

JOINT CHANNEL AND FREQUENCY OFFSET ESTIMATION USING SIGMA POINT KALMAN FILTER FOR AN OFDMA UPLINK SYSTEM

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ABSTRACT

In uplink transmissions of a coded orthogonal frequency division multiple access (C-OFDMA) system, a frequency synchronization has to be addressed. Few papers deal with this issue. Expectation - maximization (EM) approaches have been proposed but initialization strategies must be considered because the EM may converge to a local extremum. Here we propose a joint estimation of the channel and the carrier frequency offset (CFO) based on a sigma point Kalman filter. When considering an OFDMA uplink system over a Rayleigh fading channel, simulation results confirm that the proposed algorithm corresponds to a good compromise between CFO and channel estimation, bit error rate and computational cost. Since our approach has a lower computational complexity, the power consumption is lower, which is a great advantage in a wireless system.

1. INTRODUCTION

Today, orthogonal frequency division multiple access (OFDMA) is one of the most common solutions for high data rate transmission systems. OFDMA, initially used for cable TV network and for DVB-RCT standard is applied in WiMAX technology (802.16x). When considering this modulation technique, different users simultaneously transmit their own data by modulating an exclusive set of orthogonal subcarriers. Allocation algorithms [1] exploit this spectral diversity to allocate the communication resources to the different users, such as power, constellation size and necessary bandwidth to maximize the link efficiency.

In an uplink OFDMA system, orthogonality between subcarriers is not satisfied without carrier-frequency offset (CFO) compensation. Therefore, CFOs have to be estimated. This issue has been addressed in several papers such as [1], [2] and [3]. In [4], the CFO estimation is obtained by comparing the phases of two received OFDMA blocks. Zhao *et al.* [5] use an extended Kalman filter (EKF) to estimate the CFO, provided that a training sequence is known. In [6], we propose to combine a Kalman filter based method for the CFO estimation and the so-called minimum mean square error successive detector, the purpose of which is to estimate the signal sent by

each user; unlike existing approaches, no training sequence is required.

However, in the above approaches the channel is assumed to be known or estimated. Recently, the joint CFO/channel estimation has been investigated. In [7], [8] and [9], Pun *et al.* study how to obtain the joint maximum likelihood estimation of the channels and the CFOs of the multiple users. Thus in [7], a conventional expectation-maximization (EM) is first proposed: during the E-step the received signals transmitted by each user, namely the "complete data", are estimated. During the M-step, all the CFOs and the channels are jointly estimated by using these complete data. To simplify the optimization issue, the value of the channel is replaced by its expression depending on the CFO in the criterion to be minimized. Therefore, only the estimation of the CFO of each user has to be addressed. Even if the criterion is explicitly given, the authors do not mention the estimation method they use. For instance, exhaustive grid search could be considered as suggested by the same authors in [8]. To reduce the computational cost, the authors in [7] suggest using the space alternating generalized expectation-maximization (SAGE). In that case, instead of simultaneously estimating every-user parameters, one iteration of the EM algorithm is dedicated to one user. Instead of addressing a multi-dimensional optimization issue, the authors in [8] use the so-called alternating projection estimator. This method consists in iteratively estimating the CFO of one user, by means of an exhaustive grid search over the possible range of the CFO value and by setting the other CFOs to their last updated values. In [9], a suboptimal method is presented. In [10], Xiaoyu *et al.* propose two iterative estimation approaches using the SAGE method. Nevertheless, the EM-based algorithms do not necessarily converge to the global extremum. An initialization step is therefore required. Another drawback of the above methods is the high computational cost due to the iterative estimation and the exhaustive grid search.

In this paper, our contribution is the following: a sigma point Kalman filter (SPKF) [11], namely the unscented Kalman filter (UKF) or the central difference Kalman filter (CDKF), is used to simultaneously estimate the channel and the CFO for

all users. Even if the proposed estimator needs a training sequence, its computational cost is lower than the existing approach ones. This advantage is crucial because one of the goals in the design of wireless systems is to reduce the energy consumption of the system. It should be noted that our work is complementary to the study presented in [12], in which an unscented Kalman filter (UKF) makes it possible to jointly estimate the CFO and the channel, but it is designed for an orthogonal frequency division multiplexing (OFDM) single-user system.

The paper is organized as follows. The OFDMA system and the signal models are presented in section 2. Section 3 shows how to jointly estimate the CFO and the channel coefficients of each user in order to restore orthogonality among the received users' signals. Simulation results are presented in section 4 and finally conclusions are given in section 5.

