A MULTIPLE NARROWBAND EMITTERS GEOLOCATION ALGORITHM BASED ON AOA ESTIMATIONS

Jonathan Bosse[1,2]*,Anne Ferréol*[1,2] *and Pascal Larzabal*[2]

[1] : THALES Communication 160 Boulevard de Valmy 92704 Colombes, France
[2] : SATIE, ENS Cachan, CNRS, Universud 61 avenue du président Wilson, 94230 Cachan, France
email: name@satie.ens-cachan.fr

ABSTRACT

The Problem of non-cooperative narrowband sources location is often solved by estimating many parameters, among them one can cite the angle of arrival (AOA), the time of arrival (TOA) or the frequency of arrival (FOA). To the best of our knowledge most of the techniques are using separately estimated location parameters on several stations. Since they do not take into account that the observed signals on the stations comes from the same emitters, they are suboptimal. They lead to ambiguïties especially in multi-emitter context. Based on the narrowband assumption on the whole sensor network, composed of the multiple base stations, the proposed method relies on a centralised processing by means of a stacked observation vector. As we simultaneously treat all the signals collected in several stations, the corresponding criterion treats all stations together and does not exhibit ambiguities in presence of multiple emitters. For the sake of simplicity, we will consider in this paper only two stations. After having described the implementation of our algorithm, simulations and Cramér-Rao Bounds illustrate the improvement of our method compared to a classical AOA estimation based algorithm performed on each station independently.

1. INTRODUCTION

AOA estimation has always been an intensive research theme. In Radio-communication the problem of geolocation thanks to an AOA estimation has been especially studied by the array processing community [1]. As far as we know, less efforts have been focused on direct non cooperative planar geolocation. In order to locate an emitter most of the current "classical" algorithms are estimating parameters that could be AOA, TOA or FOA using different methods on separated sensor network [2][3]. Since the emitters location is characterised by at least two parameters, these parameters have then to be associated for each emitter. Thus, on the one hand such methods lead to ambiguïties (in presence of muliple emitters) and on the other hand since the estimations are performed independently, the technique is suboptimal.

As far as we know less effort has been focused on the simultaneous estimation of the location parameters, taking into account that the signals received by all stations belong to the same emitters. Previous studies [4, 5] provide new insights on the location algorithm by means of a Direct Position Determining (DPD) in presence of multiple stations. In this article, focusing on the use of two multiple sensors stations, we propose a new non-copertaive multiple emitter geolocation algorithm, on the line-of-sight context, considering that the narrowband assumption can be performed on the whole sensor network. Our goal is to highlight the advantages of using an algorithm performing a simultaneous estimation of the location parameters by means of an extending stacked observation vector. In this article a practical implementation including a second order steepest descent algorithm is proposed. Finally, simulations and Cramer-rao Bounds (CRB) underline the advantage of our algorithm compared to a classical method.

2. MODEL OF THE SIGNAL

We focus on the problem of locating multiple emitters on two stations denoted A and B composed by N_A and N_B sensors, respectively. Let $\mathbf{x}_A(t)$ and $\mathbf{x}_B(t)$ denote the observation vectors collected on the two stations, and let us consider M narrowband transmitters emitting unkown signals denoted $s_m(t)$ ($1 \leq m \leq M$). Assuming classically that both stations receive narrowband signals, that is to say the emitter bandwith of the emitter times the time delay of propagation accross each base station is small, leads to

$$\mathbf{x_A}(t) = \sum_{m=1}^{M} \rho_{A,m}\mathbf{a}(\alpha_m)s_m(t) + \mathbf{n}_A(t), \quad (1)$$

$$\mathbf{x_B}(t) = \sum_{m=1}^{M} \rho_{B,m}\mathbf{b}(\beta_m)e^{-j2\pi f_0 \tau_m}s_m(t-\tau_m) + \mathbf{n}_B(t), \quad (2)$$

where $\mathbf{a}(.)$ and $\mathbf{b}(.)$ are the steering vectors (array responses) of the stations A and B, without loss of generality, chosen so as to possess the same norm. α_m and β_m denote the AOA of the mth emitter on station A and B, respectively. $\rho_{A,m}$ and $\rho_{B,m}$ denote unknown complex parameter standing for the channel effect. $\mathbf{n}_A(t)$ and $\mathbf{n}_B(t)$ are additive gaussian noises whose covariance matrix are $\sigma^2\mathbf{I_{N_A}}$ and $\sigma^2\mathbf{I_{N_B}}$, respectively, where $\mathbf{I_N}$ denotes the $N \times N$ identity matrix. τ_m includes the TDOA of the mth emitter between stations A and B and the synchronisation time error between both stations. f_0 stands for the carrier frequency.

