IMPROVING THE PERFORMANCE OF THE SEMI-BLIND CROSS-RELATION-BASED CHANNEL ESTIMATION METHOD

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ABSTRACT

As verified experimentally in literature, the performance of the semi-blind cross-relation-based channel estimation may suffer from significant degradation when the involved channels are ill-conditioned. In this paper, we investigate this degradation and we introduce simple conditions for the training part in order to mitigate the problem. The suggested training design is based on the minimization of the mean square error of the estimation in a high SNR regime. The attained performance is studied through analytical arguments and verified via extensive simulations.

1. INTRODUCTION

An interesting problem in digital communications is the blind and semiblind identification of finite impulse response channels ([1], [2]). The methods that have been proposed for the blind case, are based only on the received signals and, in some cases, on statistical assumptions about the input and the channels. The so-called semi-blind methods originate from pure blind methods which are properly extended so as to incorporate a training part. This knowledge makes them more robust with respect to problems that are frequently encountered in purely blind methods, such as over/under modeling and existence of common roots ([3]).

Many of the methods that have been proposed in literature, formulate the problem using a multichannel model as in Fig. 1. One of the seminal works was [4], where a blind identification method was suggested based on second order statistics and the so-called Cross-Relation (CR) criterion. Later it was proved that, for the two sub-channels case, the CR criterion is equivalent to the subspace method [5]. Over the past 15 years, a number of results appeared in literature concerning either the performance or the algorithmic aspects of the CR method. Thus, in [6] asymptotic bounds for the normalized mean squared error (MSE) were derived, while in [7] approximate MSE expressions using perturbation theory and the Cramer-Rao bound were suggested. In [8], the Karhunen-Loeve expansion was used to improve the performance of the method. Recently, a new blind CR based algorithm utilizing orthogonal frequency division multiplexing has been developed in ([9]). Semi-blind versions of the CR-based blind method were presented in several papers (see [10] and the references therein). In [11], an efficient algorithm for semi-blind channel estimation was proposed utilizing a parametric model for the channel. Finally, in [3], identifiability conditions for both the pure blind and the semi-blind case were presented.

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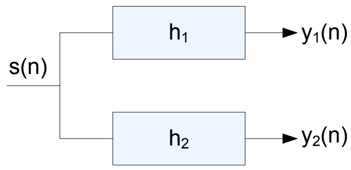


Figure 1: A single-input, two-output system

If the identifiability conditions hold, the performance of the CR method depends on whether or not the channels are ill-conditioned. In [6], performance bounds were derived that are related to the condition number of the involved channel matrix. In [7], it was shown that the MSE may degrade considerably if the sub-channels have relatively close zeros.

In this paper, we investigate a similar degradation that was observed for the semi-blind case. It was shown in [12], that the degradation of the channel estimation performance in cases of ill-conditioned channels depends on the particular structure of the multichannel model. Here we further investigate this issue and we suggest a simple MSE-based training design methodology to mitigate the observed degradation and make the CR method to perform acceptably even in ill-conditioned cases.

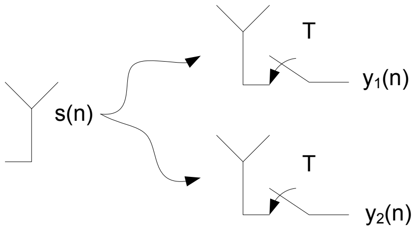
In the following, bold capital and small letters denote matrices and vectors, respectively. $\boldsymbol{A}^T$, $\boldsymbol{A}^*$ and $\boldsymbol{A}^H$ denote transposition, complex conjugation and conjugate transposition of $\boldsymbol{A}$. $\boldsymbol{I}_N$ is the identity matrix of size N, $Tr\{\boldsymbol{A}\}$ is the trace of $\boldsymbol{A}$, $\|.\|$ is the 2-norm of a vector, $\mathscr{E}\{.\}$ denotes expectation over the noise samples and $*$ denotes the operation of convolution.

In Section 2, a description of the problem and the aforementioned channel degradation is provided. In Section 3, the system model is presented and in Section 4 the equations for the semi-blind cross relation estimation are given. Section 5 describes the procedure for the training design. In Section 6, experimental results are presented and, finally, Section 7 concludes the paper.

