RECEIVER OPERATING CHARACTERISTIC FOR ARRAY-BASED GNSS ACQUISITION

Javier Arribas, Carles Fernández–Prades, and Pau Closas

Centre Tecnològic de Telecomunicacions de Catalunya (CTTC),
Parc Mediterrani de la Tecnologia (PMT) B4, Av. Carl Friedrich Gauss 7, 08860, Barcelona, Spain
email: {javier.arribas, carles.fernandez, pau.closas}@cttc.es

ABSTRACT

This paper investigates the application of the antenna array Maximum Likelihood (ML) Doppler shift and code delay estimator to the Global Navigation Satellite Systems (GNSS) acquisition problem, considering an unstructured channel model and Additive White Gaussian Noise (AWGN). We propose an acquisition test function based on the ML estimator and we derive the theoretical false alarm and detection probabilities. Furthermore, this work analyzes the effect of the acquisition bandwidth on the Receiver Operating Characteristic (ROC). Finally, we propose an implementation scheme including a baseband filter to improve the acquisition ROC. The simulation results validate the theoretical analysis.

1. INTRODUCTION

With the increasing demand of more accurate and more robust Global Navigation Satellite Systems (GNSS) services, the applications of antenna arrays to GNSS technology are focusing much attention recently. The capability of the antenna arrays to reject interferences or jamming signals and the theoretical multipath mitigation potential is usually applied to the tracking operation of GNSS receivers. In this work we investigate the application of an antenna array to the Direct-Sequence Code Division Spread Spectrum (DS-CDMA) acquisition operation. The acquisition process is in charge of estimating the DS-CDMA signal synchronization parameters, defined as the Doppler frequency and the code delay. In the literature can be found a number of works considering the array acquisition problem with the assumption of the receiver capability to estimate the Direction Of Arrival (DOA), see, e.g., [1, 2]. In these works, the DOA is estimated using either a pilot signal or including the DOA in the acquisition search grid, and then using the estimated DOA information to recombine the outputs of the correlators or matched filters. In a GNSS receiver it is difficult to estimate the DOA without acquiring the signal and the DOA-based beamforming techniques usually need a calibrated array. Here, the Maximum Likelihood (ML) estimator is derived for the synchronization parameters assuming Additive White Gaussian Noise (AWGN) and considering a receiver using an unstructured antenna array. Based on the results, a test function suitable to be used in the acquisition process is proposed and analyzed in terms of the false alarm and detection probability. This work extends the results of [3] and extracts the Receiver Operating Characteristic (ROC).

The receiver bandwidth has an important effect on the acquisition and it was analyzed in [4, 5] in terms of the correlation losses. The second part of this work is devoted to study how the receiver bandwidth affects the acquisition ROC for the Galileo E1 Multiplexed Binary Offset Carrier (MBOC) modulation, and how it can improve the ROC performance. An implementation scheme of the acquisition is also proposed.

The paper is organized as follows: Section 2 presents the antenna array signal model, Section 3 derives the ML estimator for the synchronization parameters. Section 4 proposes a new acquisition test function using the resulting estimator and analyzes its performance. The effect of the receiver bandwidth is theoretically analyzed. In Section 5 we plot theoretical results and computer simulations, comparing the performances to the single antenna ML acquisition. Finally, Section 6 concludes the paper.

2. SIGNAL MODEL

Considering a single GNSS satellite signal received with an N-element antenna array, the discrete baseband signal model is defined as:

$$\mathbf{X} = \mathbf{h}\mathbf{d}(F,\tau) + \mathbf{N}, \tag{1}$$

where

- $\mathbf{X} = [\mathbf{x}(t_0)\dots\mathbf{x}(t_{K-1})] \in \mathbb{C}^{N\times K}$ is referred to *spatiotemporal data matrix*, where $\mathbf{x}(t) = [x_1(t)\dots x_N(t)]^T$ is defined as the antenna array snapshot and K is the number of captured snapshots.
- $\mathbf{h} = [h_1\dots h_N]^T \in \mathbb{C}^{N\times 1}$ is the non-structured channel model, which models both the channel and the array response, where $|h_i|^2$ is the signal power for the *i-th* antenna element and $|\cdot|$ is the modulus operator.
- $\mathbf{d}(F,\tau) = [s(t_0-\tau)e^{j2\pi F t_0}\dots s(t_{K-1}-\tau)e^{j2\pi F t_{K-1}}] \in \mathbb{C}^{1\times K}$ is the GNSS complex baseband DS-CDMA signal with normalized power and known structure $s(t)$, received by the array with a propagation delay τ and a Doppler frequency F. In this work, we consider $s(t) = \sum_{k=-\infty}^{\infty} c_k p_k(t-kT_c)$, where c_k are the spreading code chips, p_k is a rectangular pulse of support T_c, and T_c is the spreading code chip rate [6].
- $\mathbf{N} = [\mathbf{n}(t_0)\dots\mathbf{n}(t_{K-1})] \in \mathbb{C}^{N\times K}$ is a complex, circularly symmetric Gaussian vector process with a zero-mean and temporally white.

