

STRUCTURE TENSOR FIELD REGULARIZATION BASED ON GEOMETRIC FEATURES

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ABSTRACT

This paper investigates structure tensor field regularization applied to directional textured image analysis. From previous works on tensor filtering, we demonstrate that, knowing that the structure tensor is a specific tool coding the local geometry of the image, the tensor field filtering process must be driven by a geometric dissimilarity measure to define the adaptability of the smoothing process. We propose a new dissimilarity measure combining two terms devoted respectively to the orientation and to the shape component of the tensor. This intelligible encoding exhibiting the geometric structure of the image enables us to overcome major drawbacks of conventional Euclidean and Riemannian approaches for which the dissimilarity measure emphasizes only the local manifold geometry. Finally, for seismic imaging application, our method compared to existing ones shows that relevant information can be extracted by enhancing the seismic structures identification.

1. INTRODUCTION

Data denoising is a conventional task in signal processing applications. For scalar images, many filter paradigms have been developed such as local regression, variational approaches, partial derivative methods and robust statistics techniques. Surveys have shown the connection between existing approaches [1]. Considering these pioneering works, most of proposed approaches have been extended in the framework of tensor filtering for Magnetic Resonance Imaging applications (MRI). Diffusion Tensor Magnetic Resonance Imaging (DT-MRI) associates a 3x3 real symmetric positive-definite (SPD) matrix, called tensor, with each voxel in a 3D volume. In this specific framework, non-linear filtering taking into account the manifold of the space of tensors has been derived [2],[3],[4]. Processing MRI tensor field leads to use appropriate Riemannian metric such as affine invariant tensor dissimilarity measure [3],[4], or Log-Euclidean metric [5]. Using specific tools dedicated to the geometry of the space of SPD matrices guarantees to stay onto the tensor manifold and remedies to shortcomings such as swelling effect [4].

Considering the tensor field regularization task, this paper focuses on another family of SPD matrices, i.e. the structure tensor (ST). In computer vision and image processing appli-

cations, the ST is a conventional tool based on the partial derivatives characterizing the local geometry and low-level features of the image [6], [7], ranging from local orientation, edge and corner or for coherency analysis. Taking into account the geometric nature of ST, the paper proposes to show that for ST field regularization associated with directional textured image characterization, the use of dissimilarity focused on geometric features such as shape and orientation rather than conventional Riemannian approaches is suitable. The paper is intended as a contribution in this way: by providing for directional texture, firstly, an enhanced geometric scheme to increase the relevance of the ST filtering and, secondly, by proposing novel geometric dissimilarity exhibiting tractable components in terms of orientation and shape.

The paper is organized as follows: after a brief description of the related works dedicated to the non-linear filtering in section 2 and a survey of structure tensor in section 3, a new dissimilarity measure called Shape-Orientation is presented in section 4 when section 5 discusses experiments in the seismic imaging application field in the framework of local orientation estimation.

2. TENSOR FIELD REGULARIZATION

Numerous algorithms such as M-estimators, nonlinear diffusion or bilateral filters are widely-used in image denoising. Although their formalism seems somewhat different, all these approaches have been casted into a unified framework [1] of functional minimization. Smoothness terms of this framework are briefly outlined below.

Let us consider N samples f_i , $i=1, \dots, N$ of a noisy image f . A M-estimator provides a denoised solution u by minimizing

$$E(u) = \sum_{i=1}^N \sum_{j=1}^N \psi(|u_i - f_j|^2). \quad (1)$$

where $\psi(\cdot)$ is an error function. As well-known form, the l_2 error function $\psi(s^2) = s^2$ leads to an estimation of u which is simply the average of f .