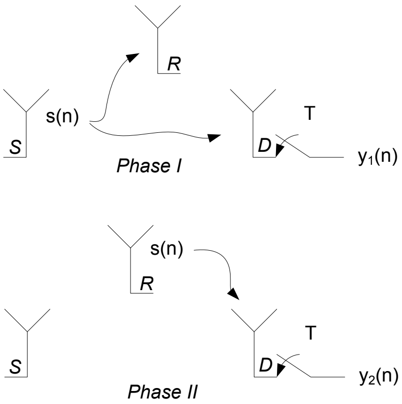
2. PROBLEM DESCRIPTION

In this paper, we are interested in communication systems that can fit into a multichannel model with a single input and two outputs (Fig. 1). Such a model is important if a (semi-)blind method, utilizing second-order statistics, is to be used and, actually, it is capable of describing many of the contemporary communication systems.

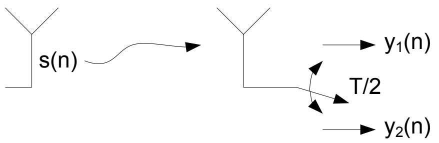
There are two distinct cases related to this model ([3]). According to the first one, the transmitted signal passes



(a) A two-antenna receiver (at symbol rate).



(b) A two-phase cooperative system (at symbol rate).



(c) A single-antenna receiver (at double the symbol rate).

Figure 2: Examples of communication systems that can fit into the multichannel model of Fig. 1.

through two (or more) different paths in space (so-called "space" oversampling case). Some examples of the first case are provided in Figs. 2(a), 2(b). Thus, for instance, the receiver of Fig. 2(a) has two antennas and fits naturally to the multichannel model of Fig. 1. Another example comes from cooperative communications where a source node S sends information to a destination node D and a relay node R assists the transmission. In Fig. 2(b), S sends the signal $s(n)$ to D during the first phase and the relay forwards the same signal at the second phase. Obviously, the two received signals constitute the upper and lower branches, respectively, of the model of Fig. 1. Note that in the "space" oversampling case the received signal is time-sampled at a symbol rate.

According to the second case, a single-antenna receiver samples the received signal at a rate higher than the symbol rate (e.g. by a factor of 2 in Fig. 2(c)). In this case (so-called "time" oversampling case), the single output of the system is split into two data streams that correspond to the outputs of two different sub-channels to the same input signal $s(n)$.

It is pointed out that, in the first case (i.e. multi-antenna receivers and cooperative systems), the procedure of "time oversampling' can, also, be applied to each separate branch, treating them as single-input and single-output systems. In this case, the model of Fig. 1 is easily transformed to the mixed model of Fig. 3 where, for instance, h_{1i}'s ($i = 1,2$) are the sub-channels of h_1.

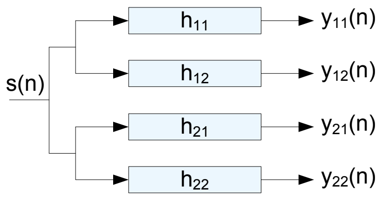


Figure 3: The multichannel model after "time" oversampling.

Is is important to emphasize that the above mentioned two cases, i.e. "space" and "time" oversampling, do not exhibit the same performance under the same conditions. This fact should be taken into account if both options are possible for a particular application. Let us see this point via a typical example. Let us assume that the involved sub-channels (either "space" or "time" sub-channels) can be modeled as multipath channels with distinct components, i.e. $h_i(t) = \sum_k a_{i,k} g(t - \tau_{i,k})$, $(i = 1,2)$, where $a_{i,k}$'s are independent random variables and $\tau_{i,k}$ is the delay introduced by the k^{th} path of the $i-th$ sub-channel. The $g(.)$ function is the combination of the transmit and receive filters (e.g. a raised cosine pulse).

In case of "space" oversampling, we have symbol rate sampling (i.e. $t = nT$) and therefore the sub-channels can easily be written in the form $\boldsymbol{h}_i = \boldsymbol{G}(\tau_i)\boldsymbol{a}_i$ and can be thought as uncorrelated if there is a sufficient antenna spacing. Matrix $\boldsymbol{G}(\tau_i)$ contains as columns delayed and scaled versions of the sampled $g(.)$ function.