3. MAXIMUM LIKELIHOOD ESTIMATOR FOR THE DOPPLER SHIFT AND DELAY

The ML estimator for Doppler shift and code delay for an antenna array receiver using signal model (1) and considering unstructured noise covariance matrix was derived in [7]. Hereafter we consider the white noise case. The negative log-likelihood function of a complex multivariate Gaussian [8] snapshot vector $\mathbf{x}$, assuming AWGN with a diagonal covariance matrix $\mathbf{Q} = \sigma^2\mathbf{I}$, neglecting the irrelevant constants can be defined as:

$$\Lambda_1(\sigma^2,\mathbf{h},F,\tau) = N\ln(\sigma^2) + \frac{\mathrm{Tr}(\mathbf{C})}{\sigma^2}, \tag{2}$$

we define matrix $\mathbf{C}$ as:

$$\mathbf{C} = \hat{\mathbf{R}}_{\mathbf{XX}} - \hat{\mathbf{r}}_{\mathbf{Xd}}\mathbf{h}^H - \mathbf{h}\hat{\mathbf{r}}_{\mathbf{Xd}}^H + \mathbf{h}\hat{R}_{\mathbf{dd}}^{-1}\mathbf{h}^H, \tag{3}$$

where the autocorrelation and the cross-correlation matrices are defined as follows[1]:

- $\hat{\mathbf{R}}_{\mathbf{XX}} = \frac{1}{K}\mathbf{X}\mathbf{X}^H$ is the estimation of the autocorrelation matrix of the array snapshots, also known as the sample covariance matrix.
- $\hat{\mathbf{r}}_{\mathbf{Xd}} = \frac{1}{K}\mathbf{X}\mathbf{d}^H$ is the estimation of the cross-correlation vector between the array snapshot matrix and the DS-CDMA signal.
- $\hat{R}_{\mathbf{dd}} = \frac{1}{K}\mathbf{d}\mathbf{d}^H$ is the estimation of the DS-CDMA signal autocorrelation.

The next step is to find the ML estimate for each parameter, which is equivalent to minimize the negative log-likelihood function:

$$\hat{\sigma}^2, \hat{\mathbf{h}}, \hat{F}, \hat{\tau}\big|_{ML} = \arg\min_{\sigma^2,\mathbf{h},F,\tau} \Lambda_1(\sigma^2, \mathbf{h}, F, \tau), \tag{4}$$

by applying the gradient with respect to σ^2, and setting it to zero we find:

$$\hat{\sigma}^2_{ML} = \frac{\mathrm{Tr}(\mathbf{C})}{N}\Big|_{\mathbf{h}=\hat{\mathbf{h}}_{ML}, F=\hat{F}_{ML}, \tau=\hat{\tau}_{ML}}. \tag{5}$$

Replacing σ^2 with $\hat{\sigma}^2_{ML}$ in (2) and neglecting the additive constant term, we obtain $\Lambda_2(\mathbf{h}, F, \tau) = \ln(\mathrm{Tr}(\mathbf{C}))$, and by applying the gradient with respect to $\mathbf{h}$ in Λ_2 and setting it to zero again, we find the ML estimator for $\hat{\mathbf{h}}_{ML}$:

$$\hat{\mathbf{h}}_{ML} = \hat{\mathbf{r}}_{\mathbf{Xd}}\hat{R}^{-1}_{\mathbf{dd}}\big|_{\sigma^2=\hat{\sigma}^2_{ML}, F=\hat{F}_{ML}, \tau=\hat{\tau}_{ML}}. \tag{6}$$

By inserting (6) in (3), we obtain a new cost function to minimize:

$$\Lambda_3(F, \tau) = \mathrm{Tr}(\hat{\mathbf{R}}_{\mathbf{XX}} - \hat{\mathbf{r}}_{\mathbf{Xd}}\hat{R}^{-1}_{\mathbf{dd}}\hat{\mathbf{r}}^H_{\mathbf{Xd}}). \tag{7}$$