The criterion of equation (1) can be minimized by gradient descent algorithm. As a result, each term u_i is iteratively estimated with the following formula

$$u_i^{k+1} = \frac{\sum_{j=1}^N \psi'(|u_i^k - f_j|^2) f_j}{\sum_{j=1}^N \psi'(|u_i^k - f_j|^2)}. \quad (2)$$

In this paper we focus on the bilateral filter [8] case: the sample f_j of the initial image f is replaced by the iterative estimation u_j^k . While the equation (2) involves a global estimate, it can be more consistent to take into account a local neighborhood. A weighting function $w(\cdot)$ depending on the distance between the positions x_i and x_j respectively of the estimated sample and the reference sample can be introduced

$$u_i^{k+1} = \frac{\sum_{j=1}^N \psi'(|u_i^k - u_j^k|^2) w(|x_i - x_j|^2) u_j^k}{\sum_{j=1}^N \psi'(|u_i^k - u_j^k|^2) w(|x_i - x_j|^2)}. \quad (3)$$

Let us consider now the tensor field framework. Equation (3) has been extended [9] to the tensor field filtering case. Using capital letters U_i to denote tensors, the iterative solution becomes:

$$U_i^{k+1} = H^{-1} \left(\frac{\sum_{j=1}^N \psi'(d(U_i^k, U_j^k)^2) w(|x_i - x_j|^2) H(U_j^k)}{\sum_{j=1}^N \psi'(d(U_i^k, U_j^k)^2) w(|x_i - x_j|^2)} \right) \quad (4)$$

where $H(\cdot)$ stands for a transformation function and $d(\cdot, \cdot)$ denotes a dissimilarity measure between two tensors.

A trivial choice for the function H is the identity

$$H(A) = A. \quad (5)$$

It is also well-suited to perform the Log-Euclidean transformation due to the specific geometry of tensor manifold:

$$H(A) = \log(A), \quad (6)$$

which ensures the symmetric definite positive property of the resulting matrix in equation (4), i.e. U_i .

The first dissimilarity measure dealing with matrix is the Frobenius norm

$$d_F(A, B) = \|A - B\|_F, \quad (7)$$

where $\|M\|_F = \sqrt{\text{trace}(M^T M)}$.

Taking into account the topology of symmetric positive definite matrices, Pennec et al [4] proposed to use a Riemannian metric known as Log-Euclidean metric to define a distance adapted to the tensor manifold

$$d_{LE}(A, B) = \|\log(A) - \log(B)\|_F, \quad (8)$$

where \log is the matrix logarithm. Some distances specifically developed for DT-MRI are also detailed by Dryden et al.[10].

Because of its edge preservation properties, the error function ψ which is considered in this paper is the Perona-Malik penalizer

$$\psi(d^2) = \lambda^2 \log \left(1 + \frac{d^2}{\lambda^2} \right), \quad (9)$$

where λ is a barrier parameter. The derivative ψ' of ψ used in equation (4) is given by

$$\psi'(d^2) = \frac{1}{1 + \frac{d^2}{\lambda^2}}. \quad (10)$$

The weight function $w(\cdot)$ defines the form of the neighborhood integration. Several choices are possible such as a uniform square, a Gaussian, a unit disk, etc.

3. STRUCTURE TENSOR

The structure tensor T_σ is defined as the covariance matrix of the first partial derivatives of I :

$$T_\sigma = (\nabla I \nabla I^t)^* G_\sigma, \quad (11)$$

where t and $*$ denotes respectively the transposition operator and the convolution operator, ∇I is the gradient of I and G_σ stands for a 2-D Gaussian averaging window of standard deviation σ . The choice of σ is crucial to getting relevant local image analysis. The higher standard deviation is, the smoother the ST is. On the contrary, a low standard deviation ensures an accurate analysis but with high sensitivity to noise.

Let T be a tensor. Its eigen decomposition is written as

$$T = P D P^{-1} \quad (12)$$

and can be developed as follows:

$$T = \begin{bmatrix} V_x^1 & V_x^2 \\ V_y^1 & V_y^2 \end{bmatrix} \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \begin{bmatrix} V_x^1 & V_y^1 \\ V_x^2 & V_y^2 \end{bmatrix}, \quad (13)$$

where $V^1 = [V_x^1 \ V_y^1]^T$ and $V^2 = [V_x^2 \ V_y^2]^T$ are the eigenvectors respectively associated with the eigenvalues β_1 and β_2 in decreasing order.