Let us consider the following stacked observation vector

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x_A}(t) \\ \mathbf{x_B}(t) \end{bmatrix}, \quad (3)$$

and the stacked noise vector

$$\mathbf{n}(t) = \begin{bmatrix} \mathbf{n_A}(t) \\ \mathbf{n_B}(t) \end{bmatrix}. \quad (4)$$

Now, we assume the narrowband assumption on the whole sensor network, that is to say the product emitter bandwith times time delay of propagation across both base station is

© EURASIP, 2010 ISSN 2076-1465

considered as small. The following expression of $\mathbf{x_B}(t)$ is obtained

$$\mathbf{x_B}(t) = \sum_{m=1}^{M} \rho_{B,m}\mathbf{b}(\beta_m)e^{-j2\pi f_0 \tau_m}s_m(t) + \mathbf{n}_B(t). \quad (5)$$

Thus,

$$\mathbf{x}(t) = \sum_{m=1}^{M} \mathbf{u}_m s_m(t) + \mathbf{n}(t), \quad (6)$$

where

$$\mathbf{u}_m = \begin{bmatrix} \rho_{A,m}\mathbf{a}(\alpha_m) \\ \rho_{B,m}e^{j2\pi f_0 \tau_m}\mathbf{b}(\beta_m) \end{bmatrix}. \quad (7)$$

In order to define the following signal to noise ratio

$$SNR = 10log\left(\frac{E[s_m(t)s_m^H(t)]}{\sigma^2}\right), \quad (8)$$

for the mth source, we will consider the following normalized stacked steering vector :

$$\mathbf{u}(\alpha,\beta,\psi,\phi) = \frac{1}{\sqrt{1+\psi^2}}\begin{bmatrix} \mathbf{a}(\alpha) \\ \psi e^{j\phi}\mathbf{b}(\beta) \end{bmatrix}, \quad (9)$$

where ψ and ϕ are unknown deterministic real parameters. Since $\mathbf{a}(.)$ and $\mathbf{b}(.)$ have been chosen so as to have an equal norm, the new stacked steering vector $\mathbf{u}(.)$ has a constant norm. So it leads to

$$\mathbf{x}(t) = \sum_{m=1}^{M} \mathbf{u}(\alpha_m,\beta_m,\psi_m,\phi_m)s_m(t) + \mathbf{n}(t). \quad (10)$$

And finally a more compact expression is obtained

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\psi},\boldsymbol{\phi})\mathbf{s}(t) + \mathbf{n}(t), \quad (11)$$

where

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 & \ldots & \alpha_M \end{bmatrix}^T, \quad (12)$$

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_1 & \ldots & \beta_M \end{bmatrix}^T, \quad (13)$$

$$\boldsymbol{\psi} = \begin{bmatrix} \psi_1 & \ldots & \psi_M \end{bmatrix}^T, \quad (14)$$

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_1 & \ldots & \phi_M \end{bmatrix}^T, \quad (15)$$

$$\mathbf{A}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\psi},\boldsymbol{\phi}) = \begin{bmatrix} \mathbf{u}(\alpha_1,\beta_1,\psi_1,\phi_1) & \ldots & \mathbf{u}(\alpha_M,\beta_M,\psi_M,\phi_M) \end{bmatrix}, \quad (16)$$

$$\mathbf{s}(t) = \begin{bmatrix} s_1(t) & \ldots & s_M(t) \end{bmatrix}^T. \quad (17)$$

According to the equation (11) our goal is to perform the simultaneous estimation of $\{(\alpha_m,\beta_m), m \in [1,M]\}$ and to deduce $\{(x_m,y_m), m \in [1,M]\}$ the location of the emitters.