On the other hand, in case of "time" oversampling at instances $t_1 = nT$ and $t_2 = nT + T/2$, the sub-channels $h_1(nT)$ and $h_2(nT + T/2)$ are created, respectively. These can also be written in matrix form as $\boldsymbol{h}_1 = \boldsymbol{G}(\tau_1)\boldsymbol{a}_1$ and $\boldsymbol{h}_2 = \boldsymbol{G}(\tau_2)\boldsymbol{a}_2$. Note however that now, since the sub-channels come from "time" oversampling of the same channel impulse response, we have $\tau_2 = \tau_1 + T/2$ and $\boldsymbol{a}_1 = \boldsymbol{a}_2$. Thus, the two sub-channels are correlated and have a covariance matrix equal to $\boldsymbol{G}(\tau_1)\mathscr{E}\{\boldsymbol{a}_1\boldsymbol{a}_1^H\}\boldsymbol{G}(\tau_2)^H$ (i.e., their correlation is mainly related to the form of the pulse shaping function $g(.)$).

For the purely blind case, it was shown in [6] that the performance is related to the condition number of the involved channels' matrix which, in turn, depends on the relation among the sub-channels. The more related the sub-channels are, the more degraded the performance is. For the semi-blind case, a similar dependence was observed experimentally for a cooperative communication system ([12]). Here, we describe a simple training design procedure based on the MSE at the high SNR regime in order to combat this degradation. Moreover, a closed-form expression for the MSE is derived which provides some intuition concerning different aspects of the performance of the semi-blind method.

3. SYSTEM MODEL

In this section, the multichannel models of Figs. 1, 3 will be described mathematically. Specifically, if the receiver samples the received signals at symbol rate (first case in Section 2), the input-output equations are

$$y_i(n) = \boldsymbol{h}_i^H \boldsymbol{s}(n) + w_i(n), \text{ with } i = 1,2. \quad (1)$$

Alternatively, if the receiver oversamples the input sig-

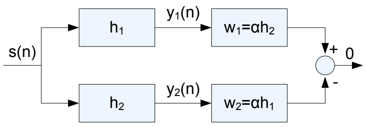


Figure 4: The blind CR criterion.

nals at a higher rate (e.g. by a factor of 2 - see second case in Section 2), the input-output equations are

$$y_{1i}(n) = \boldsymbol{h}_{1i}^H \boldsymbol{s}(n) + w_{1i}(n), \text{ with } i = 1,2, \tag{2}$$

$$y_{2i}(n) = \boldsymbol{h}_{2i}^H \boldsymbol{s}(n) + w_{2i}(n), \text{ with } i = 1,2. \tag{3}$$

In the above equations

$$\begin{aligned} \boldsymbol{h}_j^H &= [h_j^*(0), h_j^*(1) \dots h_j^*(L-1)], \\ \boldsymbol{s}(n) &= [s(n), s(n-1) \dots s(n-L+1)]^T \end{aligned}$$

where $j \in \{i, 1i, 2i\}$ and $n = 0, \dots, M+N-1$. The L and $w_j(n)$ denote the channel length and the noise samples, respectively. The latter are assumed independent and identically distributed zero mean complex Gaussian random variables with variance σ^2. The first M outputs, i.e. $n = 0 \dots M-1$, correspond to known input symbols (training part). The blind part consists of the remaining N outputs. Finally, the channels incorporate the transmit and receive filters as already explained.

4. SEMI-BLIND CR CHANNEL ESTIMATION

In the following, we focus on the input-output description of the multichannel model as given in Eq. (1). The respective derivation for Eqs. (2), (3) is straightforward and is omitted. We will start with the noise-free case (i.e. $w_i(n) = 0$ in Eq. (1)).