A 2-D tensor can be considered as an ellipse with a principal orientation and a shape factor. The orientation θ of the tensor T is determined by the eigenvector associated with the highest eigenvalue

$$\theta(T) = \tan^{-1} \left(\frac{V_y^1}{V_x^1} \right), \quad (14)$$

when the shape factor S is defined as:

$$S(T) = \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}. \quad (15)$$

As mentioned by the authors, the approaches developed by Pennec et al within the DT-MRI data do not yield satisfactory results for the ST [4]. The most likely explanation lies in the particular geometry of the ST in the case of many images such as directional textures: unlike DT-MRI, the major part of the tensor field contains very thin tensors, that is to say the first eigenvalue is much higher than the second one. In order to make DT-MRI dedicated works useable, we propose to reinforce the shape factor, i.e. to give more weight to the second eigenvalue, without changing the orientation through a nonlinear transformation of the initial ST expressed by

$$T_p = P D^p P^{-1} \quad (16)$$

with $p < 1$. Figure 1 exhibits results of such transformation



Figure 1 – Example of tensor transformation. From left to right, $p=1, 1/2, 1/3, 1/5$.

4. DISSIMILARITY MEASURES FOR STRUCTURE TENSOR

As far as we know, no distances have been specifically developed for ST. We propose to build dedicated dissimilarity measure by considering geometric properties.

The first geometric feature that distinguishes two tensors A and B is the difference of orientation φ . So a dissimilarity measure in orientation is obtained by a normalized cross product

$$d_\varphi(A, B) = |\sin(\varphi)| = \frac{\|V^{1,A} \wedge V^{1,B}\|}{\|V^{1,A}\| \|V^{1,B}\|} \quad (17)$$

where $V^{1,A}$ and $V^{1,B}$ denotes respectively the eigenvectors associated with the largest eigenvalue of A and B and \wedge the cross product of two vectors.

The angle φ is considered as the difference of orientation of eigenvectors $V^{1,A}$ and $V^{1,B}$ taking into account a $\pm\pi$ ambiguity in phase angle. Confidence in the orientation of an eigenvector is directly linked to the shape factor of equation (15). Higher the shape factor is, the more confident the orientation of the tensor is. Therefore we propose to weight the measure d_φ by the lowest shape factor of A and B so that the measure decreases as the orientation of one of the two tensors is uncertain. This new dissimilarity measure d_o is defined as

$$d_o(A, B) = |\sin(\varphi)| \min(S(A), S(B)). \quad (18)$$

A second geometric feature of a tensor is naturally shape factor. Let us define another measure d_s between tensors only based on a shape difference by computing the ratio of intrinsic shape factors

$$d_s(A, B) = \max\left(\frac{S(A)}{S(B)}, \frac{S(B)}{S(A)}\right). \quad (19)$$

This measure is an indicator that ranges from 1 (identical shapes) to infinity.

By combining the measures d_o and d_s , we define the Shape-Orientation (SO) distance:

$$d_{so}(A, B) = d_o(A, B)^q d_s(A, B)^{1-q} \quad (20)$$

where q is a parameter that varies from 0 to 1 and can adjust the weight relative of distances in orientation or in shape. Because the shape measure is higher than 1, it is clear that the SO measure is mainly dependant of the orientation measure. Moreover, in the case of a shape factor equal to zero, i.e. a circular tensor, the SO dissimilarity measure value is undetermined and must be set to zero.

Behavior of the SO measure is illustrated in Figure 2: we set a reference tensor A characterized by a null orientation $\theta(A)=0$ and a shape factor $S(A)=1/3$ when the tensor B is characterized by a variable orientation $\theta(B) \in [0, \pi/2]$ and a variable shape factor $S(B) \in [0, 1]$.