In the following, $Re\{\cdot\}$ denotes the real part of $\{\cdot\}$, $Im\{\cdot\}$ the imaginary part of $\{\cdot\}$ and \mathbf{I}_L the identity matrix of size L .

2. SYSTEM DESCRIPTION

Let us consider an OFDMA network consisting of a single base station and U simultaneously independent users. The available bandwidth B is divided among N sub-carriers, and a fair distribution of the bandwidth $B_u = B/U$ between each user is supposed.

The signal received by the base station is a superposition of the contributions from the U active users. In the following, let \mathbf{S}_u^p be the symbols emitted by the u th user with $u \in \{1, \dots, U\}$ and corresponding to the p th OFDMA symbol:

$$\mathbf{S}_u^p = [S_u^p(0), S_u^p(1) \dots S_u^p(N-1)]^T \quad (1)$$

According to the frequency allocation of each user, $S_u^p(k)$ can be non-zero if the k th carrier is allocated to the u th mobile terminal, for $k \in \{0, \dots, N-1\}$. The corresponding transmitted signal from the u th user is given by:

$$X_u^p(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_u^p(k) e^{j2\pi nk/N} \quad (2)$$

where $-N_g \leq n \leq N-1$ and $N_g < N$ is the length of the cyclic prefix.

Moreover, let us assume that the channel impulse response of the u th user and related to the p th OFDMA symbol is:

$$\mathbf{h}_u^p(n) = [h_u^p(n, 0), h_u^p(n, 1), \dots, h_u^p(n, L_u)]^T \quad (3)$$

where L_u is the length of the maximum channel delay spread and $L_u \leq N_g$ so that the cyclic prefix discards the inter block interference. We suppose a multipath quasi-static Rayleigh fading channel:

$$\mathbf{h}_u^p(n) = \mathbf{h}_u^p(n-1) = \mathbf{h}_u^p \quad (4)$$

At the receiver, due to the propagation conditions, time offset and CFO are induced into the baseband signal. By choosing an appropriate cyclic prefix length $N_g = \max_u \{\tau_u + L_u\}$,

where τ_u is the normalized spacing and timing error related to the u th user, the effects of the uplink timing errors are counteracted, i.e they are incorporated as a part of their channel responses¹.

Let us now introduce the normalized CFO to the sub-carrier spacing ϵ_u^p :

$$\epsilon_u^p(n) = \epsilon_u^p(n-1) = \epsilon_u^p \quad (5)$$

In the following, we will focus our attention on the row vectors ϵ^p and \mathbf{h}^p which contain the normalized CFO and the channel impulse response of each user respectively:

$$\epsilon^p = [\epsilon_1^p, \epsilon_2^p, \dots, \epsilon_u^p, \dots, \epsilon_U^p] \quad (6)$$

$$\mathbf{h}^p = [\mathbf{h}_1^{pT}, \mathbf{h}_2^{pT}, \dots, \mathbf{h}_u^{pT}, \dots, \mathbf{h}_U^{pT}] \quad (7)$$

The U incoming waveforms are naturally combined by the receiver antenna. After cyclic prefix removing, the resulting p th received signal can be expressed as follows:

$$\begin{aligned} \mathbf{R}^p &= [R^p(0), R^p(1), \dots, R^p(N-1)]^T \\ &= \sum_{u=1}^U \mathbf{R}_u^p + \mathbf{B} \end{aligned} \quad (8)$$

where $\mathbf{B} = [B(0), \dots, B(N-1)]^T$ is a complex white Gaussian noise vector with covariance matrix $\sigma_b^2 \mathbf{I}_N$, while \mathbf{R}_u^p , the p th signal received from the u th user, can be expressed as:

$$\begin{aligned} \mathbf{R}_u^p &= [R_u^p(0), R_u^p(1), \dots, R_u^p(N-1)]^T \\ &= \mathbf{E}_u^p(\epsilon_u^p) \mathbf{H}_u^p(\mathbf{h}_u^p) \mathbf{X}_u^p \end{aligned} \quad (9)$$

where $\mathbf{X}_u^p = [X_u^p(0), X_u^p(1) \dots X_u^p(N-1)]^T$, \mathbf{H}_u^p is a circulant matrix formed by N cyclic shifts of \mathbf{h}_u^{pT} and $\mathbf{E}_u^p = \text{diag}[1, e^{j2\pi\epsilon_u^p/N}, \dots, e^{j2\pi(N-1)\epsilon_u^p/N}]$.