Finally, by applying the trace cyclic properties and neglecting the additive and multiplicative constant terms, the ML estimate for F and τ is:

$$\hat{F}_{ML}, \hat{\tau}_{ML} = \arg\max_{F,\tau}(\hat{\mathbf{r}}^H_{\mathbf{Xd}}\hat{\mathbf{r}}_{\mathbf{Xd}}), \tag{8}$$

which is equal to the maximization of the Euclidian norm of the cross-correlation vector, formulated as $\|\hat{\mathbf{r}}_{\mathbf{Xd}}(F,\tau)\|^2$. We define the Euclidian norm as $\|\cdot\| = \sqrt{\mathbf{u}^H\mathbf{u}}$, where $(\cdot)^H$ denotes conjugate transpose. Since it is not possible to obtain a closed expression for $\hat{F}_{ML}$ and $\hat{\tau}_{ML}$, a grid based search is suitable to find the function maximum [7]. The ML array-based estimator is the natural extension to the single antenna ML estimator extensively used in single antenna GNSS receivers [9].

4. PERFORMANCE OF THE ARRAY-BASED ACQUISITION ALGORITHM BASED ON ML ESTIMATORS

The aim of this Section is the characterization of the performance of the array-based Doppler frequency and code delay ML estimation when used as an acquisition algorithm. We consider GNSS acquisition process as a grid search, evaluating a test function $T(\check{F}, \check{\tau})$ over a finite and discrete search space. Each of the possible pairs of $(\check{F}, \check{\tau})$ form a search cell [9]. The acquisition algorithm compares the output of the test function to a given threshold γ, and the result decides if there is a satellite signal present or not. As a consequence, the performance evaluation of the acquisition algorithm is based on the false alarm and the detection probabilities over two hypothesis:

- The Null Hypothesis $\mathcal{H}_0$ is defined as the absence of the satellite signal, or the misalignment of the signal with the local replica, $(\check{F}, \check{\tau})\Big|_{\mathcal{H}_0} \neq (F, \tau)$, which differs sufficiently either in frequency or code delay or both, to consider the received satellite signal $\mathbf{d}(F, \tau)$ orthogonal to the local replica.
- The Match Hypothesis $\mathcal{H}_1$ defines the case when the searched satellite is present and the detection is performed on the correct cell. In the simulations, we consider a perfect Doppler frequency and code delay alignment $(\check{F}, \check{\tau})\Big|_{\mathcal{H}_1} = (F, \tau)$.

The false alarm and detection probabilities are defined as:

$$P_{fa}(\gamma) = P(T(\check{F}, \check{\tau}) > \gamma|\mathcal{H}_0) \tag{9}$$

$$P_d(\gamma) = P(T(\check{F}, \check{\tau}) > \gamma|\mathcal{H}_1). \tag{10}$$

4.1 ML based Test Function

The proposed test function can be considered a simplification of the Generalized Likelihood Ratio Test (GLRT) detector [10]. Using the ML estimator of (8) we define the test function as:

$$T(\check{F}, \check{\tau}) = \|\hat{\mathbf{r}}_{\mathbf{Xd_L}}(\check{F}, \check{\tau})\|^2, \tag{11}$$

where a non-filtered locally generated satellite signal replica $\mathbf{d}_L(\check{F}, \check{\tau}) = \mathbf{d}(\check{F}, \check{\tau})$ is used. The test function is equivalent to perform a non-coherent ML acquisition independently over each antenna element and adding the results, expressed as:

$$T(\check{F}, \check{\tau}) = \sum_{i=1}^{N}\Big(|h_i|^2|\hat{R}_{\mathbf{dd}_L}|^2 + \tag{12}$$

$$+ \underbrace{\frac{1}{K}(\mathbf{h}_i\hat{R}_{\mathbf{dd_L}}\mathbf{d}\mathbf{N}_i^H + \mathbf{N}_i\mathbf{d}^H\hat{R}_{\mathbf{dd_L}}\mathbf{h}_i^* + \frac{1}{K}\mathbf{N}_i\mathbf{d}^H\mathbf{d}\mathbf{N}_i^H)}_{\eta_i}\Big),$$

where $\mathbf{h}_i$ denotes the *i-th* component of vector $\mathbf{h}$, $(\cdot)^*$ indicates complex conjugate, $\hat{R}_{\mathbf{dd_L}} = \frac{1}{K}\mathbf{d}\mathbf{d}_L(\check{F}, \check{\tau})$ is the cross-correlation between the received satellite signal and the local replica, and $\mathbf{N}_i = [\mathbf{n}(t_0)_i \ldots \mathbf{n}(t_{K-1})_i]$ is the noise input vector for the *i-th* antenna element. The correlation of the local replica with the input noise can be grouped in a single noise term η_i.