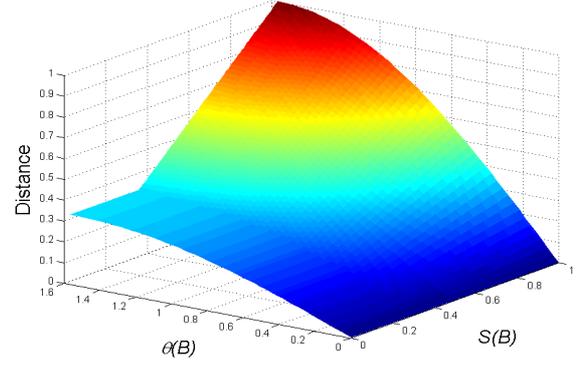


Figure 2 - Measure response from a tensor A , with $\theta(A)=0$ and $S(A)=1/3$, compared to a tensor B with $\theta(B) \in [0, \pi/2]$ and $S(B) \in [0, 1]$.

5. RESULTS

A comparative study of methods is carried out on real directional textured image in order to show the capability to enforce the saliency of the structural components highlighted in the tensor field. We conduct experiments on seismic data which are challenging data due to the fact that seismic imaging exhibits very noisy data with poorly sampling in terms of geometric structures.

Acquisition of reflection seismic data aims to provide a seismic image of acoustic impedance interfaces. These interfaces or reflectors are assumed to follow lithologic boundaries and as a consequence a seismic image can be considered as an image of subsurface geological units and structures. Thus, the goal of seismic interpretation is to recognize plausible geological patterns in seismic images. The identification of structures is critically important to oil and gas exploration activities. The structural complexity of seismic field imposes to increase continuously the relevance of algorithms used to process data for structural interpretation. These techniques include dip and azimuth estimation for delineation of fault patterns, and fault slices to evaluate juxtaposition and fault seal which can be obtained by regularization of the ST field.

We perform a comparison on real data shown in Figure 3. The sample image has been divided coarsely in three geological areas delimited by fault crossing. Illustrating the regularization impact is provided by interpreting data on three Regions Of Interested (ROI) which exhibit horizon ends and noisy patches.

The initial tensor field of the image is computed with $\sigma=1$, i.e. a 7×7 Gaussian window. Moreover, the tensor enhancing parameter p is set to $1/3$. In all experiments, the following bilateral filter parameters are fixed:

- The mapping $H(\cdot)$ considered is the Log-Euclidean.
- The weight function $w(\cdot)$ is a 3×3 square.

