COMPARED PERFORMANCES OF MF-BASED AND LOCALLY OPTIMAL-BASED MAGNETIC ANOMALY DETECTION

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ABSTRACT

Although magnetic dipole detection was mainly investigated in the eighties, new interest for the question has been recently marked in literature. The aim of this paper is to revisit the different existing approaches based on the knowledge of the physics underlying the detection problem, and to give/recall the mathematical background on which the detection scheme is based. Thus, assessment of the detection method is possible, as well as the influence of the physical parameters on detector performances. Furthermore, the paper proposes to use a 3D information (measurement of the magnetic induction anomaly) instead of the common 1D information (use of a scalar sensor providing the total field modulus), both cases being encompassed in the same formalism. Finally, the case where noise is not Gaussian is investigated and a locally optimal receiver is studied and tested.

1. INTRODUCTION

In submarine passive detection of magnetic sources, the detector uses the well known fact that the presence of a magnetic dipole in the magnetic earth induces a local spatial magnetic anomaly. Thus, for a fixed source and a moving sensor (or the opposite), such a spatial deformation is measured by the sensor as a non constant signal. Several studies about the detection of such a signal, called MAD (for magnetic anomaly detection), have been done in the eighties, generally using a scalar sensor measuring the magnetic signal modulus [1, 2]. Surprisingly, although the modeling of the magnetic anomaly and the principle of its detection are quite old, the study of the detection of such signal is still under investigation [3, 4, 5], some of the papers being reconsideration and adaptation of the ancient studies. The goal of the present work is to revisit and develop the mathematical grounding of the detection scheme on which the previous papers are generally implicitly based. Thanks to the generality of our approach, it is then possible to extend the previous studies in the three following directions: (i) generalization in the case where the sensor measures the three (euclidean) components of the magnetic anomaly, by giving the theoretical justifications and showing that the scalar case can be properly included in the same formalism; (ii) the performances of the detection scheme can be analytically expressed in the specific case where the observation is projected on true signal bases, allowing the analyze of the specific influence of each physical detection parameters (iii) in the previous studies, the noise is assumed Gaussian: after showing how the nonGaussianity affects the performance of the receiver, an alternative scheme is proposed, based on the well-known locally optimal detectors (LOD). These three incremental extensions will be illustrated and the physical parameters can affect the performance or can be "tuned" will be shortly discussed.

2. ORTHOGONAL BASIS-BASED DETECTOR

2.1 The MAD approach

As introduced, we consider in the sequel of the paper a magnetic dipole field for the target (or source), represented by its magnetic moment $\mathbf{P}$. It is well known that the presence of $\mathbf{P}$ in the Earth's magnetic field $\mathbf{H}_0$ induces a local anomaly of this Earth's field, under the form $\mathbf{\Delta H} = \frac{1}{4\pi}\frac{3\,(\mathbf{P}^t\mathbf{OM})\,\mathbf{OM}-\|\mathbf{OM}\|^2\mathbf{P}}{\|\mathbf{OM}\|^5}$ where $R = \|\mathbf{OM}\|$ is the distance between the source at position O, and the sensor at the position M [1, 3, 5]. In the sequel, as usually done, the speed and the altitude of the sensor are assumed constant, and its motion is assumed linear. We will also consider the situation where the signal is sampled at a rate T_s and that the acquisition is performed during a finite size duration denoted T. We will then denote x the position of the sensor along its direction of motion, t_0 the instant at which the sensor is at its closest position to the source (point called CPA for closest point of approach), D the smallest distance of the sensor from the source, and $u = \frac{x}{D} = \frac{V(t-t_0)}{D}$ the standardized position where time $t = nT_s$ is an integer multiple of T_s and N points are acquired, $T = (N-1)T_s$. A rapid analysis of the geometry of the problem permits then to express the anomaly under the form

$$\mathbf{\Delta H}(t) = \sum_{i=0}^{2} \mathbf{c}_i \, \frac{u^i}{(1+u^2)^{5/2}} \ , \qquad (1)$$

where the basis $f_i(t) = u^i/(1+u^2)^{\frac{5}{2}}$ is only parametrized by the two parameters of the CPA, namely D and t_0, and not by the other physical parameters, and where the 3×1 coefficients $\mathbf{c}_i$ depend on the geometrical configuration. In the sequel, we will assume also that the orientation of the 3D sensor remains the same all along the motion. Thus, coefficients $\mathbf{c}_i$ will not depend on time. This assumption is not necessary in the scalar case where measurement is independent of sensor orientation (modulus measurement). More details can be found in [1, 3, 5] where in most cases the approach is scalar but transposable to the 3D context without additional effort, under the above hypothesis. Note that these assumptions can be met when the sensor is fixed and the source moving.