The blind CR criterion is shown in Fig. 4. Specifically, the criterion states that if $y_1 * w_1 = y_2 * w_2$, then $w_1 = ah_2$ and $w_2 = ah_1$, where a is a constant. In matrix form, this criterion can be written as

$$\begin{bmatrix} \boldsymbol{Y}_2 & -\boldsymbol{Y}_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \end{bmatrix} = \boldsymbol{0}, \tag{4}$$

where

$$\boldsymbol{Y}_k = \begin{bmatrix} y_k(M+L-1) & \dots & y_k(M) \\ y_k(M+L) & \dots & y_k(M+1) \\ \dots & \dots & \dots \\ y_k(M+N-1) & \dots & y_k(M+N-L) \end{bmatrix}, \quad k = 1,2$$

is a Toeplitz matrix. It is easily verified that in $\boldsymbol{Y}_k$ the N outputs $y_k(n)$, with $n = M, \dots, N+M-1$, are used for the blind part. The training part, i.e. outputs $y_k(n)$ with $n = 0, \dots, M-1$, can be written as

$$\boldsymbol{S}_t \boldsymbol{h}_k = \boldsymbol{z}_k, \quad k = 1,2, \tag{5}$$

where

$$\boldsymbol{S}_t = \begin{bmatrix} s(0) & \dots & s(-L+1) \\ s(1) & \dots & s(-L+2) \\ \dots & \dots & \dots \\ s(M-1) & \dots & s(M-L) \end{bmatrix}$$

is a Toeplitz matrix constructed by $M+L-1$ known training symbols and $\boldsymbol{z}_k = [y_k(0), \dots, y_k(M-1)]^T$.

Finally, Eqs. (4), (5) are merged together in a single matrix form and a linear system of equations with the desired channels as unknowns is derived. The corresponding relation is the semi-blind CR based channel estimation ([11]) and is given as

$$\overbrace{\begin{bmatrix} \boldsymbol{Y}_2 & -\boldsymbol{Y}_1 \\ \boldsymbol{S}_t & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_t \end{bmatrix}}^{\boldsymbol{Y}} \overbrace{\begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \end{bmatrix}}^{\boldsymbol{h}} = \overbrace{\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \end{bmatrix}}^{\boldsymbol{z}}. \tag{6}$$

In Eq. (6), if noise is added, the relation is transformed into

$$\tilde{\boldsymbol{Y}} \tilde{\boldsymbol{h}} = \tilde{\boldsymbol{z}}, \tag{7}$$

where

$$\begin{aligned} \tilde{\boldsymbol{Y}} &= \boldsymbol{Y} + \boldsymbol{W}, \\ \tilde{\boldsymbol{h}} &= \boldsymbol{h} + \delta \boldsymbol{h}, \\ \tilde{\boldsymbol{z}} &= \boldsymbol{z} + \boldsymbol{w} \end{aligned}$$

and

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{W}_2 & -\boldsymbol{W}_1 \\ \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{w} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{w}_1 \\ \boldsymbol{w}_2 \end{bmatrix}.$$

The matrices $\boldsymbol{W}_k$ and the vectors $\boldsymbol{w}_k$ are defined similarly to $\boldsymbol{Y}_k$ and $\boldsymbol{z}_k$, respectively, by replacing $y_k(n)$ with $w_k(n)$. Finally, $\delta \boldsymbol{h} = [\delta \boldsymbol{h}_1^T \quad \delta \boldsymbol{h}_2^T]^T$ is the estimation error.

The channel estimator that is used, is based on Eq. (7) and is given by

$$\hat{\boldsymbol{h}} = \tilde{\boldsymbol{Y}}^{\dagger} \tilde{\boldsymbol{z}}, \tag{8}$$

where $\boldsymbol{A}^{\dagger} = (\boldsymbol{A}^H \boldsymbol{A})^{-1} \boldsymbol{A}^H$ is the pseudo-inverse of $\boldsymbol{A}$. Finally, using Eq. (6) in Eq. (7), the equation

$$\delta \boldsymbol{h} = \tilde{\boldsymbol{Y}}^{\dagger} (\boldsymbol{w} - \boldsymbol{W} \boldsymbol{h}) \tag{9}$$

is derived for the error $\delta \boldsymbol{h}$.

5. PERFORMANCE ENHANCEMENT

The performance of the estimator of Eq. (8) is related to the condition of the $\boldsymbol{Y}$ matrix which, in turn, is related to the desired channels involved in matrix $[\boldsymbol{Y}_2 \ -\boldsymbol{Y}_1]$. In this section, we will follow a simple training design procedure in order to enhance the performance of the estimator. From the experiments in the next section, we will observe that this training design actually makes the estimator independent of the relation between the sub-channels involved in any of the two cases that were described in Section 2 (i.e., "space" and "time" oversampling cases).