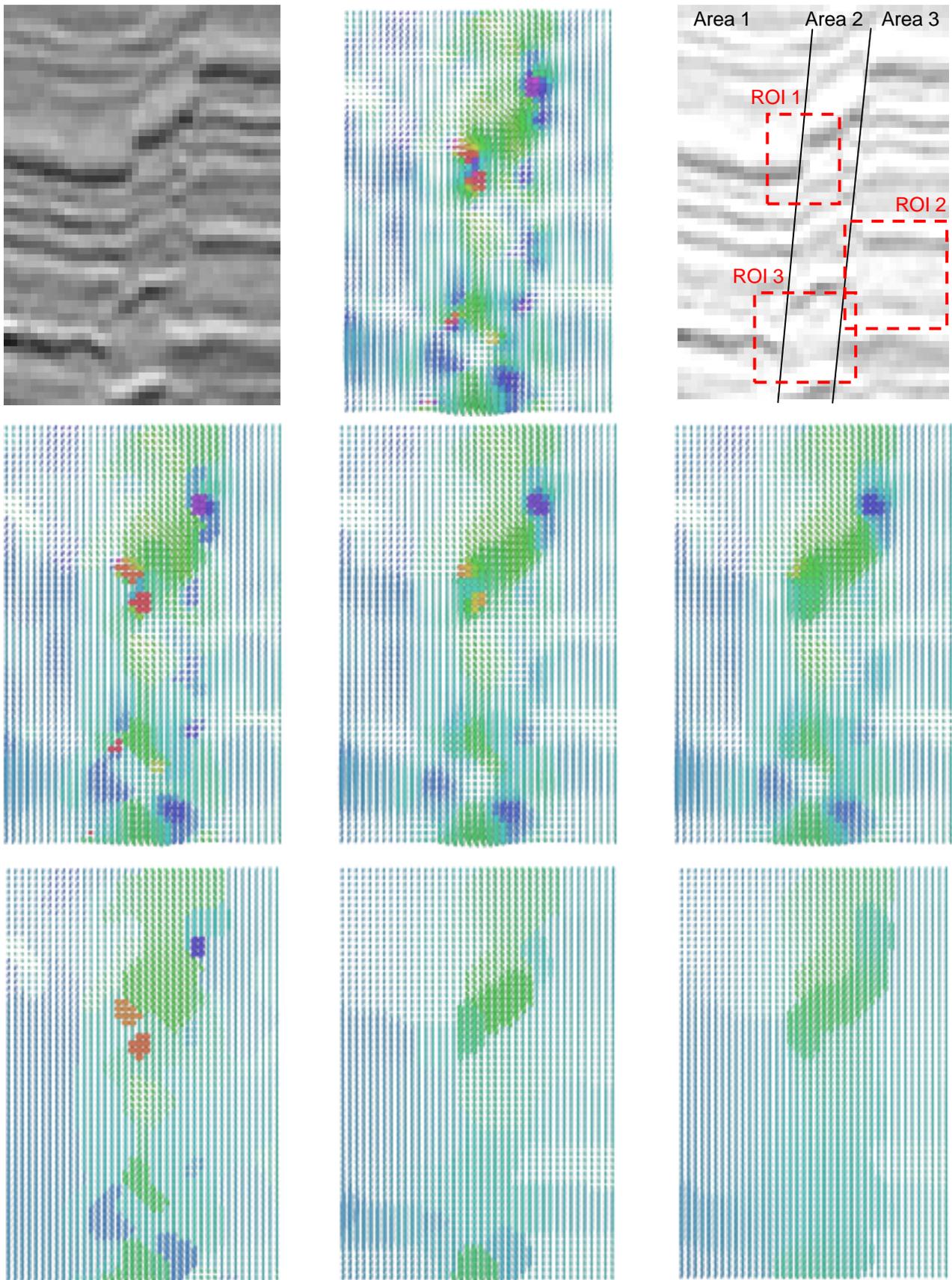


Figure 3 - **Up Row, Left:** Original Image, **Middle:** Original ST field. **Right:** Scheme of areas and ROI in the seismic image.
Middle Row, Left to Right: Filtering results after 10 iterations with SO, Frobenius, Log-Euclidean distance.
Bottom Row, Left to Right: Filtering results after 100 iterations with SO, Frobenius, Log-Euclidean distance.

- The error function is the Perona-Malik penalizer of equation (9).

The Shape-Orientation, the Log-Euclidean and the Frobenius distances are compared.

The choice of the penalizer parameter λ is crucial for the regularization performance. A low value will result in an unchanged tensor field whereas a high value will completely smooth the data and provide a blurry tensor field. Moreover, according to the used distance, the value of λ can be completely different, which makes the comparison not trivial. A set of values has been chosen according to each distance histogram (Figure 4). Indeed, λ values have been selected for each method, by observing equivalent discontinuities response values on the barrier images resulting from the error function $\psi'(\cdot)$. Thus, a comparative table of filtered tensor field is shown in Figure 3 where rows 2 and 3 correspond respectively to 10 and 100 iterations.

In ROI 1 and ROI3, we observe that the fault information, i.e. horizon ends, has been completely removed by the Log-Euclidean and Frobenius distances whereas the SO dissimilarity measure preserves them. Moreover, with the SO measure, the ROI 2 after filtering exhibits more homogenous content and have well defined boundaries. Because the SO measure does not take into account the tensor energy, the resulting tensor field does not exhibit energy discontinuities like in the Frobenius and Log-Euclidean cases, and in low energy areas, the SO measure provides more accurate orientation.

