FULL-RATE FULL-DIVERSITY SPACE-FREQUENCY BLOCK CODING FOR DIGITAL TV BROADCASTING

Iker Sobron, Mikel Mendicute and Jon Altuna

Signal Theory and Communications Area, University of Mondragon
Loramendi 4, 20500, Mondragon, Spain
phone: + (34) 943739422, email: isobron@eps.mondragon.edu
web: www.mondragon.edu/tsc

ABSTRACT
The goal of the future terrestrial and mobile digital video broadcasting standards is to combine diversity and spatial multiplexing in order to fully exploit the multiple-input multiple-output (MIMO) channel capacity. Full-rate full-diversity (FR-FD) space-time codes (STC) such as the Golden code, that maximize the rate preserving the diversity gain, are studied for that purpose. Most of them present a high-complexity detection problem which results in a handicap for their hardware implementation. Therefore, we analyze the performance of a sub-optimal full-rate full-diversity 2x2 STC, whose design reduces the complexity of the detection, as an alternative for future MIMO broadcasting TV systems. The assessment of the proposed STC code is carried out over the bit-interleaved coded modulation (BICM) scheme of DVB-T2 system which includes low-density parity check (LDPC) codes for error correction. Finally, soft STC list detection is studied in order to generate soft information for LDPC decoding.

1. INTRODUCTION
Multiple-input multiple-output (MIMO) wireless communication systems are based on signal processing with multiple antennas at both transmitter and receiver side. MIMO processing can increase the limits of the channel capacity, adding spatial diversity or canceling interference. This versatility has caused MIMO techniques to be incorporated in many of the current wireless communications systems. The second generation of terrestrial digital video broadcasting (DVB-T2) [1] has been the first broadcasting standard to include a multi-antenna system as an optional technique. This consists of a multiple-input single-output (MISO) transmission scheme for two transmit antennas based on the Alamouti transmit diversity technique [2]. On the other hand, the DVB-T2 MISO technique is a pure diversity approach which forms a subset of MIMO since it only includes two antennas at the transmitter side. Therefore, DVB-T2 does not fully exploit the capacity of MIMO channels.

The proposal for the future generations of terrestrial, portable and mobile digital video broadcasting (DVB-NGH) is the combination of the advantages of the Alamouti scheme and the multiple-plexing gain in order to approach the full MIMO diversity-multiplexing frontier. The family of space-time codes (STC) called full-rate full-diversity (FR-FD) are the best candidates for that purpose since they make the most of MIMO channel capacity maximizing the diversity order. There exist several FR-FD STCs in the literature but the named Golden code [3] is considered the 2x2 FR-FD STC that offers the best performance. In fact, its implementation in broadcasting digital TV systems has been studied in [4]. The main disadvantage of Golden code arises from its very high decoding complexity, which is proportional to the fourth power of the constellation size. Consequently, we propose a different FR-FD 2x2 code introduced in [5, 6] as an alternative for future broadcasting TV systems since the decoding complexity is reduced at least to the second power of the constellation size.

The current bit-interleaved coded modulation (BICM) systems proposed for wireless communications standards such as DVB-T2 include low density parity check (LDPC) codes for forward error correction (FEC). That is due to the fact that LDPC codes allow us reliable communications close to the Shannon limit. The decoding of this channel code is performed using soft information from the conditional probabilities for all possible transmitted symbols. Therefore, we need to detect and decode the aforementioned STCs giving a soft solution. In this paper, we propose a list version of the maximum likelihood (ML) detection algorithm based on [7] in order to analyze the behavior for DVB-T2 of a 2x2 MIMO system using different STCs: Alamouti scheme, Golden code and low complexity FR-FD. We provide simulation results that show the performance of these codes in typical TV broadcasting simulation environments, such as Rayleigh and Rician channels.

This paper is organized as follows: in the next section, we present the evaluated MIMO schemes. Section 3 describes the complete DVB-T2 BICM system and the soft ML detection problem. Later, in section 4, bit error rate (BER) performances are shown and finally, the conclusions are drawn.

2. DESCRIPTION OF MIMO SCHEMES
In this section, we focus on the MIMO schemes which are evaluated in this research work. First of all, we define the analyzed system with \( M = 2 \) transmit and \( N = 2 \) receive antennas using matrix notation. Next, the Alamouti transmit diversity scheme proposed in DVB-T2 is described and finally, the FR-FD codes are presented after stating the optimal STC design criteria for Rayleigh channels.