The proposed training design is based on the minimization of the $MSE = \mathscr{E}\{\|\delta \boldsymbol{h}\|^2\}$ at a high SNR regime. Specifically, following similar arguments as in [13], we write Eq. (9) as

$$\begin{aligned} \delta \boldsymbol{h} &= \boldsymbol{Y}^{\dagger} (\boldsymbol{w} - \boldsymbol{W} \boldsymbol{h}) \\ &\quad + (\tilde{\boldsymbol{Y}}^{\dagger} - \boldsymbol{Y}^{\dagger})(\boldsymbol{w} - \boldsymbol{W} \boldsymbol{h}) \\ &\overset{\text{a}}{\approx} \boldsymbol{Y}^{\dagger} (\boldsymbol{w} - \boldsymbol{W} \boldsymbol{h}). \end{aligned} \tag{10}$$

The approximation in Eq. (10) is reasonable at a high SNR regime, where $\boldsymbol{W}$ and $\boldsymbol{w}$ contain relatively small elements.

The MSE, using the approximation of Eq. (10), becomes

$$\begin{aligned} MSE &= \mathscr{E}\{\|\delta \boldsymbol{h}\|^2\} \\ &= Tr\{\boldsymbol{Y}^{\dagger}\boldsymbol{R}\boldsymbol{Y}^{\dagger H}\}, \end{aligned} \tag{11}$$

where $\boldsymbol{R} = \mathscr{E}\{(\boldsymbol{w}-\boldsymbol{W}\boldsymbol{h})(\boldsymbol{w}-\boldsymbol{W}\boldsymbol{h})^H\}$. The matrix $\boldsymbol{R}$ can be written as

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_2 \end{bmatrix}, \tag{12}$$

where $\boldsymbol{R}_2 = \sigma^2 \boldsymbol{I}_{2M}$, while concerning matrix $\boldsymbol{R}_1$, it can be shown that it is a symmetric and diagonally dominant matrix with its diagonal elements equal to $\sigma^2\|\boldsymbol{h}\|^2$. To simplify the subsequent analysis, we approximate $\boldsymbol{R}_1$ as $\boldsymbol{R}_1 \approx \sigma^2\|\boldsymbol{h}\|^2\boldsymbol{I}_{N-L+1}$. This choice is fully justified by the experiments in Section 6. We also define

$$\boldsymbol{Y}_b = [\ \boldsymbol{Y}_2 \quad -\boldsymbol{Y}_1\ ], \quad \boldsymbol{S}_b = \begin{bmatrix} \boldsymbol{S}_t & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_t \end{bmatrix}. \tag{13}$$

Using Eqs. (12) and (13), the MSE in Eq. (11) becomes

$$\begin{aligned} MSE &= Tr\{(\boldsymbol{Y}^H\boldsymbol{Y})^{-1}\boldsymbol{Y}^H\boldsymbol{R}\boldsymbol{Y}(\boldsymbol{Y}^H\boldsymbol{Y})^{-1}\} \\ &= Tr\{(\boldsymbol{Y}_b^H\boldsymbol{Y}_b+\boldsymbol{S}_b^H\boldsymbol{S}_b)^{-1} \\ &\quad \cdot(\sigma^2\|\boldsymbol{h}\|^2\boldsymbol{Y}_b^H\boldsymbol{Y}_b+\sigma^2\boldsymbol{S}_b^H\boldsymbol{S}_b) \\ &\quad \cdot(\boldsymbol{Y}_b^H\boldsymbol{Y}_b+\boldsymbol{S}_b^H\boldsymbol{S}_b)^{-1}\}. \end{aligned} \tag{14}$$