4.2 Null Hypothesis $\mathcal{H}_0$

In order to statistically characterize the function $T(\check{F}, \check{\tau})\Big|_{\mathcal{H}_0}$ we consider (12) as the sum of N independent random variables with the same Probability Density Function (pdf). When the satellite signal is present, but not correctly aligned, $E[\hat{R}_{\mathbf{dd}_L}]\Big|_{\mathcal{H}_0} \simeq 0$ due the misalignment of the local replica. The remaining term η_i is a central χ^2_2 random variable with underlying Normal distribution $\mathcal{N}(0, \sigma^2)$, where $\sigma^2 = \frac{P_n}{2K}$, and P_n is the noise power at the correlator input [11]. Applying the definition of the Chi-Square distribution [12], we found that the sum of N χ^2_2 random variables is distributed as χ^2_{2N}. As a consequence, $T(\check{F}, \check{\tau})\Big|_{\mathcal{H}_0} \sim \chi^2_{2N}$.

Using (9) the false alarm probability can be expressed as $P_{fa}(\gamma) = 1 - P_{\mathcal{H}_0}(T(\check{F}, \check{\tau}) \leq \gamma)$, where $P_{\mathcal{H}_0}(T(\check{F}, \check{\tau}) \leq \gamma)$ is the χ^2_{2N} Cumulative Density Function (cdf). Finally, applying the definition of χ^2_{2N} cdf we find:

$$P_{fa}(\gamma) = \exp\Big\{\frac{-\gamma}{2\sigma^2}\Big\}\sum_{k=0}^{N-1}\frac{1}{k!}\Big(\frac{\gamma}{2\sigma^2}\Big)^k. \tag{13}$$

[1]For the sake of simplicity of the notation, we drop the $\mathbf{d}(F, \tau)$ dependency on F and τ.

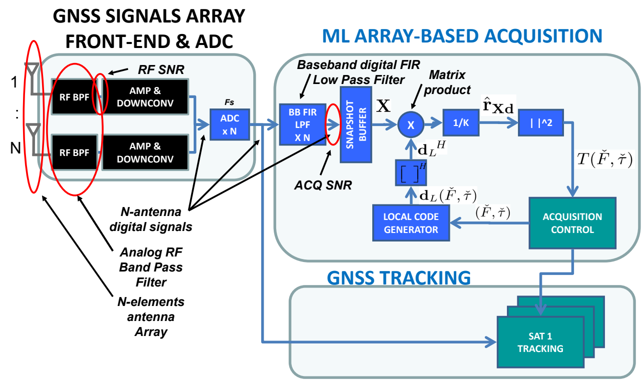

Figure 1: Implementation scheme for the proposed ML array-based acquisition.

4.3 Match Hypothesis $\mathcal{H}_1$

Considering $T(\check{F},\check{\tau})\Big|_{\mathcal{H}_1}$, the correlator output for the *i-th* antenna element can be expressed as the sum of N noncentral χ_2^2 random variables with the noncentrality parameter due to the despreading gain:

$$\lambda_i^2 = \left(|h_i||\hat{R}_{\mathbf{dd_L}}|\right)^2 \simeq |h_i|^2, \tag{14}$$

where we assume $E[\hat{R}_{\mathbf{dd}_L}]\Big|_{\mathcal{H}_1} \simeq 1$. Using the Chi-Square properties, it can be shown that $T(\check{F},\check{\tau})\Big|_{\mathcal{H}_1}$ is a noncentral χ_{2N}^2 with the noncentrality parameter $\lambda^2 = \sum_{i=1}^N \lambda_i^2$.