Let us build now the $3\times N$ matrix $\mathbf{\Delta H}$, by concatenating the N acquired measurements, $\mathbf{F}$ the $3\times N$ matrix of the N points for the 3 basis functions, and $\mathbf{C}$ the 3×3 matrix of the coefficients, $\mathbf{C}^t = [\mathbf{c}_0\ \mathbf{c}_1\ \mathbf{c}_2]$. Thus, the acquisition writes

 ISSN 2076-1465

equivalently in the compact form $\mathbf{\Delta H} = \mathbf{C}^t\mathbf{F}$. Since the basis of the $f_i(u)$ is not orthogonal, i.e. $\mathbf{F}\mathbf{F}^t \not\propto I$, the previous expression is generally rewritten using an orthonormal basis, e.g. via a Gram-Schmidt orthonormalization procedure [6] that obviously does only (potentially) depend on D and t_0 and not on the physical parameters. Let us denote $\mathbf{E}$ the considered orthonormal basis and $\mathbf{S}_3$ the coefficients in this new base, leading finally to the compact expression of the anomaly

$$\mathrm{s}_3 = \mathbf{S}_3^t\mathbf{E} \tag{2}$$

Finally, in the scalar context where the sensor measures the modulus of the total field (Earth's field and anomaly), it is very common to use the assumption that the anomaly is far smaller than the Earth field, and its angle to this field far small: the total field modulus is then approximated by the sum of the Earth's field modulus and the anomaly projection onto the direction of the Earth field [1, 3], leading to the measured signal $s_1 \stackrel{\mathrm{d}}{=} \mathbf{h}_0^t\mathrm{s}_3$,

$$s_1 = \mathbf{S}_1^t\mathbf{E} \quad \text{where} \quad \mathbf{S}_1 = \mathbf{S}_3\mathbf{h}_0 \quad \text{and} \quad \mathbf{h}_0 = \mathbf{H}_0/\|\mathbf{H}_0\|. \tag{3}$$

As our approach is general, in the sequel of the paper, we will use the notation s and $\mathbf{S}$ for the 3D context and for the scalar context as well.

2.2 Optimal and near-optimal detection

In practice, the sensor measurement is affected by noise (e.g. electronic noise, etc.). Thus, to detect the presence of the anomaly we are faced to the classical binary decision problem [7, 8]: from the observation of N samples of an acquisition, we have to decide whether a signal is present in the observation, versus the observation is noise only,

$$\left\{ \begin{array}{lcccl} \mathrm{H}_0 & : & \mathbf{r} & = & \xi \\ \mathrm{H}_1 & : & \mathbf{r} & = & \mathrm{s}+\xi \end{array} \right. . \tag{4}$$

Signal s is the magnetic anomaly to detect (the Earth's field is assumed removed), assumed deterministic, while ξ denotes the matrix-variate observation noise. The probability density function (pdf) of ξ, denoted f_ξ is assumed known and ξ is assumed zero-mean and of covariance matrix proportional to the identity $\sigma^2\mathbf{I}_d \otimes \mathbf{I}_3$ in the dD context ($d = 1$ or 3), where σ^2 represents then the variance of each spatio-temporal component of the noise, where $\mathbf{I}_k$ is the $k\times k$ identity matrix and where $\otimes$ is the Kronecker product of the temporal covariance and the spatial one (see e.g. [9, th. 2.3.10] for more details on matrix variates random variables).