2.1 Description of the 2x2 MIMO system
Due to the fact that DVB-T2 is an orthogonal frequency division multiplexing (OFDM) system, STC coding is carried out in frequency and space domains. If the codeword is transmitted combining two adjacent carriers of an OFDM symbol, it is named space-frequency block coding (SFBC). On the other hand, if the codeword is transmitted at the same carrier of two consecutive OFDM symbols, it is called space-time block coding (STBC). In our case, the evaluated STC codes are implemented as SFBC. If we assume the fading channel to be quasi-static in two adjacent carriers, a 2x2 MIMO transmission can be written in the form:

\[
Y = HX + Z, \tag{1}
\]

where \( Y \) denotes the 2x2 received symbol matrix for two adjacent carriers and two receive antennas, \( H \) denotes the 2x2 complex channel matrix, \( X \) is any codeword matrix with \( M = 2 \) rows and \( T = 2 \) columns, and finally, \( Z \) denotes the zero-mean additive complex Gaussian noise matrix whose complex coefficients fulfill \( CN(0, 2\sigma^2) \) being \( \sigma^2 \) the variance per real component.

The 2x2 codeword \( X \) can be expressed generically as:

\[
X = \begin{bmatrix}
x_{11} \\
x_{21} \\
x_{12} \\
x_{22}
\end{bmatrix} = f(s_1, \ldots, s_Q), \tag{2}
\]

where \( X \) is formed by a linear combination of \( Q \) symbols. As a result, we can define the spatial rate as:
Therefore, we can observe that SFBC codes vary the raw bit rate by a factor of $L$ in comparison to the single-input single-output (SISO) system.

2.2 The SFBC DVB-T2 system

As we have stated in the introduction, the DVB-T2 standard [1] includes an Alamouti MISO transmission scheme as an optional technique. We can observe the corresponding transmission structure in Fig. 1. Using matrix notation, the pairwise processing of symbols $s = (s_1, s_2)$ can be written generically as:

$$X_{af} = \begin{bmatrix} s_1 \\ -s_2^* \end{bmatrix}$$

(4)

where $X_{af}$ defines the 2x2 codeword matrix and $*$ denotes complex conjugate. The DVB-T2 SFBC technique provides the same diversity order as maximal ratio combining (MRC) with one transmit antenna and two receive antennas maintaining the same spatial rate as a SISO system. The model can be easily generalized to $N$ receive antennas providing a diversity order of $2N$.

2.3 Full Rate-Full Diversity SFBC codes

2.3.1 STC design criteria

The code optimization is based on the analysis of pairwise error probability (PEP) $P(X \rightarrow \hat{X})$ which is the probability of transmitting $X$ and detecting $\hat{X}$ at the receiver. Assuming ideal channel state information (CSI), Chernoff bound analyses proposed in [8, 9] lead to two criteria in order to minimize PEP for Rayleigh fading channels:

1. Rank criterion: In order to achieve maximum diversity, the matrix $\Delta = X - \hat{X}$ has to be full rank for any codewords $X$ and $\hat{X}$. Then, the code is said to have full diversity.

2. Determinant criterion: In order to obtain maximum coding gain, the minimum of the determinants of the matrices $\Delta^H \Delta$ has to be maximized for all pairs of different codewords $X$ and $\hat{X}$.

The forthcoming STC schemes have full diversity and a large coding gain. They are also considered full-rate since they send as many data symbols as the number of the system’s degrees of freedom $MT$.

2.3.2 The Golden code

The Golden code is the best full-rate full-diversity 2x2 STC which achieves the maximal coding gain [3]. In this case, a group of four symbols $s = (s_1, s_2, s_3, s_4)$ is transmitted as follows:

$$X_s = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha (s_1 + \theta s_3) \\ i\alpha (s_2 + \theta s_4) \\ \alpha (s_2 + \theta s_4) \\ \alpha (s_1 + \theta s_3) \end{bmatrix},$$

(5)

with $\theta = 1 + 2\sqrt{5}$ and $\alpha = 1 + i - \theta$ and $\bar{\alpha} = 1 + i - \theta$. The main drawback of the Golden code lies on the decoding complexity. In order to choose a detected vector $\hat{s}$, we need a maximum likelihood detector which solves

$$\hat{s}_{ml} = \arg\min_s \|Y - HX\|^2,$$

(6)

performing an exhaustive search for all symbol vectors $s = (s_1, s_2, s_3, s_4)$ which form the codeword $X$. The complexity of computing $\hat{s}_{ml}$ is exponential in the length of the symbol vector $s$, i.e. $\Theta(P^4)$, which is prohibitive for large constellation sizes $P$.

