency-domain structures separately. Thus, signal processing applications become much more powerful and accurate.

Time-frequency analysis tools such as short–time Fourier transform (STFT), spectrogram, continuous wavelet transform, Wigner Distribution (WD) and its derivatives have been used for this purpose for a long time. WD generates sharp and well-localized time-frequency representations of single–component signals, however it becomes inefficient for multi-component signals as it introduces cross-terms on the time-frequency plane [1]. Being a linear distribution, STFT is a reliable choice for multi-component signals. Moreover, to avoid the resolution problem of wavelet transform at high frequencies, STFT becomes an attractive alternative providing uniform resolution for all frequency bands on the time-frequency plane.

STFT of a signal $x(n)$ is defined as

$$STFT_x(t,f) = \int x(t')h(t'-t)e^{-j2\pi f t'}dt' \qquad (1)$$

where $h(t)$ is a low-pass unit-energy window function. Up until now, STFT has been improved by various techniques, i.e., in [2], an instantaneous frequency (IF) estimation technique is added to STFT to choose the window function appropriately. In [3, 4], fractional Fourier transform (FrFT) is used to obtain high–resolution STFT images. Moreover, Durak *et. al.* introduce the generalized time-bandwidth product (GTBP)-optimal STFT by using FrFT and time-bandwidth product (TBP) of signals in fractional Fourier domains [5, 6]. This way, it has been possible to adapt the STFT of signals to both IF and fractional Fourier domain TBP of the signal simultaneously. In the simulations they presented the improvement imposed by the GTBP–optimal STFT on linear and quadratic FM signals. The GTBP definition provides a rotation-invariant measure of signal support on the time-frequency plane. Then, the optimal STFT kernel providing the most compact representation is obtained by considering the GTBP of signals The proposed time-frequency analysis is shown to be equivalent to an ordinary STFT analysis conducted in a scaled fractional Fourier domain [5, 6].

On the other hand, the discrete evolutionary transform (DET) is introduced for non-stationary signal–analysis in [7]. Two different DET models have been defined as sinusoidal basis-DET and chirp basis-DET, previously. The appropriate DET can be selected depending on the application. If the signal is narrow-band, sinusoidal bases are more suitable to represent in time-frequency plane. On the other hand, chirp basis-DET is more capable when the signal contains wide-band components. Time and/or frequency dependence is inherently included in selecting the window type. For example, Malvar-based windows are both time and frequency dependent orthogonal bases, on the contrary Gabor-based windows are time-dependent. In the DET analysis, Malvar-based and Gabor-based windows are used in [8]. The DET is used in various applications such as a jammer excision algorithm [9], estimation of multipath fading and frequency selective channels [10].

The goal of this paper is to prove that the GTBP–optimal STFT is a special case of DET. The remainder of this paper is organized as follows. In Section 2, preliminary information is given on discrete evolutionary transform, time-frequency localization, GTBP–optimal STFT and instantaneous frequency (IF) estimation procedure. Section 3 shows the link between the optimal STFT and the DET. Simulation results are given in Section 4. Finally, conclusions are drawn in Section 5.

2. PRELIMINARIES

2.1 Discrete Evolutionary Transform

The DET is a time-frequency method that provides a representation of non-stationary signals as well as their spectra.

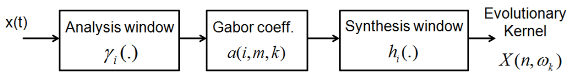


Figure 1: Block diagram of Gabor–based DET calculation.

The DET can be defined by using sinusoidal or chirp basis. When sinusoidal basis are used, the evolutionary kernel of the DET becomes

$$X(n,f_k)=\sum_{l=0}^{N-1}x(l)W_k(n,l)e^{-j2\pi f_k l},\ 0\le k\le K-1 \qquad (2)$$

where $W_k(n,l)$ represents the time–frequency dependent window function. The inverse DET is stated as

$$x(n)=\sum_{k=0}^{K-1}X(n,f_k)e^{j2\pi f_k n},\ 0\le n\le N-1. \qquad (3)$$

If chirp bases are used, the kernel is changed to

$$X_p(n,f_k)=\sum_m x_p(m)W_k(m,n)e^{-j(2\pi f_k m+\phi_p(m))} \qquad (4)$$

where $x_p(n)$ and $\phi_p(n)$ represent each of the individual signal components and phases, respectively. In case of chirp basis, the inverse transform is defined by

$$x(n)=\sum_p^{P-1}\sum_{k=0}^{K-1}X_p(n,f_k)e^{j(2\pi f_k n+j\phi_p(n))}\ . \qquad (5)$$