When the signal to be detected is known, the optimal receiver (Neyman-Pearson criterion, Bayes criteria, etc.) is the well known likelihood ratio test (LRT) $\Lambda_{\mathrm{lr}} = f_\xi(\mathbf{r}-\mathrm{s})/f_\xi(\mathbf{r}) \gtrless \eta$ [7, 8], where $\gtrless$ means that if Λ_{lr} is higher than the decision threshold η, hypothesis H_1 is decided, and H_0 otherwise and where the decision threshold η is determined by the chosen detection criterion [7, 8]. In the particular case where f_ξ is a matrix-variate Gaussian pdf (here of covariance proportional to the identity), the log-likelihood ratio (LLR) turns out to be the well-know matched filter (MF), that writes here in the matrix-variate form $\Lambda_{\mathrm{mf}} = \mathrm{Trace}(\mathrm{s}\,\mathbf{r}^t)$. Remind that this correlator receiver is also the linear receiver that maximizes the so-called deflection $D_\Lambda = (E[\Lambda|_{\mathrm{H}_1}] - E[\Lambda|_{\mathrm{H}_0}])/VAR[\Lambda|_{\mathrm{H}_0}]$, regardless the noise statistics [7, 8].

2.2.1 *Known (or assumed known) orthonormal basis*

The signal to be detected is generally unknown, and thus the (L)LR or the MF cannot be used. However, in the special physical context under study here, the signal subspace is known, up to the two parameters D and t_0: s decomposes in the basis $\mathbf{E}$ via (2).

When the basis is known (i.e. D and t_0), which may arise in a cooperative context, but also in a non-cooperative context provided a localization has been already made while the signal itself is unknown (e.g. lack of knowledge on the physical parameters of the source), the classical approach consists then in the so-called generalized likelihood ratio test approach (GLRT): the unknown parameters $\mathbf{S}$ of the signal $\mathrm{s} = \mathbf{S}^t\mathbf{E}$ to be detected are replaced by their maximum likelihood estimation (MLE) [7, 8, 10]. In particular, when the noise ξ is Gaussian, one can easily show [1, 10] that the signal coefficients $\mathbf{S}$ that maximize the likelihood $f_\xi(\mathbf{r}-\mathbf{S}^t\mathbf{E})$ is the projection of the observation $\mathbf{r}$ onto the orthonormal basis $\mathbf{E}$,

$$\widehat{\mathbf{S}} = \mathbf{E}\mathbf{r}^t. \tag{5}$$

This estimator is also the linear one that minimizes the residual error $E[\|\mathbf{r}-\mathbf{S}^t\mathbf{E}\|^2]$ (least square error approach), regardless the noise pdf.

Using the estimated signal in the matched filter, it becomes $\Lambda_{\mathrm{ba}} = \mathrm{Trace}(\widehat{\mathbf{S}}^t\mathbf{E}\mathbf{r}^t)$, which is, from (5),

$$\Lambda_{\mathrm{ba}} = \mathrm{Trace}\left(\widehat{\mathbf{S}}^t\widehat{\mathbf{S}}\right) = \left\|\widehat{\mathbf{S}}\right\|^2, \tag{6}$$

where $\|\cdot\|$ denotes the Froebenius norm. As for the 1D context, it also corresponds to the energy of the projected observation to the orthonormal basis [1].

2.2.2 *Unknown parameters of the basis*

In the most realistic noncooperative context, the basis itself is unknown (D and t_0 unknown). Thus, to be able to estimate properly the signature s from the observation, one needs to estimate the orthonormal basis, i.e. to estimate the unknown parameters D and t_0. Again, a GLRT-like approach consists in replacing D and t_0 by the values that maximize the likelihood ratio. Here, it is difficult to extract explicitly the optimal estimators of D and t_0, thus the principle is to implement directly

$$\Lambda_{\widehat{\mathrm{ba}}} = \max_{D,t_0} \left\|\widehat{\mathbf{S}}\right\|^2 \tag{7}$$

using an optimization procedure to track D and t_0. The most usual approach consists in considering a bank of receivers (6), built for several D and t_0 [1, 3]. Here, a stochastic optimization tool has been used, namely a genetic algorithm [11], in order to both avoid a fixed grid for search space and a convergence to a possible local maximum. In our tests it appears that such an approach allowed to achieve the same performance than that given for large bank of receivers, but for a smaller computational time cost. Since such an optimization goes beyond the goal of this paper, we will not go further in such an optimization.

3. LOCALLY OPTIMAL RECEIVER

The Gaussian assumption is very often made and justified via the central limit theorem (sum of large number of noise sources). However, because of the geologic noise and the possible presence of various magnetic small sources, an impulsive noise model should be more adequate in such a situation. When the noise is nonGaussian, the LR does not give the MF, but the MF-based detector can still be implemented and when the basis is known, the performance will not be changed (see further). However, the nonGaussianity can have an impact when the basis has also to be estimated. Furthermore, the MF is no more optimal and other approaches may improve the performance.