Let \mathbf{Y}^p be the $2 \times N$ observation matrix that stores the real and the imaginary parts of the received signal \mathbf{R}^p :

$$\begin{aligned} \mathbf{Y}^p &= [Y^p(0), Y^p(1), \dots, Y^p(N-1)] \\ &= \mathbf{A}^p(\epsilon^p, \mathbf{h}^p) + \mathbf{V} \end{aligned} \quad (10)$$

where

$$\begin{aligned} \mathbf{A}^p(\epsilon^p, \mathbf{h}^p) &= [A^p(0, \epsilon^p, \mathbf{h}^p), \dots, A^p(N-1, \epsilon^p, \mathbf{h}^p)] \\ &= \begin{bmatrix} Re \left\{ \sum_{u=1}^U \mathbf{R}_u^p(\epsilon_u^p, \mathbf{h}_u^p) \right\}^T \\ Im \left\{ \sum_{u=1}^U \mathbf{R}_u^p(\epsilon_u^p, \mathbf{h}_u^p) \right\}^T \end{bmatrix} \end{aligned} \quad (11)$$

and $\mathbf{V} = \begin{bmatrix} Re \{\mathbf{B}\}^T \\ Im \{\mathbf{B}\}^T \end{bmatrix}$ which is a white Gaussian noise

vector, the covariance matrix of which is $(\sigma_b^2/2)\mathbf{I}_2$.

In order to restore orthogonality among each user sub-carrier, both the synchronization error vector ϵ^p and the channel vector \mathbf{h}^p have to be estimated, given \mathbf{Y}^p . Due to the non-linear feature of the estimation, a new method based either on the EKF or the SPKF is proposed in the next section.

¹Since this paper deals with the CFO/channel estimation, τ_u and L_u are supposed to be known, for $u \in \{1, \dots, U\}$.

3. FREQUENCY OFFSET AND CHANNEL ESTIMATION

When dealing with a non-linear state-space representation of a system, local methods such as the extended Kalman filter (EKF) and global methods including UKF and CDKF can be considered [11]. In the SPKF, the state distribution is approximated by a Gaussian. It is then characterized by a set of points lying along the main eigenaxes of the Gaussian random variable covariance matrix. Then, these so-called sigma-points propagate through the non-linear system. A weighted combination of the resulting values makes it possible to estimate the mean and the covariance matrix of the transformed random vector, i.e. the random variable (RV) that undergoes the non linear transformation. On the one hand, the UKF is based on the unscented transformation. When the density is odd, the weights are chosen to provide the 2nd order Taylor expansion around the mean of the RV. On the other hand, the CDKF is based on the 2nd order Sterling polynomial interpolation formula. The difference between CDKF and UKF stands in the way the mean and the covariance matrix of the transformed RV are calculated. According to various studies [6], [11] and [13], there is a very slight difference between UKF and CDKF. The advantages of the SPKF over the EKF is that does not require calculations of Jacobians or Hessians and that the EKF is more likely to diverge. SPKF makes it possible to recursively estimate the following state vector:

$$\begin{aligned} \mathbf{x}^p(n) &= \left[\epsilon^p(n) \quad \text{Re}\{\mathbf{h}^p(n)\} \quad \text{Im}\{\mathbf{h}^p(n)\} \right]^T \\ &= \left[\mathbf{x}_1^p(n) \quad \mathbf{x}_2^p(n) \quad \mathbf{x}_3^p(n) \right]^T \end{aligned} \quad (12)$$

Given (4), (5) and (12), $\mathbf{x}^p(n)$ satisfies the following state-equation², which is the representation of what happens for an OFDMA symbol:

$$\mathbf{x}^p(n) = \mathbf{x}^p(n-1) \quad \forall n \in [0, N-1] \quad (13)$$

Given (10) and (11), the measurement equation is defined as follows:

$$Y^p(n) = A^p(n, \mathbf{x}^p(n)) + V(n) \quad \forall n \in [0, N-1] \quad (14)$$