3. A NEW LOCATION ALGORITHM

Considering the model (11) a MUSIC [6] algorithm approach is proposed. Let us first denote :

$$\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)], \quad (18)$$

$$\mathbf{R_s} = E[\mathbf{s}(t)\mathbf{s}^H(t)], \quad (19)$$

we have then

$$\mathbf{R} = \mathbf{A}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\psi},\boldsymbol{\phi})\mathbf{R_s}\mathbf{A}^H(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\psi},\boldsymbol{\phi}) + \sigma^2\mathbf{I}, \quad (20)$$

So we propose to minimize the following criterion :

$$C(\alpha,\beta,\psi,\phi) = \frac{\mathbf{u}^H(\alpha,\beta,\psi,\phi)\mathbf{\Pi}_b\mathbf{u}(\alpha,\beta,\psi,\phi)}{\mathbf{u}^H(\alpha,\beta,\psi,\phi)\mathbf{u}(\alpha,\beta,\psi,\phi)}, \quad (21)$$

where $\mathbf{\Pi}_b = \mathbf{I} - \mathbf{U_s}\mathbf{U_s}^H$ and where the $(N_A+N_B)\times M$ matrix $\mathbf{U_s}$ consists of the M eigenvectors of the matrix corresponding to the M largest eigenvalues of $\mathbf{R}$. This would lead to a 4-Dimentionnal search whereas ψ and ϕ are undesired nuisance parameters. But it can be noticed that since

$$\mathbf{u}(\alpha,\beta,\psi,\phi) = \mathbf{U}(\alpha,\beta)\boldsymbol{\rho}(\psi,\phi), \quad (22)$$

where

$$\mathbf{U}(\alpha,\beta) = \begin{bmatrix} \mathbf{a}(\alpha_m) & \mathbf{0} \\ \mathbf{0} & \mathbf{b}(\beta_m) \end{bmatrix}, \quad (23)$$

$$\boldsymbol{\rho}(\psi,\phi) = \frac{1}{\sqrt{1+\psi^2}}\begin{bmatrix} 1 \\ \psi e^{j\phi} \end{bmatrix}, \quad (24)$$

a well known linear algebra result allow us to propose the following reduced criterion :

$$C_2(\alpha,\beta) = \lambda_{min}\left\{\mathbf{U}^H(\alpha,\beta)\mathbf{\Pi}_b\mathbf{U}(\alpha,\beta)[\mathbf{U}^H(\alpha,\beta)\mathbf{U}(\alpha,\beta)]^{-1}\right\}, \quad (25)$$

where $\lambda_{min}(.)$ stands for the minimal eigenvalue. For the sake of computational cost [7] and to avoid the full numerical optimisation of the criterion (25) we prefer to compute the following equivalent criteria :

$$C_r(\alpha,\beta) = \frac{|\mathbf{U}^H(\alpha,\beta)\mathbf{\Pi}_b\mathbf{U}(\alpha,\beta)|}{|\mathbf{U}^H(\alpha,\beta)\mathbf{U}(\alpha,\beta)|}, \quad (26)$$

where $|.|$ stands for the determinant.

Indeed, as we will see in the section 4 the expression (26) is well suited to obtain closed-form expresssion for the gradient and the Hessian requiered in a second order steepest descent algorithm. The optimisation is performed through a 2-Dimensional search, giving us for each emitter the estimated location parameter couples $\left\{(\hat{\alpha}_m,\hat{\beta}_m), m \in [1,M]\right\}$. Once the AOA have been estimated the locations are estimated thanks to following geometrical relations in the $(\mathbf{AB},\mathbf{AC})$ plane (where $\mathbf{AC}$ is orthonormal to $\mathbf{AB}$) :

$$\left\{\begin{array}{lcl} x & = & x_A + ||AE||cos(\alpha) \\ y & = & y_A + ||AE||sin(\alpha) \end{array}\right., \quad (27)$$

where

$$||AE|| = \frac{||AB||}{cos(\alpha) - \frac{sin(\alpha)}{tan(\beta)}}, \quad (28)$$

and (x_A,y_A) and (x_B,y_B) denotes the location of the stations A and B.

Our extended observation based method appears to be unambiguous. Indeed, in presence of M emitters, when two independant MUSIC algorithms are processed on each station, M AOA are estimated on both stations leading to M^2 possible emitter location. On the contrary the criterion (26) exhibit directly only M solutions.