2.3.3 Low complexity FR-FD codes

The following STC scheme, which we call FR-FD code from now on, is designed in such a way that maximizes the aforementioned STC design criteria and allows us to perform an optimum detection with lower complexity, which can be described in the worst case as $\Theta(P^2)$. Although this sort of STC first appeared in [10], its low-complexity detection property was not analyzed until [5, 6]. Despite FR-FD coding gain is lower than Golden coding gain, its low-complexity detection property was not analyzed until [5, 6].

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3. THE MIMO-BICM SYSTEM

This section describes the complete DVB-T2 system analyzed in this work. On one hand, we describe the transmission scheme and on the other hand, we detail the necessary soft STC detection for channel decoding.

3.1 Transmission system

One of the main current research topics in digital TV concerns the optimization of the MIMO-OFDM schemes in order to improve the spectrum efficiency for high-definition television (HDTV) services. So far, DVB-T2 [1] is the only standard in broadcasting TV systems which includes multi-antenna processing. Consequently, we have implemented a MIMO-OFDM system over the basic chain of DVB-T2 in order to analyze the STC codes for future proposals. As it is depicted in Fig. 2, the main feature of DVB-T2 is the joint processing of mobile reception possible. After all the interleaving blocks, SFBC processing is included before OFDM modulation. On the other hand, DVB-T2 allows us to increase the capacity using high modulation orders (up to 256-QAM) and low LDPC code rates $R$. The applicability of MIMO techniques in broadcasting digital TV systems is based on the deployment of single-frequency networks (SFN). In SFN, multiple transmitters transmit synchronously the same signal at the same frequency carrier. The SFN networks are...
currently formed by SISO systems. However, DVB-T2 defines a distributed MISO scheme where every transmit antenna of the system is a transmitter of the SFN network. In the case of MIMO, SFBC encoding is carried out between two adjacent cells of the SFN network while the receive antennas are sited at the terminal as is shown in Fig. 3.

Figure 2: Block diagram of DVB-T2 system.

Figure 3: Distributed MIMO transmission in SFN networks.

For a correct reception of the depicted transmission scheme, the MIMO detector must provide soft-information to the LDPC decoder. That is achieved by means of a soft detector that we explain in the following subsection.

3.2 Soft Detection

3.2.1 MAP Detection

The main idea of the LDPC decoding algorithm is to exchange soft information between nodes that are linked by a parity check matrix. The method to obtain this soft information is called maximum a posteriori (MAP) detection and consists of taking a posteriori probability (APP) information expressed in the form of log-likelihood ratios (LLRs) (i.e. L-values [12]). The LLR of a bit $b_k$ is defined as the logarithm of the ratio of the probabilities of the bit taking its two possible values and can be expressed as:

$$L(b_k) = \ln \frac{\Pr [b_k = +1]}{\Pr [b_k = -1]},$$  \hspace{1cm} (8)

where the values of the bits are taken to be +1 and -1, representing logical ‘1’ and ‘0’, respectively. We can assume that the information bits are scrambled by means of several interleavers in such a way that the bits within $Y$ may be considered statistically independent. Therefore, using Bayes’ rule, Eq. (8) can be expressed as follows:

$$L_D(b_k|Y) = \ln \frac{\Pr [b_k = +1|Y]}{\Pr [b_k = -1|Y]} = L_A(b_k) + L_E(b_k|Y),$$  \hspace{1cm} (9)

where $L_A(b_k)$ and $L_E(b_k|Y)$ denote the a priori and extrinsic information, respectively and $k = 0, \ldots, MT \log_2 P - 1$. The extrinsic information conditioned to the received vector $Y$ can be written as:

$$L_E(b_k|Y) = \ln \frac{\sum_{b \in B_{k+1}} p(Y|b) \exp \left( \sum_{j \in J_{k,b}} L_A(b_j) \right)}{\sum_{b \in B_{k-1}} p(Y|b) \exp \left( \sum_{j \in J_{k,b}} L_A(b_j) \right)},$$  \hspace{1cm} (10)