Using $\boldsymbol{Y}_b = \boldsymbol{U}\Sigma\boldsymbol{V}^H$, i.e. the singular value decomposition of $\boldsymbol{Y}_b$, and the identity $Tr\{\boldsymbol{A}\boldsymbol{B}\boldsymbol{A}^{-1}\} = Tr\{\boldsymbol{B}\}$ ([14]), the MSE in Eq. (14) becomes

$$\begin{aligned} MSE &= Tr\{(\Sigma^2+\boldsymbol{V}^H\boldsymbol{S}_b^H\boldsymbol{S}_b\boldsymbol{V})^{-1} \\ &\quad \cdot(\sigma^2\|\boldsymbol{h}\|^2\Sigma^2+\sigma^2\boldsymbol{V}^H\boldsymbol{S}_b^H\boldsymbol{S}_b\boldsymbol{V}) \\ &\quad \cdot(\Sigma^2+\boldsymbol{V}^H\boldsymbol{s}_b^H\boldsymbol{s}_b\boldsymbol{V})^{-1}\} \quad (15) \\ &= Tr\{\boldsymbol{Q}^{-1}\boldsymbol{P}\boldsymbol{Q}^{-1}\}, \quad (16) \end{aligned}$$

where matrices $\boldsymbol{Q}$ and $\boldsymbol{P}$ are defined by the respective quantities of Eq. (15).

In order to minimize the MSE, we use the bound ([15])

$$Tr\{\boldsymbol{A}^{-1}\} \geq \sum_i [\boldsymbol{A}]_{ii}^{-1}, \tag{17}$$

which holds if the matrix $\boldsymbol{A}$ is positive definite. $[\boldsymbol{A}]_{nm}$ stands for the element of $\boldsymbol{A}$ in position (n,m). Now, the use of the above bound in Eq. (16) results in

$$\begin{aligned} MSE &= Tr\{(\boldsymbol{Q}\boldsymbol{P}^{-1}\boldsymbol{Q})^{-1}\} \\ &\geq \sum_{i=1}^{2L}[\boldsymbol{Q}\boldsymbol{P}^{-1}\boldsymbol{Q}]_{ii}^{-1}. \end{aligned} \tag{18}$$

The equality holds when the matrix $\boldsymbol{Q}\boldsymbol{P}^{-1}\boldsymbol{Q}$ is diagonal. By a simple inspection of Eq. (15), we can see that the matrix is diagonal when the condition $\boldsymbol{S}_b^H\boldsymbol{S}_b = \beta\boldsymbol{I}_{2L}$ holds true or, equivalently, when $\boldsymbol{S}_t^H\boldsymbol{S}_t = \beta\boldsymbol{I}_L$ (see Eq. (13)). In order for this to be possible, $M \geq L$ should be satisfied. Here, we use the constant amplitude zero autocorrelation sequences that are described in [16] in oder to construct the training matrix $\boldsymbol{S}_t$. We, also, assume $M = L$, i.e. the smallest possible value. Finally, because the symbols that are used for training have magnitude equal to one, the constant β is equal to M because M symbols participate in the inner products of $\boldsymbol{S}_t^H\boldsymbol{S}_t$.

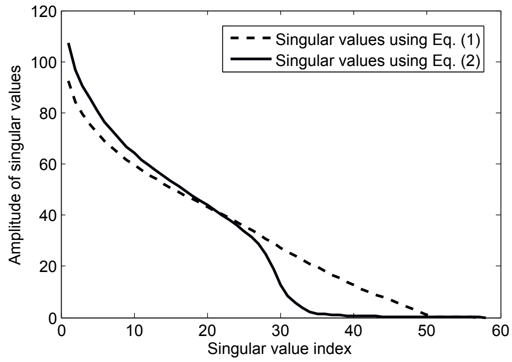


Figure 5: The singular values of matrix $\boldsymbol{Y}_b$ in decreasing order for the models described by Eqs. (1) and (2), respectively.

Using the condition $\boldsymbol{S}_b^H\boldsymbol{S}_b = M\boldsymbol{I}_{2L}$ in Eq. (15), the following analytical expression is derived.

$$MSE = \sigma^2 \sum_{i=1}^{2L} \frac{\|\boldsymbol{h}\|^2\sigma_i^2 + M}{(\sigma_i^2+M)^2}, \tag{19}$$

where the σ_i's are the singular values of $\boldsymbol{Y}_b$. The main observation in Eq. (19) is that the denominator grows faster with M as opposed to the numerator and, hence, the MSE tends to zero as M increases.