4. PRACTICAL IMPLEMENTATION

The reduced criterion (26) suits well to the use of a second order steepest descent algorithm. Denoting $\boldsymbol{\Theta} = [\begin{array}{cc} \alpha & \beta \end{array}]^T$ the parameter vector, at each step i we obtain an iterative estimate

$$\boldsymbol{\Theta}_{i+1} = \boldsymbol{\Theta}_i - \lambda \mathbf{H}^{-1}(\boldsymbol{\Theta}_i)\boldsymbol{\nabla}(\boldsymbol{\Theta}_i), \tag{29}$$

where λ stands for the step size, $\mathbf{H}(\boldsymbol{\Theta}_i)$ and $\boldsymbol{\nabla}(\boldsymbol{\Theta}_i)$ are respectively the Hessian and the gradient of the criterion (26) at the point $\boldsymbol{\Theta}_i$. Well known formulas provide [1]:

$$[\boldsymbol{\nabla}(\boldsymbol{\Theta})]_k = \frac{|\mathbf{M}|}{|\mathbf{U}^H(\boldsymbol{\Theta})\mathbf{U}(\boldsymbol{\Theta})|} Tr\left[\mathbf{M}^{-1}\mathbf{M}_k\right], \tag{30}$$

$$\begin{aligned}[\mathbf{H}(\boldsymbol{\Theta})]_{kl} \quad = \quad & \frac{|\mathbf{M}|}{|\mathbf{U}^H(\boldsymbol{\Theta})\mathbf{U}(\boldsymbol{\Theta})|}\left(Tr\left[\mathbf{M}^{-1}\mathbf{M}_l\right] Tr\left[\mathbf{M}^{-1}\mathbf{M}_k\right]\right. \\ & -Tr\left[\mathbf{M}^{-1}\mathbf{M}_k\mathbf{M}^{-1}\mathbf{M}_l\right] \\ & \left.+Tr\left[\mathbf{M}^{-1}\mathbf{M}_{kl}\right]\right) \end{aligned} \tag{31}$$

with

$$\mathbf{M} = \mathbf{U}^H(\boldsymbol{\Theta})\boldsymbol{\Pi}_b\mathbf{U}(\boldsymbol{\Theta}), \tag{32}$$

$$\mathbf{M}_k = \frac{\partial \mathbf{M}}{\partial \Theta_k}, \tag{33}$$

$$\mathbf{M}_{kl} = \frac{\partial^2 \mathbf{M}}{\partial \Theta_k \partial \Theta_l}, \tag{34}$$

where $1 \le k, l \le 2$ and Θ_k denotes the kth component of $\boldsymbol{\Theta}$.

In order to initialize this second order steepest descent algorithm we propose to take the result of two independant MUSIC algorithm processed on each station independently, providing us M^2 possibilities. The M most appropriate couples are those providing the M lowest value of (26).

5. CRAMER-RAO BOUNDS

5.1 Cramer-Rao bounds on AOA for the stacked observations

We focus on unknown deterministic signals. Let us note the following unknown parameter vector, where the term σ^2 is discarded :

$$\boldsymbol{\xi} = [\begin{array}{ccccc} \mathbf{s}^T & \boldsymbol{\alpha}^T & \boldsymbol{\beta}^T & \boldsymbol{\psi}^T & \boldsymbol{\phi}^T \end{array}], \tag{35}$$

where $\boldsymbol{\psi} = [\begin{array}{ccc} \psi_1 & \ldots & \psi_M \end{array}]^T$, $\boldsymbol{\phi} = [\begin{array}{ccc} \phi_1 & \ldots & \phi_M \end{array}]^T$, $\mathbf{s}$ is the source signal vector $\mathbf{s} = [\begin{array}{ccc} \mathbf{s}^T(1) & \ldots & \mathbf{s}^T(T) \end{array}]^T$ and T the number of samples. We can show by means of [8] and [9] that the CRB of the stacked model (11), where $\boldsymbol{\eta} = [\begin{array}{cccc} \boldsymbol{\alpha}^T & \boldsymbol{\beta}^T & \boldsymbol{\psi}^T & \boldsymbol{\phi}^T \end{array}]$, writes :

$$CRB_s^{-1}(\boldsymbol{\eta}) = \frac{2}{\sigma^2}\sum_{t=1}^{T} Re[\mathbf{S}^H(t)\mathbf{D}^H\boldsymbol{\Pi}_A\mathbf{D}\mathbf{S}(t)], \tag{36}$$

$$\boldsymbol{\Pi}_A = \mathbf{I} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H, \tag{37}$$

$$\mathbf{S}(t) = \mathbf{I}_4 \otimes diag(\mathbf{s}(t)), \tag{38}$$

$$\mathbf{D} = [\begin{array}{cccc} \mathbf{D}_\alpha & \mathbf{D}_\beta & \mathbf{D}_\psi & \mathbf{D}_\phi \end{array}], \tag{39}$$