where $p(Y|b)$ represents the likelihood function. Defining $K_k = MT \log_2 P$, $B_{k+1}$ represents the set of $2^K_k$ -1 bit vectors $b$ having $b_k = +1$, so that,

$$B_{k+1} = \{ b | b_k = +1 \}, B_{k-1} = \{ b | b_k = -1 \},$$  \hspace{1cm} (11)

and $J_{k,b}$ is the set of subindexes that can be written as

$$J_{k,b} = \{ j | j = 0, \ldots, K_b - 1, j \neq k, b_j = +1 \}.$$  \hspace{1cm} (12)

3.2.2 Likelihood function for MAP detection

The most important part of the calculation of $L_D$ in (9) is the likelihood function $p(Y|b)$. Considering our system in (1) and following [11], we rewrite (1) in such a way that we can observe an equivalence with a MIMO system with $M=4$ and $N=4$ where there is no channel interference between the sets of transmit antennas $\{1,2\}$, $\{3,4\}$ and the sets of receive antennas $\{3,4\}$, $\{1,2\}$ respectively. First, we define the column vectors $x = [x_{11}, x_{21}, x_{12}, x_{22}]^T$, $\bar{x} = [\bar{x}_{11}, \bar{x}_{21}, \bar{y}_{12}, \bar{y}_{22}]^T$ and $\bar{z} = [\bar{z}_{11}, \bar{z}_{21}, \bar{z}_{12}, \bar{z}_{22}]^T$, whose elements are taken column-wise from matrices $X$, $Y$ and $Z$ respectively. Next, we define the real-value column vector

$$\tilde{s}_R = [\Re \{x_{11}\}, \Im \{x_{11}\}, \Re \{x_{21}\}, \ldots, \Im \{x_{22}\}].$$  \hspace{1cm} (13)

The described STC codes belong to the class of linear dispersion codes [13] so they can be written as $\tilde{s}_R = G\tilde{s}_R$, where $\tilde{s}_R$ corresponds to the symbol vector $s = [s_1, s_2, s_3, s_4]^T$ expressed as (13) and $G$ is the real generator matrix of the STC. Then, using the new notation, the likelihood function $p(Y|b)$ can be written as:

$$p(Y|\tilde{s}_R = \text{map}(b)) = \frac{\exp \left( \frac{||\tilde{s}_R - \tilde{G}\bar{x}_R||^2}{2\sigma^2} \right)}{(2\pi\sigma^2)^{N+1}},$$  \hspace{1cm} (14)

where $\tilde{s}_R = \text{map}(b)$ is the mapping of the vector $b$ into the symbols of column vector $s$ and the equivalent channel $\tilde{H}$ is obtained from channel matrix $H$ as $\tilde{H} = (1/2) I_2 \otimes (H \otimes E + \hat{H} \otimes E^*)$, where $\otimes$ corresponds to the Kronecker product and $E = \begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix}$. Only the term inside the exponent in (14) is relevant for the calculation of the $L_E$-value, and the constant factor outside the exponent can be omitted. Therefore, with the Max-log approximation, the extrinsic value $L_E$ (10) becomes
provides a good performance with lower detection complexity. FR-FD is 0.3 dB worse than Golden code. Therefore, FR-FD code η full-rate codes for 16 complexity FR-FD can be expressed as $O(2^n)$ The computation complexity of each $M$ ired SFBC codes over the DVB-T2 system using a soft list de-

η achieve a higher gain than the DVB-T2 MIMO system for Table 1.

η respectively. Two different bit rates have been used in the analysis: schemes using the DVB-T2 system over TU6 and RA6 channels, 

η been assessed using bit error rate (BER) curves for systems with diversity order is the same for all the systems. The performance has 

evaluated STC codes. Therefore, in the case of the DVB-T2 MISO technique, we have included a MRC at the receiver side so that the 

η on [15].

η DVB-T2 parameters used in the simulations are the following:

• Length of LDPC block: 64800 bits
• LDPC code rate: $R = 2/3$
• Constellation sizes: QPSK, 16-QAM and 256-QAM
• FFT size: 2048 carriers (2K)
• Guard interval: 1/4

η The simulations have been carried out over a Rayleigh channel (Typical Urban of six path, TU6) and a Ricean channel (Rural Area of six paths, RA6), both commonly used as simulation environ-

ments of terrestrial digital television systems [14]. We consider perfect CSI at the receiver and non-iterative MIMO detection. Therefore, there is no a priori information $L_A(b_k)$ and hence (17) is sim-

η. The number of candidates has been set to $N_{\text{cand}} = 100$ based on [15].