Hence, (13) and (14) define the state space representation of the system making it possible to estimate both the channels and the CFOs. The resulting SPKF estimator is given by:

$$\hat{\mathbf{x}}^p(n) = \hat{\mathbf{x}}^p(n-1) + \text{Re}\left\{ \mathbf{K}^p(n) \tilde{Y}(n) \right\} \quad (15)$$

where $\hat{\mathbf{x}}^p(n) = \left[\hat{\mathbf{x}}_1^p(n) \quad \hat{\mathbf{x}}_2^p(n) \quad \hat{\mathbf{x}}_3^p(n) \right]^T$ is the estimation of $\mathbf{x}^p(n)$, $\tilde{Y}(n) = Y^p(n) - \hat{Y}^p(n)$ is the so-called innovation and \hat{Y}^p is the estimation of Y^p , obtained by using a weighted combination of the sigma points. In addition

$$\mathbf{K}^p(n) = \left[\mathbf{P}_{\epsilon \tilde{Y}}^p(n) \right] \left[\mathbf{P}_{\tilde{Y}}^p(n) \right]^{-1} \quad (16)$$

is the filter gain, where $\mathbf{P}_{\epsilon \tilde{Y}}^p(n)$ is the covariance matrix between the state prediction error and the innovation, and $\mathbf{P}_{\tilde{Y}}^p(n)$

²It should be noted that transition matrix is equal to I_M , where $M = U + 2UL_u$ and there is no state noise.

is the covariance matrix of the innovation. Both are estimated by using a weighted combination of the sigma points. For details about the SPKF algorithm description, the reader is referred to [11]. After some recursions, the algorithm can provide an "accurate" joint estimation of the CFO value and the channel impulse response for the u th user, which is denoted as $\hat{\epsilon}_u^p(n)$ and $\hat{\mathbf{h}}_u^p(n)$ respectively with $u \in \{1, \dots, U\}$.

4. SIMULATION RESULTS

In the following, a comparative study is carried out between our approach based on EKF, CDKF or UKF and the method presented in [7] where a grid search approach is used to update the CFO estimation.

Simulation protocol: we performed 500 Monte-Carlo runs. We consider an OFDMA uplink system, which is composed of $U = 4$ users sharing $N = 128$ sub-carriers and with cyclic prefix $N_g = N/8 \geq L_u$. We suppose a transmission over a Rayleigh quasi-static frequency selective channel composed of $L_u = 3$ multi-paths. QPSK is used to modulate the information bits. The carrier frequency is at $f_c = 2.5\text{GHz}$ and the channel bandwidth is set to $W = 20\text{MHz}$. The duration of an OFDMA symbol is $T_s = N/W$. The users' normalized CFO errors satisfy: $\epsilon_u = N \frac{v_u}{c} f_c T_s$, where v_u is the user speed and c is the light speed. In addition they are randomly and uniformly generated in the interval $[-0.3, 0.3]$. We define $\text{Eb/No} = \frac{\sigma_u^2}{\sigma_b^2}$, where σ_u^2 is the mean power of the received signal from the u th user. Here, the EM algorithm proposed in [7] is based on $\alpha = 20$ iterations and a grid search precision equal to 10^{-3} ; this means that $\beta = \frac{2 \times 0.3}{10^{-3}} + 1 = 601$ values of CFO are studied in the grid search algorithm. For the CFO, the initialization parameters of the algorithm is $\hat{\epsilon}_u^p(0) = 0$ when $p = 1$ and $\hat{\epsilon}_u^p(0) = \epsilon_u^{p-1}$ otherwise.

Performances: first of all, our approach provide similar results when UKF and CDKF are used. Therefore, in the following, we speak of SPKF filtering performances. We focus our attention on the first user in the system. Figure 1 and 2 show the results in terms of CFO and channel minimum mean square error (MMSE).

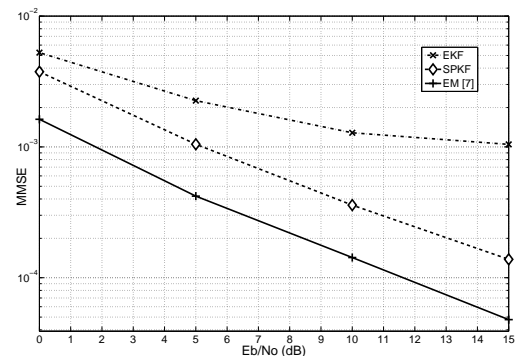


Fig. 1. CFO estimation performances, 20 iterations performed when using the EM [7].