When Gabor–based DET is calculated, both the analysis function $\gamma(.)$ and its dual pair synthesis function $h(.)$ are employed. Figure 1 shows the block diagram that illustrates the calculation of the evolutionary kernel of sinusoidal–DET for $x(t)$ when Gabor-bases are used. Gabor–coefficients $a(m,k)$ are obtained as

$$a(m,k)=\sum_{n=0}^{N-1}x(n)\gamma^*(n-mL)e^{-j2\pi\ nf_k}\ . \qquad (6)$$

Then, the evolutionary kernel becomes

$$X^G(n,f_k)=\sum_{m=0}^{M-1}a(m,k)h(n-mL) \qquad (7)$$

the time-varying DET window is defined as

$$w(l,n)=\sum_{m=0}^{M-1}\gamma^*(n-mL)h(n-mL)\ . \qquad (8)$$

The main difference between the ordinary STFT and the DET is that in DET the analysis window varies by time. The time-varying window has been expressed as a function of a set of orthogonal functions in [7]. The evolutionary spectrum is defined as the magnitude square of the DET kernel as

$$S_E(n,f_k)=|X(n,f_k)|^2. \qquad (9)$$

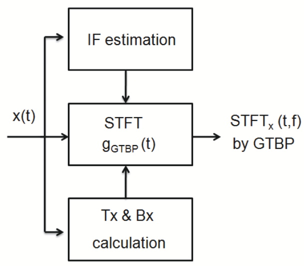


Figure 2: Block diagram of GTBP–opt. STFT.

In Malvar-based DET, the analysis–window length is chosen with respect to the analyzed signal by using a cost-function optimization. However, Gabor–based DET does not depend on the signal. Thus, it can be said that the DET provides a signal representation and its corresponding spectrum without using any of the characteristics of the signal, and thus it can be improved. Moreover, if the signal consists of multi–components, DET requires the separation of each component by using a mask in an offline–procedure. For this task, instantaneous frequency of each of the signal components must be estimated. Therefore, each mask represents a region of a single component. Then, each of the components can be analyzed individually.

2.2 Time–Frequency Localization of Signals

Localization of a signal on time–frequency domain gives information about the signal support. According to the well–known uncertainty principle, there is a lower bound on the spread of a signal's energy in both time and frequency domains together. This concentration may be measured by the time-bandwidth product (TBP), which is defined as the product of time-width T_x and bandwidth B_x, and it is bounded by [11, p.50]

$$T_xB_x\ge\frac{1}{4\pi} \qquad (10)$$

where

$$T_x=\frac{\left[\int(t-\eta_t)|x(t)|^2\mathrm{d}t\right]^{\frac{1}{2}}}{||x||} \qquad (11)$$

$$B_x=\frac{\left[\int(f-\eta_f)|X(f)|^2\mathrm{d}f\right]^{\frac{1}{2}}}{||x||}\ . \qquad (12)$$

and η_t, η_f and $||.||$ are the time and frequency mean values and the norm operator, respectively. $X(f)$ is the Fourier transform of $x(t)$. The Gaussian function is the best localized function in both time and frequency domain having, the lowest TBP which equals to $1/(4\pi)$.

2.3 GTBP–Optimal STFT Definition

GTBP–optimal STFT is introduced as a signal dependent representation by Durak *et. al.* [5, 6]

$$\mathrm{D}_x(t,f)=e^{-j\pi\psi}\int x(\tau)\ g^*_{GTBP}(\tau-t)\ e^{-j2\pi f\tau}\ d\tau \qquad (13)$$

where $\psi = (t^2 - f^2)\sin\phi_0 \cos\phi_0 + 2tf\sin^2\phi_0$ and the optimal kernel is

$$g_{GTBP}(\tau) = K\, e^{-j\pi\tau^2 \frac{\cot\phi_0(\gamma^2-1)}{\gamma^2+\cot^2\phi_0}}\; e^{-\pi\tau^2 \frac{\gamma\csc^2\phi_0}{\gamma^2+\cot^2\phi_0}} \quad (14)$$