The presented test function is a non-coherent detector, and it is affected by twice the noise power than the coherent detector [13]. Assuming $|h_i|^2 = P_s \,\forall\, i$, comparing the array Signal-to-Noise Ratio (SNR) gain with respect to the single coherent acquisition SNR, the array gain is defined as:

$$G_{ARRAY} = 10\log\left(\frac{\frac{\lambda^2}{\sigma^2}}{\frac{2P_s}{\sigma^2}}\right) = 10\log\left(\frac{N}{2}\right). \tag{15}$$

When a calibrated planar array with equal antenna array elements gain patterns is used, the most significant differences between the signals received by the antenna array elements are located in the signal phase [14], thus the test function is not affected by the DOA-dependant phase-shifts. Obviously, the DOA affects the received signal power due to the individual antenna array elements gain patterns.
Finally, using (10), the detection probability can be computed as $P_d(\gamma) = 1 - P_{\mathcal{H}_1}(T(\check{F},\check{\tau}) \leq \gamma)$, where $P_{\mathcal{H}_1}(T(\check{F},\check{\tau}) \leq \gamma)$ is the cdf of a noncentral Chi-square χ_{2N}^2:

$$P_d(\gamma) = \mathrm{Q}_N\left(\frac{\sqrt{\lambda^2}}{\sigma}, \frac{\sqrt{\gamma}}{\sigma}\right), \tag{16}$$

where Q_N is the generalized Marcum Q-function [12] of order N and σ was defined in Section 4.2.

4.4 The effect of the receiver bandwidth

The proposed implementation scheme can be found in Fig. 1. From left to right, the array receiver N-channel RF front-end has a limited RF bandwidth given by the RF Band Pass Filter (BPF), which for the sake of simplicity, is considered an ideal BPF covering all the signal bandwidth. After the amplification and the conversion to baseband, an N-channel analog-to-digital converter (ADC) is in charge of digitizing the received signal with a sampling frequency $F_s = 2B_{RF}$, where B_{RF} is the BPF bandwidth. At this point, the signal is sent both to the acquisition block and to the tracking block. The acquisition block has a pre-conditioning baseband Finite Impulse Response (FIR) Low Pass Filter (LPF) with bandwidth B_{bb} which improves the SNR. The snapshot matrix container can be implemented using a Random Access Memory, and the operations required by the test function can be implemented using a Field Programmable Gate Array device [15].

In order to characterize the effect of the acquisition bandwidth we consider equal received signal power over all the antenna array elements and we define the SNR after the LPF as:

$$\rho_{acq} = \frac{P_s'}{P_n'}, \tag{17}$$

where P_s' and P_n' are the satellite signal and the noise power after the LPF. Using the convolution property $x(t) * y(t) = X(f)Y(f)$ and the Parseval's theorem [12], P_s' can be computed:

$$P_s' = P_s \int_{-\frac{1}{2}}^{+\frac{1}{2}} S_s(f)|H_{LPF}(f)|^2 df, \tag{18}$$

where $S_s(f)$ is the Power Spectral Density (PSD) of the satellite signal and $H_{LPF}(f)$ is the Fourier transform of the LPF impulse response $h_{LPF}[n]$. Considering the Galileo E1 MBOC(6,1,1/11) [6], the analytical expression for the PSD can be found in [16]:

$$S_s(f) = \frac{10}{11}G_{BOC(1,1)} + \frac{1}{11}G_{BOC(6,1)} \tag{19}$$

$$G_{BOC(m,n)}(f) = \frac{1}{T_c}\left(\frac{\sin(\frac{\pi f F_s T_c}{N_B})\sin(\pi f F_s T_c)}{\pi f F_s \cos(\frac{\pi f F_s T_c}{N_B})}\right)^2,$$

where $N_B = 2\frac{m}{n}$ is the BOC(m,n) modulation index relation. Using the same approach, the noise power can be expressed as:

$$P_n' = N_0 B_{RF} \int_{-\frac{1}{2}}^{+\frac{1}{2}} |H_{LPF}(f)|^2 df, \tag{20}$$

where N_0 W-Hz is the antenna noise density.

Fig. 2 shows the theoretical and the simulated dependance of the SNR with the LPF cutoff frequency.

Considering now a band-limited satellite signal $\mathbf{d}' = \mathbf{d} * \mathbf{h}_{LPF}$, using the Wiener-Khinchine theorem and the convolution properties [12, 5], we can compute the cross-correlation between $\mathbf{d}'$ and $\mathbf{d}_L$:

$$r_{\mathbf{d}'\mathbf{d}_L}[n] = \int_{-\frac{1}{2}}^{+\frac{1}{2}} S_{\mathbf{s}}(f)|H_{LPF}(f)|^2 e^{j2\pi f n} df, \tag{21}$$

where the despreading gain is equal to the filtered signal power $r_{\mathbf{d}'\mathbf{d}_L}[0] = P_s'$. Clipping the bandwidth of the received satellite signal makes the correlation peak wider. Fig. 3 shows the evolution of MBOC(6,1,1/11) $r_{\mathbf{d}'\mathbf{d}_L}[n]$ function for different baseband bandwidths. We obtain the 80% of the despreading gain using $B_{bb} = 2$ MHz, which is the minimum usable baseband bandwidth given by $B_{bb} \geq \frac{1}{2T_c}$ according to Nyquist-Shannon criterium [12]. Considering an acquisition search grid size of $\tau = \pm 0.5$ chips and $F = \pm 250$ Hz [9],