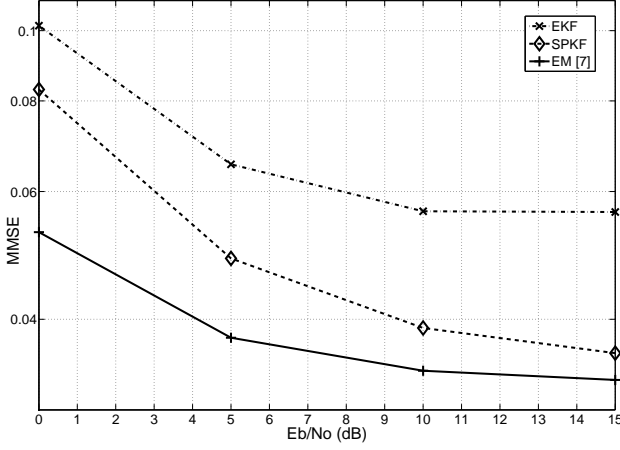


Fig. 2. Channel estimation performances, 20 iterations when using the EM [7].

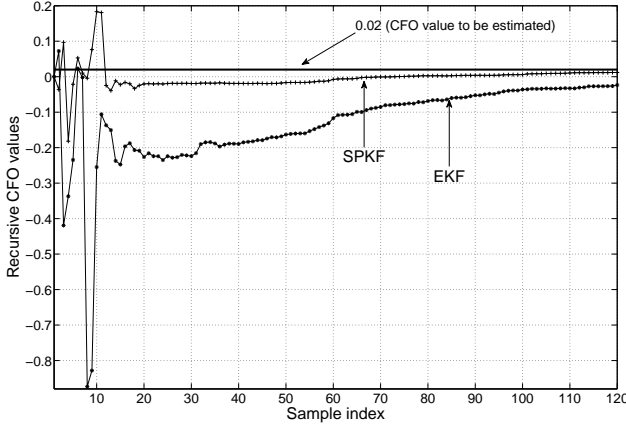


Fig. 3. Recursive CFO estimation using EKF and SPKF.

The SPKF algorithms give a better estimation of the CFO and the channel than the EKF. Figure 3 shows that the SPKF seem to require less observations than the EKF to estimate the CFOs. The EM-based algorithms proposed in [7] provide better performances in terms of MMSE. Nevertheless, the estimation error of our algorithm is small enough to guarantee bit error rates (BERs) that are similar to the ones obtained when [7] is used. See table 1.

Computational complexity: in the following let us have a look at the computational cost of both algorithms. The EM-based algorithm [7] and the SPKF provide similar results in terms of BER, but the EM algorithm has a higher computational complexity due to the exhaustive grid search over the possible range of CFOs and the iterative estimation.

Table 2 shows the number of arithmetic operations performed by the EM; N_{ob} represents the number of observations for the algorithms and it is set to $N_{ob} = N$. The computational cost of the EM approach depends on the number of α iterations and on the number of β tested values. The M-step is based on the inversions of U matrices of size $L_u \times L_u$. In addition, the

Eb/No (dB)	theoretical BER	BER (EM)	BER (SPKF)
0	1.464×10^{-1}	1.469×10^{-1}	1.584×10^{-1}
5	6.418×10^{-2}	6.452×10^{-2}	6.681×10^{-2}
10	2.327×10^{-2}	2.361×10^{-2}	2.395×10^{-2}

Table 1. BER performance.

decision test has to be done to decide which is the best value of the CFO. Table 3 shows the number of arithmetic operations performed by the SPKF. The first step of the algorithm, corresponding to the selection of the sigma points requires the Cholesky decomposition of a matrix of size $M \times M$, where $M = U + 2UL_u$. The measurement update also requires the inversion of a 2×2 matrix.

EM algorithm		
E-step		
additions and subtractions	$\alpha N_{ob}^2 U + \alpha N_{ob} U(2L_u + 6)$	1.4336×10^6
multiplications and divisions	$\alpha N_{ob}^2 U + \alpha N_{ob} U(U + 2L_u)$	1.4131×10^6
M-Step		
add./sub. for the grid search	$\alpha \beta N_{ob}^2 U L_u + \alpha \beta N_{ob} U(3UL_u + L_u^2) - \alpha \beta U(3UL_u + L_u + 1)$	2.6382×10^9
mult./div. for the grid search	$\alpha \beta U + \alpha \beta U N_{ob}^2 (3UL_u + L_u) + \alpha \beta U N_{ob} (L_u^2 + 15U + 2L_u + 1)$	3.1190×10^{10}
other add./sub.	$\alpha N_{ob}^2 U L_u + \alpha N_{ob} U(3UL_u + L_u^2) - 3\alpha U^2 L_u$	4.3901×10^6
other mult./div.	$\alpha N_{ob}^2 U(3UL_u + L_u) + \alpha U N_{ob} (L_u^2 + 15U + L_u)$	5.1855×10^7
Total arithmetic operations		3.3887×10^{10}

Table 2. Number of arithmetic operations performed by the EM algorithm, 601 tested CFO values and 20 iterations.