$$\mathbf{D}_\mu = \left[\begin{array}{ccc} \frac{\partial \mathbf{u}(\alpha,\beta,\psi,\phi)}{\partial \mu}|_{\mu_1} & \ldots & \frac{\partial \mathbf{u}(\alpha,\beta,\psi,\phi)}{\partial \mu}|_{\mu_M} \end{array}\right], \tag{40}$$

where $\boldsymbol{\mu} \in \{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\psi}, \boldsymbol{\phi}\}$ and $diag(\mathbf{v})$ denotes a diagonal matrix where $[diag(\mathbf{v})]_{ij} = \delta_{ij}v_i$ and where δ_{ij} denotes the kroenecker symbol. Straightforward calculations provide :

$$\frac{\partial \mathbf{u}(\alpha,\beta,\psi,\phi)}{\partial \alpha}|_{\alpha_m} = \frac{1}{\sqrt{1+\psi_m^2}}\left[\begin{array}{c} \dot{\mathbf{a}}(\alpha_m) \\ \mathbf{0} \end{array}\right], \tag{41}$$

$$\frac{\partial \mathbf{u}(\alpha,\beta,\psi,\phi)}{\partial \beta}|_{\beta_m} = \frac{1}{\sqrt{1+\psi_m^2}}\left[\begin{array}{c} \mathbf{0} \\ \psi e^{j\phi}\dot{\mathbf{b}}(\beta_m) \end{array}\right], \tag{42}$$

$$\frac{\partial \mathbf{u}(\alpha,\beta,\psi,\phi)}{\partial \psi}|_{\psi_m} = \frac{1}{(1+\psi_m^2)^{3/2}}\left[\begin{array}{c} -\psi_m\mathbf{a}(\alpha_m) \\ e^{j\phi_m}\mathbf{b}(\beta_m) \end{array}\right], \tag{43}$$

$$\frac{\partial \mathbf{u}(\alpha,\beta,\psi,\phi)}{\partial \phi}|_{\phi_m} = \frac{1}{\sqrt{1+\psi_m^2}}\left[\begin{array}{c} \mathbf{0} \\ j\psi_m e^{j\phi_m}\mathbf{b}(\beta_m) \end{array}\right], \tag{44}$$

where $\dot{\mathbf{a}}(\alpha_m) = \frac{\partial \mathbf{a}(\alpha)}{\partial \alpha}|_{\alpha_m}$ and $\dot{\mathbf{b}}(\beta_m) = \frac{\partial \mathbf{b}(\beta)}{\partial \beta}|_{\beta_m}$.Thanks to (36) we can compute the stacked observation based CRB ($CRB_s(\boldsymbol{\alpha}, \boldsymbol{\beta})$) by extracting the corresponding upper-corner of $CRB_s(\boldsymbol{\eta})$.

5.2 Cramer-Rao bounds on AOA for independant observations

As classical estimation of the position can be achieved through the independent estimation of both angles α and β considering the equations (1) and (2), that does not requiere the narrowband assumption, we can examine the corresponding CRB. Now, the considered unknown parameter vector is

$$\boldsymbol{\xi}' = [\begin{array}{cccc} \mathbf{s}_A^T & \boldsymbol{\alpha}^T & \mathbf{s}_B^T & \boldsymbol{\beta}^T \end{array}], \tag{45}$$

where we define

$$\mathbf{s}_A = [\begin{array}{ccc} \mathbf{s}_A^T(1) & \ldots & \mathbf{s}_A^T(T) \end{array}]^T, \tag{46}$$

$$\mathbf{s}_B = [\begin{array}{ccc} \mathbf{s}_B^T(1) & \ldots & \mathbf{s}_B^T(T) \end{array}]^T, \tag{47}$$

$$\mathbf{s}_A(t) = [\begin{array}{ccc} s_{A,1}(t) & \ldots & s_{A,M}(t) \end{array}]^T, \tag{48}$$

$$\mathbf{s}_B(t) = [\begin{array}{ccc} s_{B,1}(t) & \ldots & s_{B,M}(t) \end{array}]^T, \tag{49}$$

and where :

$$s_{A,m}(t) = \frac{1}{\sqrt{1+\psi_m^2}} s_m(t), \tag{50}$$

$$s_{B,m}(t) = \frac{\psi_m e^{j\phi_m}}{\sqrt{1+\psi_m^2}} s_m(t-\tau_m). \tag{51}$$