with $K = \sqrt{\frac{1+j\cot\phi_0}{\gamma+j\cot\phi_0}}$ and $\gamma = B_{x_{a_0}}/T_{x_{a_0}}$. $B_{x_{a_0}}$ and $T_{x_{a_0}}$ denote the bandwith and time–width of the a_0th–order fractional Fourier domain signal, respectively. Additionally, ϕ_0 is defined as $a_0\frac{\pi}{2}$ and represents the orientation of the signal on the time-frequency plane with the corresponding fractional Fourier order a_0. Since the phase ψ can be ignored in $D_x(t,f)$, it is easy to see that the desired representation in Eq.(13) has the form of an ordinary STFT with kernel $g_{GTBP}(\tau)$. Except the fractional order a_0 determination, the computational complexity of Eq.(13) is the same as the computational complexity of the ordinary STFT.

The discretized-version of the optimal STFT is

$$\mathrm{D}_x(m,k) = e^{-j\pi\psi} \sum_{n=0}^{N-1} x(n)\; g^*_{GTBP}(n-m)\; e^{-j\frac{2\pi}{N}kn} \quad (15)$$

where N is the number of frequency bins. Furthermore when the second term of the discrete version of Eq. (14) is included in the exponential term in Eq. (15), the $D_x(m,k)$ can be recognized as a generalized discrete Fourier transform (GDFT), which has recently introduced by Akansu [15]. Figure 2 shows the block diagram of the optimal STFT computation. The algorithm first obtains the appropriate FrFT order by using an IF estimation technique. Then, the signal $x(t)$ is transformed to the fractional Fourier domain where the transformed signal has a minimum TBP. Hence, TBP and GTBP of $x_a(t)$ are equal to each other. The definition of the FrFT is given in Appendix A.

In [5], the optimal STFT is derived as follows. At first, TBP is chosen as a suboptimal measure of support and TBP–optimal STFT kernel is obtained by using the following optimization scheme.

$$\min_{T_g,B_g;\; T_g\cdot B_g \geq \frac{1}{4\pi}} \left(T_x^2 + T_g^2\right)^{1/2} \cdot \left(B_x^2 + B_g^2\right)^{1/2} . \quad (16)$$

It is shown that the TBP–optimal solution $g(t)$ must be the Gaussian kernel

$$g_{TBP}(t) = e^{-\pi t^2 B_x/T_x} . \quad (17)$$

By using GTBP–optimal STFT technique an appropriate analysis window is determined for the signal $x(t)$ that varies in time, so that time-frequency distribution with the maximum concentration is obtained. The desired time-frequency representation of $x(t)$ can equivalently be obtained as the counter-clockwise rotation of the optimal STFT for $x_{a_0}(t)$ by an angle of ϕ_0 where

$$\mathrm{STFT}_{x_{a_0}}(t,f) = \int x(\tau) \left[\int h(t'-t) e^{j2\pi f t'} B_{-a_0}(t',\tau) dt'\right]^* d\tau \quad (18)$$

with optimal Gaussian kernel $h(t) = e^{-\pi\gamma\, t^2}$. The desired representation of $x(t)$ is expressed as

$$\begin{aligned} \mathrm{D}_x(t,f) &= R_{\phi_0}\{\mathrm{STFT}_{x_{a_0}}(t,f)\} \quad (19)\\ &= \int x(\tau)\, R_{\phi_0}\left\{\int h(t'-t)\; e^{j2\pi f t'}\; B_{-a_0}(t',\tau) dt'\right\}^* d\tau . \end{aligned}$$

where the kernel function $B_a(t,t')$ and a two-dimensional $(2-D)$ the rotation operator function $D(u,v)$ are defined as

$$B_a(t,t') = \frac{e^{-j(\pi \mathrm{sgn}(sin\phi)/4+\phi/2)}}{|sin\phi|^{1/2}} e^{j\pi(t^2 cot\phi - 2tt' csc\phi + t'^2 cot\phi)} \quad (20)$$

$$R_\phi\{D(u,v)\} = D(ucos\phi + vsin\phi, -usin\phi + vcos\phi). \quad (21)$$

In Eq. (19), the expression in the brackets can be recognized as the $-a_0^{th}$-order FrFT of $h(t'-t)\; e^{j2\pi f t'}$ which is simply the time and frequency shifted form of the kernel $h(t)$. In [5] its shown that Eq. (19) is equivalent to Eq.(13).