η In Fig. 4 and Fig. 5 we compare the aforementioned STC schemes using the DVB-T2 system over TU6 and RA6 channels, 

η. Two different bit rates have been used in the analysis: $\eta = 8/3$ and $\eta = 16/3$. These correspond to the configurations of Table 1.

η As we can observe in Fig. 4, FR-FD code and Golden code achieve a higher gain than the DVB-T2 MIMO system for $\eta = 16/3$. However, the DVB-T2 scheme has a better behavior than the full-rate codes for $\eta = 8/3$. On the other hand, the performance of FR-FD is 0.3 dB worse than Golden code. Therefore, FR-FD code provides a good performance with lower detection complexity.

In Fig. 5, where we have a channel with line of sight (LOS), both FR-FD code and Golden code provide a lower performance than the DVB-T2 MIMO scheme. In this case, the gain between FR-FD and Golden codes is also maintained.

4. ASSESSMENT OF FR-FD IN DVB-T2

This section presents the simulation results for the previously described SFBC codes over the DVB-T2 system using a soft list detector. The numbers of antennas are $M = 2$ and $N = 2$ for all the evaluated STC codes. Therefore, in the case of the DVB-T2 MISO technique, we have included a MRC at the receiver side so that the diversity order is the same for all the systems. The performance has 

been assessed using bit error rate (BER) curves for systems with the same bit rate $\eta = RL \log_2 P$ and the same bit energy $E_b$. The DVB-T2 parameters used in the simulations are the following:

• Length of LDPC block: 64800 bits
• LDPC code rate: $R = 2/3$
• Constellation sizes: QPSK, 16-QAM and 256-QAM
• FFT size: 2048 carriers (2K)
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5. CONCLUSIONS AND FUTURE WORK

In this paper, we have analyzed the performance of a sub-optimal 2x2 FR-FD code with low complexity detection for its possible implementation in future broadcasting TV systems. This code fully exploits the MIMO channel capacity comparing to the cu-

rrent MISO scheme defined in DVB-T2. The result comparisons of MIMO-BICM systems with the same bit rate and bit energy show that FR-FD code achieves a higher performance gain in Rayleigh channels than the current DVB-T2 MISO system combined with a MRC technique at the receiver for high bit rates and that is reduced in low bit rates. Furthermore, it provides similar performance as Golden code with lower detection complexity. On the other hand, both FR-FD and Golden codes provide a reduction of the performance in LOS environments, being worse than the DVB-T2 Alamouti scheme with MRC for the analyzed bit rates. This is due to the fact that these codes have been designed for Rayleigh channels
Table 1: Configurations according to the bit rate.

<table>
<thead>
<tr>
<th>Bit rate (η)</th>
<th>STC scheme</th>
<th>Spatial rate (L)</th>
<th>Code rate (R)</th>
<th>Constellation size (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16/3</td>
<td>Alamouti</td>
<td>1</td>
<td>2/3</td>
<td>256-QAM</td>
</tr>
<tr>
<td>16/3</td>
<td>Golden</td>
<td>2</td>
<td>2/3</td>
<td>16-QAM</td>
</tr>
<tr>
<td>16/3</td>
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<td>2/3</td>
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</tr>
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<td>8/3</td>
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<td>QPSK</td>
</tr>
</tbody>
</table>

following the STC design criteria. Therefore, they could be optimized for Ricean channels.

Despite the reduction of performance in LOS scenarios in comparison to the DVB-T2 MIMO scheme, FR-FD code allows us to use higher constellation sizes and hence, to increase the capacity for HDTV system reducing the detection complexity.

As future work lines, we propose the definition of FR-FD codes for Ricean channels, which are very common in terrestrial broadcasting TV scenarios, as well as the evaluation of the behavior of the system in SFN networks with different transmitter power. Moreover, we can analyze the number of candidates $N_{\text{cand}}$ that is necessary to obtain a good performance and the inclusion of an iterative process between the STC and LDPC blocks decoding in order to reach near-capacity.

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