Thanks to [8] the classical unconditionnal Cramer Rao Bound CRB_i (for independent observation vectors) is straigthforwardly given by :

$$CRB_i^{-1}(\boldsymbol{\alpha}, \boldsymbol{\beta}) = \left[\begin{array}{cc} CRB_i^{-1}(\boldsymbol{\alpha}) & \mathbf{0} \\ \mathbf{0} & CRB_i^{-1}(\boldsymbol{\beta}) \end{array}\right], \tag{52}$$

where

$$CRB_i^{-1}(\boldsymbol{\alpha}) = \frac{2}{\sigma^2}\sum_{t=1}^{T} Re[\mathbf{S}_\alpha^H(t)(\mathbf{D}'_\alpha)^H\boldsymbol{\Pi}_\alpha\mathbf{D}'_\alpha\mathbf{S}_\alpha(t)], \tag{53}$$

$$CRB_i^{-1}(\boldsymbol{\beta}) = \frac{2}{\sigma^2}\sum_{t=1}^{T} Re[\mathbf{S}_\beta^H(t)(\mathbf{D}'_\beta)^H\boldsymbol{\Pi}_\beta\mathbf{D}'_\beta\mathbf{S}_\beta(t)], \tag{54}$$

where we define

$$\mathbf{\Pi}_\alpha = \mathbf{I} - \mathbf{A}_\alpha(\mathbf{A}_\alpha^H \mathbf{A}_\alpha)^{-1}\mathbf{A}_\alpha^H, \quad (55)$$

$$\mathbf{\Pi}_\beta = \mathbf{I} - \mathbf{A}_\beta(\mathbf{A}_\beta^H \mathbf{A}_\beta)^{-1}\mathbf{A}_\beta^H, \quad (56)$$

$$\mathbf{A}_\alpha = [\ \mathbf{a}(\alpha_1) \ \ \dots \ \ \mathbf{a}(\alpha_M)\ ], \quad (57)$$

$$\mathbf{A}_\beta = [\ \mathbf{b}(\beta_1) \ \ \dots \ \ \mathbf{b}(\beta_M)\ ], \quad (58)$$

$$\mathbf{D}_\alpha^{'} = [\ \dot{\mathbf{a}}(\alpha_1) \ \ \dots \ \ \dot{\mathbf{a}}(\alpha_M)\ ], \quad (59)$$

$$\mathbf{D}_\beta^{'} = [\ \dot{\mathbf{b}}(\beta_1) \ \ \dots \ \ \dot{\mathbf{b}}(\beta_M)\ ], \quad (60)$$

and where $\mathbf{S}_\alpha(t) = diag(\mathbf{s}_A(t))$ and $\mathbf{S}_\beta(t) = diag(\mathbf{s}_B(t))$. This CRB (52) can also be computed when the signals are considered narrowband for the whole sensor network ($s_m(t-\tau_m) = s_m(t)$).

5.3 Cramer-Rao Bounds on location errors

Once the *CRB* for $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is obtained, the *CRB* for (x,y) immediatly follows [1] :

$$CRB(\boldsymbol{x}, \boldsymbol{y}) = \mathbf{J}^T CRB(\boldsymbol{\alpha}, \boldsymbol{\beta})\mathbf{J}, \quad (61)$$

where $\mathbf{J}$ is the following Jacobian matrix [1] :

$$[\mathbf{J}]_{ij} = \frac{\partial[\boldsymbol{\gamma}]_j}{\partial[\boldsymbol{\theta}]_i}, \quad (62)$$

where $\boldsymbol{\theta} = [\ \alpha_1 \ \ \dots \ \ \alpha_M \ \ \beta_1 \ \ \dots \ \ \beta_M\ ]$ and $\boldsymbol{\gamma} = [\ x_1 \ \ \dots \ \ x_M \ \ y_1 \ \ \dots \ \ y_M\ ]$.

Once the CRB in (x,y) is obtained we can compute for each emitter the following expressions in meters

$$CRB_s = \sqrt{CRB_s(x_m) + CRB_s(y_m)}, \quad (63)$$

$$CRB_i = \sqrt{CRB_i(x_m) + CRB_i(y_m)}. \quad (64)$$

This expressions are standing for the Cramer-Rao Bounds in distance for the considered algorithms.