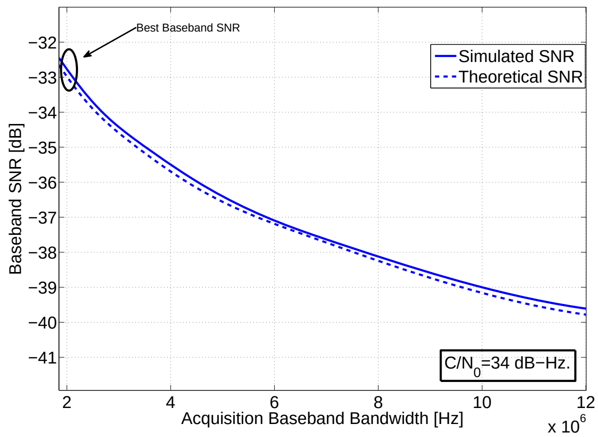

Figure 2: Theoretical and simulated Galileo E1 MBOC(6,1,1/11) SNR versus the baseband bandwidth.

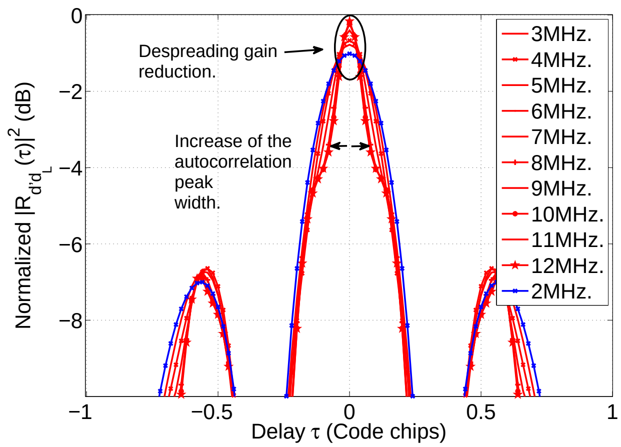

Figure 3: Autocorrelation of Galileo E1 MBOC(6,1,1/11) signal with the local replica versus the baseband bandwidth.

all the despreading gain can be contained in a single grid cell.

Recalling Section 4.2 and 4.3 the limited baseband bandwidth affects both the P_{fa} and P_d and consequently the ROC is affected according to the values of:

$$(\lambda^2)' = NP_s' \tag{22}$$

$$(\sigma^2)' = \frac{P_n'}{2K}, \tag{23}$$

where $(\lambda^2)'$ and $(\sigma^2)'$ are the new values of λ^2 and σ^2, respectively. It is useful to express the dependence in terms of the ρ_{acq}:

$$\frac{(\lambda^2)'}{(\sigma^2)'} = 2KN\rho_{acq}, \tag{24}$$

which implies that the maximization of ρ_{acq} maximizes the acquisition ROC performance. The usual threshold setting criterion for a GNSS receiver is to maximize P_d for a given P_{fa}, and γ should be computed using an estimation of the filtered noise power and the P_{fa} equation (13).

5. SIMULATION RESULTS

In order to verify the theoretical study of the array-based ML acquisition, the false alarm and the detection probabilities of the test functions are evaluated by means of Monte Carlo (MC) simulations. The results are valid for a single cell acquisition and can be easily extended to multiple cell search strategies [17]. We simulated a single Galileo satellite on the E1 MBOC(6,1,1/11) carrier signal [6] impinging on an 8-elements circular isotropic antenna array with half-wavelength separation between elements. The channel vector $\mathbf{h}$ was generated with equal power $|h_i|^2 = \frac{P_s}{2} \, \forall i$ and random DOA. The sampling frequency F_s and T_{acq} was set to $F_s = 50\frac{1}{T_c} = 51.150$ MHz and $T_{acq} = 4$ ms, respectively. The ideal RF BPF was set to have a bandwidth of $B_{RF} = 24.552$ MHz. The FIR LPF implementation was a Butterworth type with 5 coefficients. Each of the MC simulations contains 2000 realizations.