2.4 Estimation of the Instantaneous Frequency

IF gives information about the frequency variation of a signal by time. Instantaneous frequency (IF) of a signal $x(t) = A(t)e^{j\phi(t)}$ can be defined as $f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t)$. There are a lot of IF estimation techniques in the literature [2,12–14]. In [2] STFT has been employed to estimate IF of the signal. Two different IF estimation algorithms are proposed by using an optimization scheme, which makes use of the maximum fractional time-bandwidth ratio and a minimum essential bandwidth which is expressed as the minimum sum of the bandwidths of the separate signal components [12]. Genetic algorithms are employed to determine the IF of the signal components. Image processing techniques are used in [13] for multi-components LFM signals. Except these, one way of determining the IF of an LFM signal component is to search for the peaks of the FrFT magnitudes computed at various fractional orders. This method makes use of the relationship between the Radon–Wigner transform (RWT) of a signal and its corresponding FrFT [14]. Figure 3 shows the FrFT order estimation by searching the maximum peak values among all FrFTs of a chirp signal with a chirp rate of 0.5. The analyzed signal includes AWGN noise of $5dB$ SNR. Thus, this IF procedure is robust against the noise.

3. REPRESENTATION OF THE ADAPTIVE–DET

The DET uses time and time-frequency dependent windows to analyze the signal $x(t)$. Eq. (4), which belongs to the classical DET, looks like Eq. (15). In Eq. (15), the optimal window is determined according to time–width and bandwidth of the signal on the appropriate fractional Fourier domain. The optimum window, which contains the appropriate chirp component of the analyzed signal, is given in Eq. (14) in continuous time domain. This component is considered as a time–varying window. Under these circumstances, it can be said that the optimal STFT is a special case of the DET. Moreover, this technique improves the DET by taking into account of signal–specific information such as T_x, B_x and IF. Thanks to the RWT–FRFT relations, IF values are estimated robustly for each of the components when a multi–component signal is analyzed.

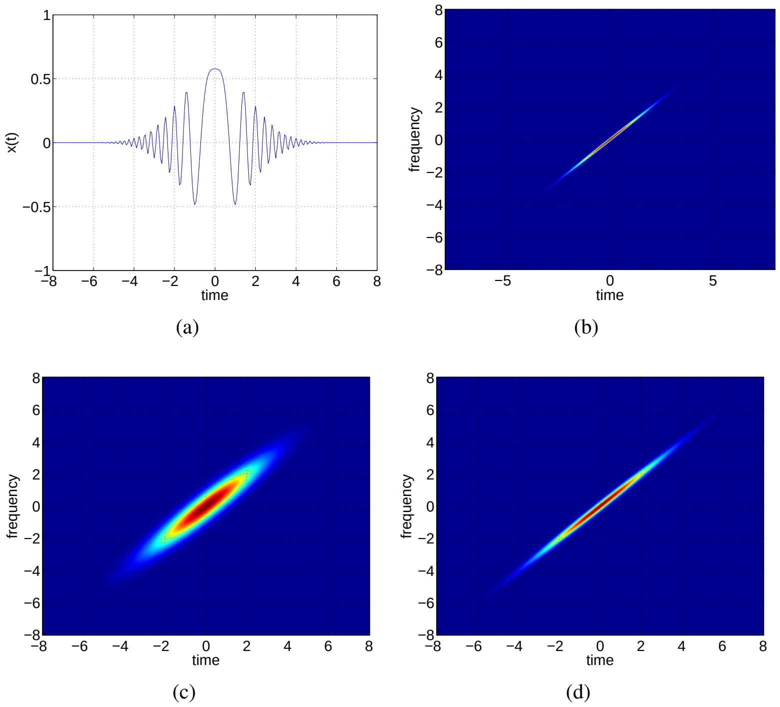


Figure 4: LFM signal (a) in time, (b) its WD (c)its TBP–optimal STFT and (d) the adaptive DET

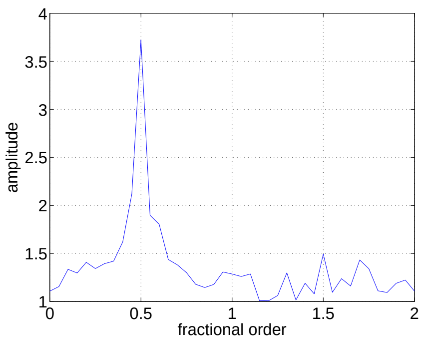


Figure 3: FrFT order estimation with respect to the IF value of the signal.