6. SIMULATIONS

In this section we consider two base stations A and B whose coordinates are (x_A, y_A) and (x_B, y_B), respectively. They both are composed of a three sensor uniform circular array which radius is 0.5 wavelength ($N_A = N_B = 3$). $\mathbf{a}(.)$ and $\mathbf{b}(.)$ are chosen with a norm equal to $\sqrt{N_A}$. The signals are complex, deterministic and unknown. The carrier frequency is $f_0 = 100$ MHz. The number of sample is $T = 500$. The signal to noise ratio is defined on equation (8). In this section we choose all ψ_m equal to 1 and ϕ_m equal the geometrical phase delay. The scenario describing the position of the emitters and the base stations is decribed the Figure 1. The performances of our algorithm are studied trough the root mean square error (RMS) of the miss distance defined in meters :

$$\varepsilon = \sqrt{\frac{1}{K}\sum_{k=1}^{K}(x_m - \hat{x}_k)^2 + (y - \hat{y}_k)^2}, \quad (65)$$

where K is the number of Monte-Carlo runs and $\hat{x}_k$ and $\hat{y}_k$ denote the kth estimation of the true position (x,y) of the emitter. When ε is computed through our algorithm that simultaneously estimates (stacked observation vector based) the two AOAs, ε is denoted ε_s and when ε is computed with two classical independent MUSIC at both stations ε is denoted ε_i.

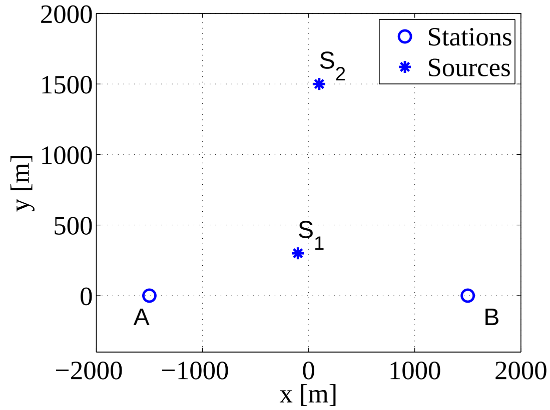


Figure 1: Considered scenario : 2 emitters are in $S_1(100,300)$ and $S_2(-100,1500)$ and two base stations A and B respectively in $(-1500,0)$ and $(1500,0)$.

Both ε_s and ε_i are compared each other and to their respective CRB : CRB_s and CRB_i on the Figure 2, according to the scenario defined in Figure 1. The proposed scenario consists in two sources, one beeing close to the stations (Figure2 (a)) and the other beeing further away (Figure 2(b)). As we can see in both cases CRB_s, the CRB of the model (11) lies under CRB_i. It underlines the potential significant gain of our method based on an extanded stacked vector. On both cases the computation of the empirical RMS error show that the stacked obervation vector algorithm outperforms the classical one (Figure 2).

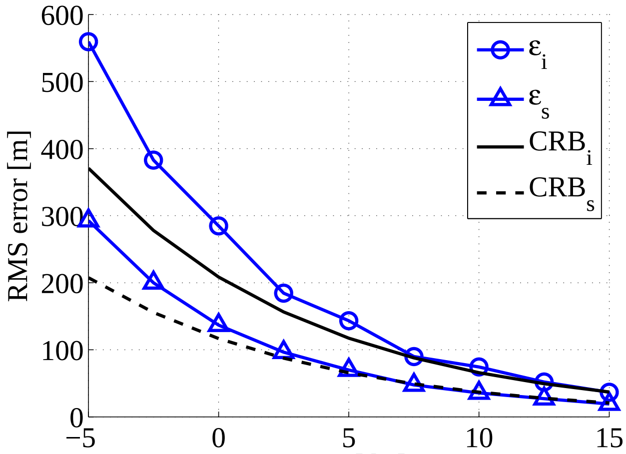
On the Figure 3 we study the performance of our algorithm when the product “emitter bandwith $\times$ time delay of arrival between the two stations” (called $B \times \tau$ in the sequel) varies. Let us define