5.1 ROC evolution for different acquisition baseband bandwidths

The effect of the acquisition bandwidth was simulated for a constant $C/N_0 = 25$ dB-Hz and bandwidth sweep $2 \leq B_{bb} \leq 13$ MHz. Using the MC results, the P_{fa} and P_d curves and the ROC can be found in Fig. 4 and Fig. 5 respectively.

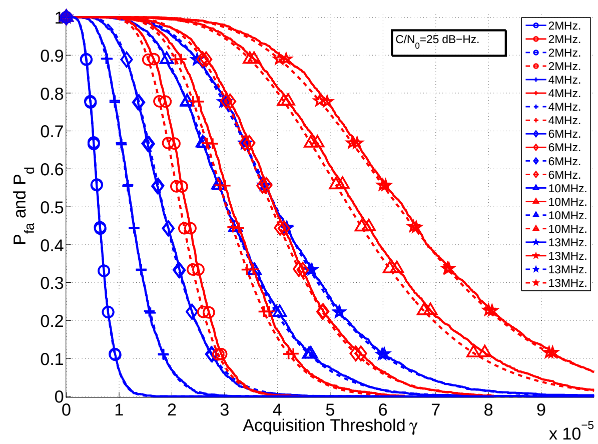

Figure 4: Theoretical and simulated P_{fa} (blue) and P_d (red) for 8-elements antenna array ML-based acquisition over different baseband bandwidths.

The theoretical performance was calculated using the analytical expressions for the P_{fa} and P_d defined in (13) and (16), respectively. Notice there are a slightly differences between the theoretical curves and the MC curves. The main reason is the differences between the implemented LPF filter frequency response and the theoretical LPF frequency response used in (18) and (20). Despite this effect, the results were aligned with the theory and the acquisition with the minimum bandwidth obtained the best performance.

5.2 ROC evolution for different C/N_0

Considering a fixed $B_{bb} = \frac{B_{RF}}{2}$ MHz, we performed a sweep of the impinging satellite signal C/N_0 from 25 dB-Hz to 33 dB-Hz. Fig. 6 shows the theoretical and simulated ROC for the array acquisition compared to a single antenna acquisition. The theoretical models for false alarm and detection probabilities are aligned with the simulations. The improvement of the array based non-coherent ML acquisition over the single antenna non-coherent acquisition is $G_{array} = 10\log(8) - 3 \simeq 6$ dB (see Section 4.3).

6. CONCLUSIONS

In this paper, we investigated the application of the antenna array ML Doppler shift and code delay estimator to

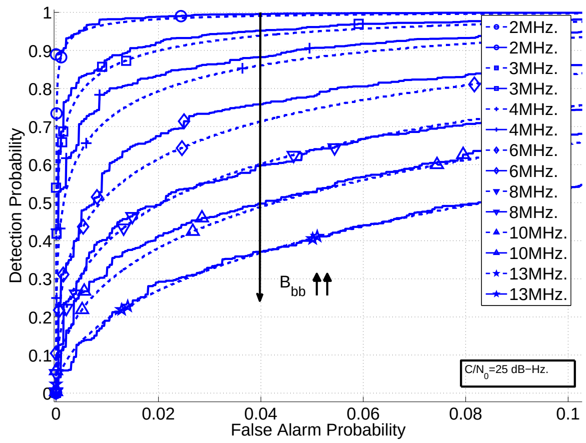

Figure 5: Theoretical and simulated ROC for 8-elements antenna array ML-based acquisition over different baseband bandwidths.

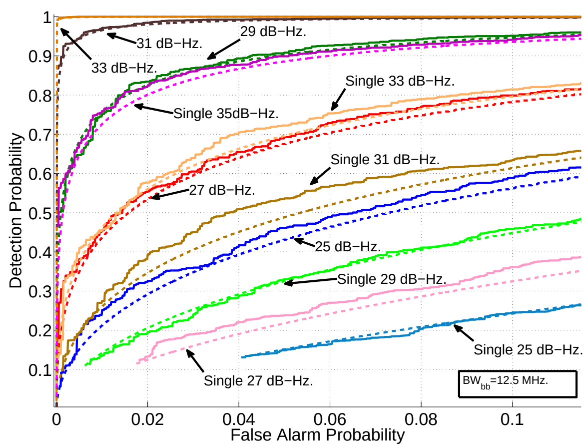

Figure 6: Theoretical and simulated ROC for single antenna and 8-elements antenna Array ML-based acquisition over different C/N_0 values.