4. SIMULATIONS

Time-frequency domain localization by using the adaptive-DET of a synthetic LFM signal and a real bat echolocation signal with multiple components are computed in simulations. Figure 4 shows a synthetic LFM signal. Its IF value is estimated as 0.5 by using IF estimation algorithm. Its time variation and WD are given in Figure 4 (a) and (b). When the signal is mono-component, WD provides a sharp time-frequency representation. Figure 4 (c) and (d) represent the STFT and adaptive–DET images. The performance of adaptive–DET is very high and almost equivalent to WD's.

The bat echolocation signal includes four non-linear chirp components. First, we obtained discretized STFT of the signal by using a Gaussian window $h(n) = e^{-\pi n^2}$. Then, time–width and bandwidth of the signal are calculated in the fractional Fourier domain which gives the minimum TBP. By using these values to construct the optimal window, TBP–optimal STFT is obtained. It is shown in Figure 5(c). Finally, $g_{opt}(n)$, which is given in Eq. (14), is used to calculate the adaptive–DET. It can be easily seen that, adaptive–DET images have higher concentrations than ordinary–STFT's.

5. CONCLUSIONS

In this paper, we link up between the classical DET and the GTBP–optimal STFT. The optimal STFT enriches the DET by taking into account of the three parameters related to signals. These are time–width and bandwidth terms in a fractional Fourier domain and the IF parameter. The signal–adaptive DET results are presented by an LFM signal and a bat echolocation signal. As future work, we will present high resolution adaptive DET images for quadratic chirp signals.

6. APPENDIX A

The a^{th}-order FrFT of $x(t)$ is defined as

$$x_a(t) = F^a\{x(t)\} = \int B_a(t,t')x(t')dt \qquad (22)$$

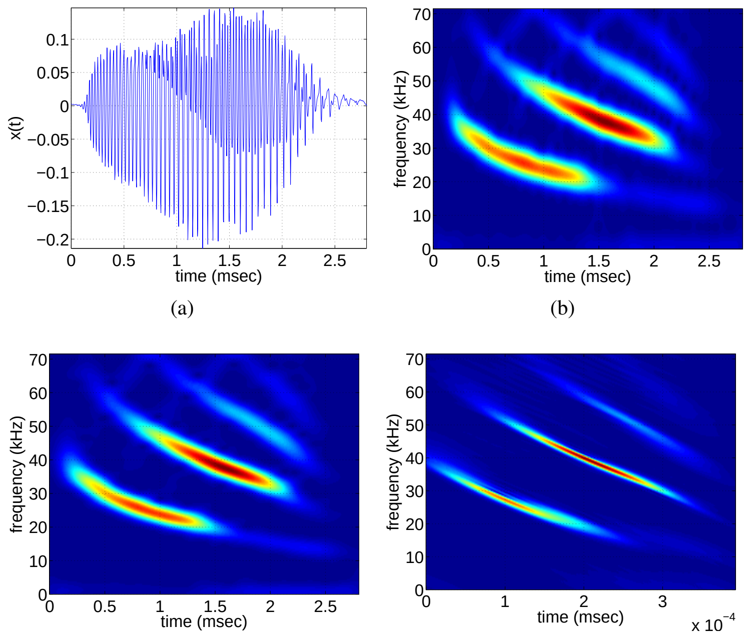
where $0 < |a| < 2$ and the transformation kernel $B_a(t,t')$ is

$$B_a(t,t') = A_\varphi e^{-j\pi(t2\cot(\varphi)-2tt'\csc(\varphi)+t'2\cot(\varphi))} \qquad (23)$$

with

$$A_\varphi = e^{-j\pi sgn(\sin(\varphi))/4+j(\varphi)/2}/|\sin(\varphi)|1/2 \qquad (24)$$

where the transform angle (φ) and the FrFT order is related by $\varphi = \alpha\pi/2$. Discrete FrFT definitions are also developed by many researchers [16, 17].