If, for intuitive reasons, we assume that $\|\boldsymbol{h}\|^2 = 1$ then Eq. (19) is simplified to

$$MSE = \sigma^2 \sum_{i=1}^{2L} \frac{1}{\sigma_i^2+M}. \tag{20}$$

Eq. (20) provides a more intuitive interpretation regarding the impact of the training design. In a way, the training part increases the magnitude of the singular values and assuming (reasonably) that $M \geq 1$, then all denominators are greater than 1. This mitigates the degradation that is caused by the singular values that are less than one. In Fig. 5, the values of the σ_i's (averaged over a number of realizations) of the matrix $\boldsymbol{Y}_b$ created by Eqs. (1) and by Eqs. (2) are plotted in decreasing order. As it can be seen, the training is expected to be more beneficial to the channel estimation method based on Eqs. (2), i.e. the model with "time" oversampling because there are more singular values near zero.

6. SIMULATION RESULTS

In this section, we present simulation results obtained for the channel estimator of Eq. (8). The channel estimator was based on the models of *(a)* Eqs. (1) ("space" oversampling) and *(b)* Eqs. (2) ("time" oversampling). In either case, the training sequence was constructed using *(a)* the procedure of Section 5 and *(b)* the training symbols were selected randomly from a quadrature phase shift keying (QPSK) constellation.

The multipath model $h_i(t) = \sum_k a_k g(t-\tau_{i,k}), (i=1,2)$, was used for the channels as already described in Section 2. The a_k's were assumed to be independent and identically

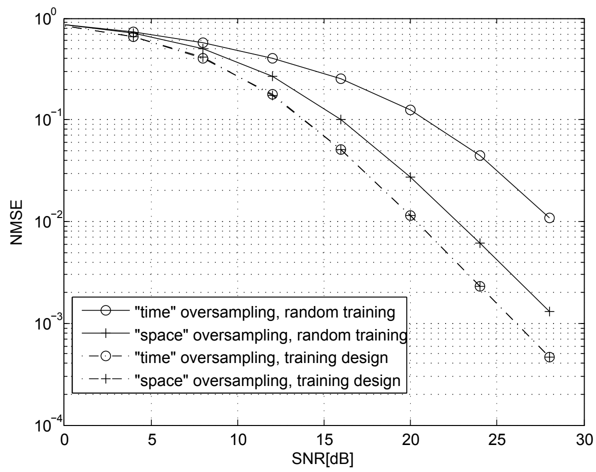


Figure 6: NMSE curves.

distributed zero mean complex Gaussian random variables with variance 2. The function $g(.)$ was the combination of the transmit and receive filters. Here, we used a raised cosine pulse extending to two symbols at each side and roll-off factor equal to 0.3. Both channels $h_i(t)$, with $i = 1,2$, are assumed to have four paths and the corresponding delays are assumed independent and identically distributed random variables following $U(0,4)$, where $U(a,b)$ denotes the uniform distribution in the interval $[a,b]$. The information symbols were drawn from a QPSK constellation. Finally, it was assumed that N=150 and M=L.

Fig. 6 shows the normalized MSE $\|\hat{\boldsymbol{h}} - \boldsymbol{h}\|^2/\|\boldsymbol{h}\|^2$ curves versus SNR for the two estimators with and without training design. The results are in accordance with the conclusions drawn in Section 5, namely, the estimator based on Eq. (1) ("space" oversampling) is better when random training is used because the involved channel matrix $\boldsymbol{Y}_b$ is better conditioned. Additionally, the estimator based on Eq. (2) ("time" oversampling) is the one that is mostly benefited from the training design and its performance is similar to the other, which implies that the dependence on the channels has been highly mitigated.

7. CONCLUSION

In this paper, a simple training design procedure is suggested in order to enhance the performance of the semi-blind CR channel estimation due to ill-conditioned channels. The main conclusions are two. First, if a system can be described by either Eqs. (1) or (2), then the first one is preferable because in this case (a) the channels are well-conditioned, and (b) no restrictions are imposed on the training sequence. Second, in a SISO system, where the use of Eqs. (2) is the only possible choice, the training sequence should be designed as proposed in Section 4 in order to overcome ill-conditioning.

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