the GNSS acquisition problem, considering an unstructured channel model and AWGN. The proposed acquisition test function was analyzed in terms of the detection and false alarm probabilities, and closed-form expressions were obtained. We found the theoretical improvement of the ROC performance with respect to a single antenna acquisition, and we also showed the dependance on the acquisition bandwidth. In particular, in order to obtain the best acquisition performance, the acquisition bandwidth needs to be reduced to maximize the SNR. Limiting the acquisition bandwidth we increase the detection probability and we reduce the false alarm probability. We proposed also an implementation scheme including a baseband LPF to improve the ROC performance according with the model. In particular, for the Galileo E1 MBOC(6,1,1/11) signal, the receiver acquisition baseband bandwidth should be 2 MHz to obtain the best ROC performance.

7. ACKNOWLEDGMENTS

This work has been partially supported by the Spanish Science and Technology Commission: CENIT2007-2002 (TIMI) and TEC2008-02685/TEC (NARRA), and by the European Commission in the framework of the FP7 Network of Excellence in Wireless COMmunications NEWCOM++ (contract n. 216715) and COST Action IC0803 (RFCSET).

REFERENCES

[1] B. Wang and H. M. Kwon, "PN code acquisition using smart antenna for spread-spectrum wireless communications part I," *IEEE Transactions on Vehicular Technology*, vol. 52, no. 1, pp. 142–149, Jan 2003.

[2] H. Puska, H. Saarnisaari, J. Iinatti, and P. Lilja, "Serial search code acquisition using smart antennas with single correlator or matched filter," *IEEE Transactions on Communications*, vol. 56, no. 2, pp. 299–308, Feb 2008.

[3] W. H. Ryu, M. K. Park, and S. K. Oh, "Code acquisition schemes using antenna arrays for DS-SS systems and their performance in spatially correlated fading channels," *IEEE Transactions on Communications*, vol. 50, no. 8, pp. 1337 – 1347, Aug 2002.

[4] J.Curran, D.Borio, C. Murphy, and G. Lachapelle, "Reducing front-end bandwidth may improve digital GNSS receiver performance," *IEEE Transactions on Signal Processing*, Accepted 2010.

[5] D. Borio, *A statistical theory for GNSS signal acquisition*, Ph.D. thesis, Politecnico Di Torino, Torino, Italy, May 2008.

[6] Galileo Joint Undertaking, "Galileo Open Service. Signal In Space Interface Control Document (OS SIS ICD) Draft 1," Tech. Rep., European Space Agency / European GNSS Supervisory Authority, February 2008.

[7] C. Fernández-Prades, *Advanced Signal Processing Techniques for GNSS Receivers*, Ph.D. thesis, Universitat Politècnica de Catalunya (UPC), Barcelona, Spain, May 2006.

[8] S. M. Kay, *Fundamentals of statistical signal processing. Estimation theory*, Prentice Hall, 1993.

[9] J. Bao and Y. Tsui, *Fundamentals of Global Positioning System Receivers. A Software Approach*, John Willey & Sons, Inc., 2000.

[10] S. M. Kay, *Fundamentals of statistical signal processing v. 2 Detection theory*, Prentice Hall, 1998.

[11] D. Borio, C. O'Driscoll, and G. Lachapelle, "Coherent, noncoherent, and differentially coherent combining techniques for acquisition of new composite GNSS signals," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 45, no. 3, pp. 1227–1240, July 2009.

[12] J. G. Proakis, *Digital communications*, McGraw-Hill, 2000.

[13] S. Haykin, *Digital communications*, Wiley, 1988.

[14] R. A. Monzingo and T. W. Miller, *Introduction to adaptive arrays*, John Wiley & Sons, 1980.

[15] J. Arribas, D. Bernal C., Fernández-Prades, P. Closas, and J. A. Fernández-Rubio, "A novel real-time platform for digital beamforming with GNSS software defined receivers," in *Proceedings of the ION GNSS 2009, Georgia (USA)*, September 2009.

[16] J. W. Betz, "The offset carrier modulation for GPS modernization," in *Proceedings of the ION GNSS 1999 (USA)*, September 1999.

[17] D.Borio, L.Camoriano, and L.Lo Presti, "Impact of the acquisition searching strategy on the detection and false alarm probabilities in a CDMA receiver," in *Position, Location, And Navigation Symposium, 2006 IEEE/ION*, 25-27, 2006, pp. 1100–1107.