(a) (b)
(c) (d)

Figure 5: Real Bat echolocation signal (a) in time , (b) its STFT (c) TBP–opt.STFT and (d) adaptive DET

Acknowledgements

The authors are supported by the Scientific and Technological Research Council of Turkey, TUBITAK under the grant of Project No. 105E078. The authors thank Curtis Condon, Ken White, and Al Feng of the Beckman Institute of the University of Illinois for the bat data and for permission to use it in this paper.

REFERENCES

[1] F. Hlawatsch and G. F. Boudreaux-Bartels, "Linear and quadratic time-frequency signal representations," IEEE Signal Processing Magazine, vol. 9, no. 2, pp. 21–67, April 1992.

[2] H. K. Kwok and D. L. Jones, "Improved instantaneous frequency estimation using an adaptive short-time Fourier transform," IEEE Trans. Signal Process., vol. 48, pp. 2964-2972, Oct. 2000.

[3] A. T. Catherall, and D. P. Williams, "High resolution spectrograms using a component optimized short-term fractional Fourier transform," Signal Processing, no. 90, 1591-1596, 2010.

[4] C. Capus and K. Brown, "Short-time fractional Fourier methods for the time-frequency representation of chirp signals," J. Acoust. Soc. Am. 113 (6), June 2003.

[5] L. Durak and O. Arikan, "Short-time Fourier transform: Two fundamental properties and an optimal implementation," IEEE Trans. on Signal Process., c.51, no.5, s.1231–1242, May, 2003.

[6] L. Durak, *Time-Frequency Analysis: Novel Techniques for Deterministic Signals*, VDM Verlag, 2009.

[7] A. S. Kayhan, A. El-Jaroudi, and L.F. Chaparro, "Evolutionary periodogram for non-stationary signals," IEEE Trans. Signal Process. 42, pp. 1527–1536, 1994.

[8] R. Suleesathira, L.F. Chaparro, and A. Akan, "Discrete evolutionary transform for time-frequency signal analysis," Journal of the Franklin Institute, 337 (4), pp. 347–364, 2000.

[9] L. F. Chaparro and R. Suleesathira, "Nonstationary jammer excision in spread spectrum using the Discrete Evolutionary and the Hough transforms," Signal Process., vol. 83, pp.1117–1133, Aug. 2003.

[10] A. Akan and L. F. Chaparro, "Modeling And Estimation Of Wireless OFDM Channels By Using Time-Frequency Analysis," Circuits, Systems, and Signal Process., vol. 25, no.3, pp. 389-403, 2006.

[11] H. M. Ozaktas, Z. Zalevski, and M.A. Kutay, The fractional Fourier transform with applications in optics and signal processing, Wiley and Sons, New York, 2001.

[12] A. Serbes, L. Durak, "Optimum signal and image recovery by the method of alternating projections in fractional Fourier domains," Communications in Nonlinear Science and Numerical Simulation, vol 15, Issue 3, pp. 675–689, March 2010.

[13] L. Rankine, M. Mesbah, and B. Boashash "IF estimation for multicomponent signals using image processing techniques in the time-frequency domain," Source Signal Processing, vol. 87, issue 6 (June 2007), pp. 1234–1250, 2007.

[14] A. W. Lohmann, B.H. Soffer, "Relationships between the Radon-Wigner and fractional Fourier transforms," J. Opt. Soc. Am. A, 11(6), pp. 1798–1801, 1994.

[15] A. N. Akansu and H. Agirman-Tosun, "Improved Correlation of Generalized Discrete Fourier Transform with Nonlinear Phase for OFDM and CDMA Communications," Proc. EUSIPCO European Signal Proces. Conference, Glasgow, Aug. 2009.

[16] C. Candan, "On higher order approximations for Hermite-Gaussian functions and discrete fractional Fourier transforms," IEEE Signal Process. Lett., 14(10), 199–702, Oct. 2007.

[17] A. Serbes and L. Durak, "Efficient Computation of DFT Commuting Matrices by a Closed-Form Infinite Order Approximation to the Second Differentiation Matrix," accepted for publication in Signal Proces., 2010.