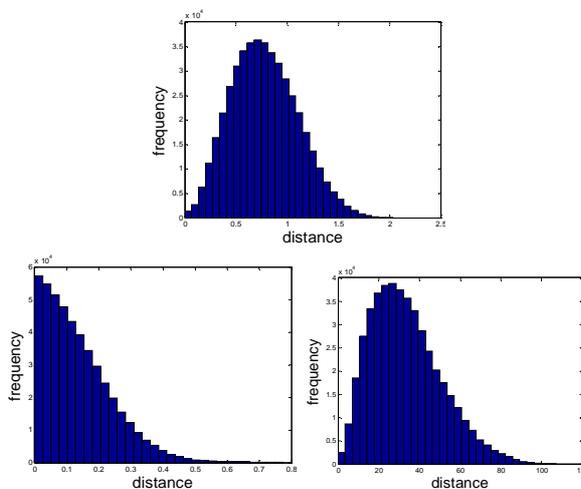


Figure 4 – Distance Histograms. **Up:** Log-Euclidean Distance. **Bottom Left:** SO dissimilarity measure. **Bottom Right:** Frobenius Distance

6. CONCLUSION

Because early works on tensor regularization do not deal great with structure tensor, we proposed two improvements: a shape reinforcement and a geometric based dissimilarity measure between tensors called Shape-Orientation dissimilarity measure. Applied in a structure tensor field smoothing process within bilateral filter, the obtained results proved their benefits for seismic images structure analysis. Future

works will concern extension of other classical filters, like anisotropic diffusion filter, to structure tensor case. The interest of our proposition for other applications, such as fingerprint recognition, corner detection or optical flow, will be investigated.

REFERENCES

- [1] P. Mrázek, J. Weickert, A. Bruhn, “On robust estimation and smoothing with spatial and tonal kernels”. In R. Klette, R. Kozera, L. Noakes, J. Weickert (Eds.): *Geometric Properties from Incomplete Data*, 335-352, Springer, Dordrecht, 2006.
- [2] C. Chefd’hotel, D. Tschumperlé, R. Deriche, and O. Faugeras, “Regularizing flows for constrained matrix-valued images”. *J. Math. Im. Vis.*, 20(1-2):147–162, 2004.
- [3] C. A. Castaño-Moraga, C. Lenglet, R. Deriche, J. Ruiz-Alzola, “A Riemannian approach to anisotropic filtering of tensor fields.” *Signal Processing* 87(2): 263-276, 2007.
- [4] P. Fillard, V. Arsigny, N. Ayache, X. Pennec: “A Riemannian framework for the processing of tensor-valued images”. In: Fogh Olsen, O., Florack, L.M.J., Kuijper, A. (eds.) *DSSCV 2005*. LNCS, vol. 3753, pp. 112–123. Springer, Heidelberg, 2005.
- [5] V. Arsigny, P. Fillard, X. Pennec, N. Ayache, “Log-Euclidean Metrics for Fast and Simple Calculus on Diffusion Tensors.” *Magnetic Resonance in Medicine*, 56(2):411-421, 2006.
- [6] J. Bigun and G. Granlund "Optimal Orientation Detection of Linear Symmetry". First int. *Conf. on Computer Vision, ICCV*, (London): 433-438, Piscataway: IEEE Computer Society Press, Piscataway, 1987.
- [7] H. Knutsson, “Representing local structure using tensors,” in *The 6th Scandinavian Conference on Image Analysis*, Oulu, Finland, pp. 244–251,1989.
- [8] R.L. Stevenson, B.E. Schmitz, E.J. Delp. “Discontinuity preserving regularization of inverse visual problems”. *IEEE Transactions on Systems, Man and Cybernetics*. 24:455-469, 1994.
- [9] L. Pizarro, B. Burgeth, S. Didas, J. Weickert, “A generic neighbourhood filtering framework for matrix” fields. In D. Forsyth, P. Torr, A. Zisserman (Eds.): *Computer Vision – ECCV 2008. Lecture Notes in Computer Science*, Vol. 5304, 521-532. Springer, Berlin, 2008.
- [10] I.L. Dryden, A. Koloydenkon D. Zhou, “Non-Euclidean statistics for covariance matrices, with applications to diffusion tensor imaging” *Annals of Applied Statistics*, volume 3, Number 3, 1102-1123. 2009.