SPKF algorithm		
step 1: calculation of the sigma points		
additions and subtractions	$2N_{ob}U^2(4L_u^2 + 4L_u + 1)$	2.0070×10^5
multiplications and divisions	$2N_{ob}U^2(4L_u^2 + 4L_u + 1)$	2.0070×10^5
step 2: estimation update		
additions and subtractions	$2N_{ob}U(8U^2L_u^3 + 12U^2L_u^2 + 6U^2L_u + 12UL_u^2 + U^2 + 12UL_u + 3U + 2L_u + 1)$	6.2290×10^6
multiplications and divisions	$N_{ob}U(16U^2L_u^2 + 24U^2L_u + 12U^2L_u + 20UL_u^2 + 2U^2 + 20UL_u + 5U + 4L_u + 2)$	6.1286×10^6
step 3: measurement update		
additions and subtractions	$2N_{ob}^3U(4UL_u + 2U + 1) + 2N_{ob}^2U(4UL_u + 2U + 1) + 20N_{ob} + N_{ob}U(28UL_u^2 + 36UL_u + 11U + 96L_u + 50)$	9.6471×10^8
multiplications and divisions	$2N_{ob}^3U(4UL_u + 2U + 1) + 2N_{ob}^2U(4UL_u + 2U + 4L_u + 3) + 18N_{ob} + 2N_{ob}U(4UL_u^2 + 4UL_u + U + 46L_u + 23)$	9.6601×10^8
Total arithmetic operations		1.9435×10^9

Table 3. Number of arithmetic operations performed by the SPKF algorithm.

Table 4 shows the number of Giga-operations per second (Go/s) performed by the EM for different grid search precisions.

Table 5 shows the number of Go/s for different numbers of users in the system with a channel composed of 3 multi-paths. In addition, table 5 shows the number of Go/s for different number of channel multi-paths with 4 users in the system.

grid search precision	β	Go/s
10^{-1}	7	0.4531
10^{-2}	61	3.4925
10^{-3}	601	33.8871
10^{-4}	6001	337.8323

Table 4. Number of Go/s performed by the EM for different grid search precisions.

4 users in the system			3 multi-paths		
Multipaths	Go/s(EM)	Go/s(SP KF)	Number of users	Go/s(EM)	Go/s(SP KF)
1	11.5235	0.8477	2	9.6945	0.4935
2	22.6930	1.3934	3	19.9786	1.0984
3	33.8871	1.9435	4	33.8871	1.9435
4	45.1058	3.0628	6	72.5776	4.3584
6	67.6173	3.6352	7	97.3597	5.9304
7	78.9101	4.2183	8	125.7662	7.7467
8	90.2274	4.8136	9	157.7972	9.8086
9	101.5695	5.4226	10	193.4528	12.1169
10	112.9362	6.0469	11	232.7328	14.6728
11	124.3276	6.6882	12	275.6373	17.4773
12	135.7436	7.3479	13	322.1664	20.5314

Table 5. Number of Go/s performed by the EM and the SPKF algorithm.

The computational complexity of the EM increases faster when the number of users (or the number of channel multi-paths) increases. Indeed the EM works in blocks in an iterative way whereas the SPKF is recursive.

We clearly see by the results, that the computational complexity of the EM is higher than the one of the SPKF. When 4 users in the system and a channel composed of 3 multi-paths are considered, the SPKF algorithm requires only 6% of the number of operations required by the EM. In addition due to the grid search, the implementation of the EM is relatively difficult in real environments.

5. CONCLUSIONS

The architecture of the proposed receiver requires a training sequence, but it does not need an initialization step. In addition, its computational cost is lower than the EM-based methods. This advantage is crucial because one of the goals in the design of wireless systems is to reduce the energy consumption of the system. In current wireless communication systems only a few Go/s are dedicated to the channel estimation and the synchronization. The proposed approach is hence completely applicable to practical environments, unlike the EM-based methods.

6. ACKNOWLEDGEMENTS

This work was partially supported by the National Bureau of Science, Technology and Innovation of Panama (SENACYT).

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