$$\gamma = \frac{\varepsilon_i}{\varepsilon_s}. \quad (66)$$

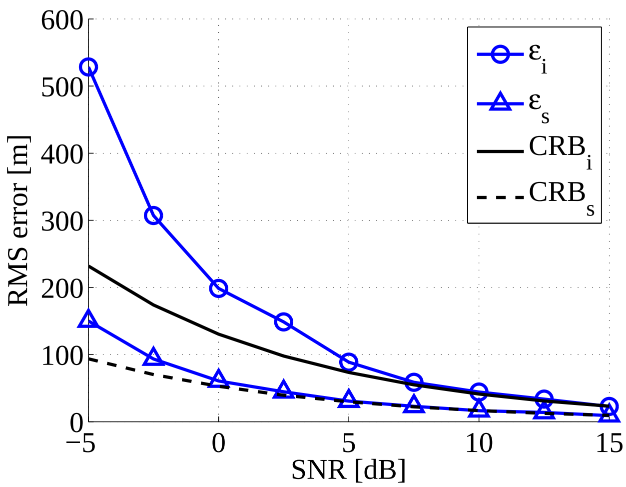
On Figure 3 we compare the evolution of γ with $B \times \tau$ and we clearly see that provided that $B \times \tau$ is rather small, we have $\gamma \geq 1$ meaning that the algorithm based on a extended stacked vector still outperforms the classical one for both sources. On the contrary when $B \times \tau$ becomes larger, $\gamma \leq 1$: the modelling error are not negligible and lead to a decrease of the performance of the proposed algorithm.

7. CONCLUSION

Based on the narrowband assumption on the whole sensor network, we provided an original location algorithm. By means of an extended stacked observation vector it provides a simultaneous estimation of the location parameters. The CRB calculation so as the simulations underline the theoretical and practical improvement compared to a classical location algorithm where the parameter are estimated independently from each other. We illustrate the fact that provided that the product “emitter bandwith $\times$ time delay of arrival between the two stations” is small enough, our algorithm exhibits still better performance. A more complete sensibility analysis so as its extension to more complex scenarios will



(a) source in S_1



(b) source in S_2

Figure 2: Error Distance for the source in S_1 and the source 2 in S_2. number of Monte-Carlo runs = 200.

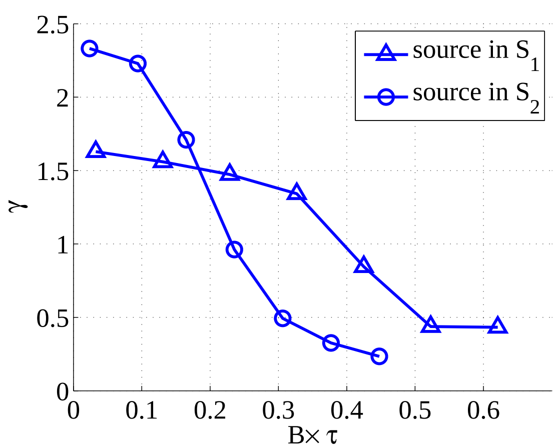


Figure 3: Evolution of γ defined in (66) with the product $B \times \tau$ for the two sources in S_1 and S_2 (see Figure 1), SNR=10dB, number of Monte-Carlo runs = 300.

be given in the future.

Acknowledgments

This work was partially supported by the network of excellence in wireless communication NEWCOM++ under the contract number 216715 funded by the european commission FP7-ICT-2007-1.

REFERENCES

[1] H. L. Van Trees, *Optimum Array Processing*. John Wiley & Sons, 2002.

[2] M. Porretta, P. Nepa, G. Manara, and F. Giannetti, "Location, location, location," *IEEE Vehicular Technology Magazine*, June 2008.

[3] A. Amar and A. Weiss, *Direct Position Determination: A Single-Step Emitter Localization Approach*. Classical and Modern Direction-of-Arrival Estimation, Editors Tuncer, T.E. and Friedlander, B. Academic Press, 2009.

[4] A. Weiss, "Direct position determination of narrow-band radiofrequency transmitters," *IEEE signal processing letters*, vol. 11, no. 5, pp. 513–516, 2004.

[5] A. Amar and A. Weiss, "Direct position determination of multiple radio signals," *EURASIP J.Appl. Signal Process.*, vol. 1, pp. 37–49, 2005.

[6] R.-O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propogation*, vol. 34, no. 3, pp. 281–290, 1986.

[7] A. Ferréol, E. Boyer, and P. Larzabal, "Low cost algorithm for some bearing estimation methods in presence of separable nuisance parameters," *ELSEVIER Electronic letters*, vol. 40, no. 15, 2004.

[8] P. Stoica and A. Nehorai, "Music, maximum likelihood and cramer rao bound," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 37, pp. 720–741, 1989.

[9] E. Cekli, H. Cirpan, and E. Dilavergolu, "Cramer rao bounds for direction of arrival and range estimation of near field sources," *ECCTD 01- European conference on circuit theory and design*, vol. 3, pp. 105–108, 2001.