Evaluating the LR is difficult in the nonGaussian context. But when the signal to be detected is known, a useful approach consists in approximating the LLR by its first order Taylor expansion leading to

$$\Lambda_{\text{lo}} = -\,\text{Trace}\left(\mathbf{s}\,\frac{\nabla^t f_\xi(\mathbf{r})}{f_\xi(\mathbf{r})}\right), \tag{8}$$

which is known as locally optimal (LO) receiver [7, §6.7 & app. 6E], [8, chap. 2], [12, 13] [14, §19.4], [15, §2.3], [16, chap. 2] where ∇ stands for the gradient operator (i.e. of size $d \times T$ here) and where the nonlinearity $\frac{\nabla f_\xi(\mathbf{r})}{f_\xi(\mathbf{r})}$ applied to the observation is known to be the score function [7, 8]. This approach is well-suited in the assumption that the signal is small compared to the noise level. Note that among the correlator receivers of the form $\text{Trace}(\mathbf{s}\mathbf{h}(\mathbf{r})^t)$, the receiver that maximizes the deflection is precisely the LO receiver ($\mathbf{h}$ is the score function): among the correlator receivers, the LO is optimal in the maximum deflection sense, regardless the noise pdf.

Once again, in a real context, $\mathbf{s}$ is generally unknown. As for the Gaussian case, when the basis is known, in a GLRT spirit one has to determine the optimal estimation of $\mathbf{S}$, in the likelihood sense (i.e. the MLE). This is a difficult task since we are faced to a nonlinear problem. An optimization approach can provide a high computational cost since the the maximization has to be performed over 9 variables (in the 3D context). The genetic approach is difficult to tune, and due to the dimension a bank over the $S_{i,j}$ may be too costly. Instead, we turn out to the use of the least square estimator (5), leading to receiver

$$\Lambda_{\text{loba}} = -\,\text{Trace}\left(\mathbf{r}\mathbf{E}^{\mathbf{t}}\mathbf{E}\,\frac{\nabla^t f_\xi(\mathbf{r})}{f_\xi(\mathbf{r})}\right). \tag{9}$$

Such an approach is suboptimal, however, recall that it remains the best linear estimator of $\mathbf{S}$ in terms of mean square error.

Finally, in the case of unknown basis, as for (7), the receiver will be chosen to be the maximum of Λ_{loba} over D and t_0 (e.g. bank of LO-based receivers for several D and t_0 or genetic algorithm).

4. PERFORMANCE OF THE DETECTORS

In this section, we give the sketch of the calculus allowing to evaluate the performance of the detectors, when it is tractable. Having analytical expression is important since it allows to determine the decision threshold, and further to be able to determine the influence of the physical parameters or how to tune them for particular tasks.

4.1 MF-based approach and known basis

From eq. (5), under hypothesis H_k, the estimation of $\mathbf{S}$ writes

$$\widehat{\mathbf{S}} = \mathbf{E}\xi^t + k.\mathbf{E}\mathbf{s}^t = \mathbf{E}\xi^t + k.\mathbf{S}\ . \tag{10}$$

When the noise is Gaussian, this matrix is Gaussian of mean $k.\mathbf{S}$ and of covariance $\sigma^2(\mathbf{E}\,\mathbf{E}^t)\otimes\mathbf{I}_d = \sigma^2\mathbf{I}_3\otimes\mathbf{I}_d$ (see e.g. [9, th. 2.3.10]). Evoking the central limit theorem (if N is large), the Gaussian assumption of $\widehat{\mathbf{S}}$ still applied (asymptotically) for a nonGaussian noise (of finite variance). Thus, under hypothesis H_k, the squared norm $\|\widehat{\mathbf{S}}\|^2/\sigma^2$ has a noncentral chi square distribution with $3d$ degrees of freedom and with noncentral parameter $k.SNR = k.\frac{\|\mathbf{S}\|^2}{\sigma^2}$ [17, chap. 29]. Under these conditions, the detection and false alarm probabilities are:

$$P_{\text{fa}} = 1 - F_{\chi^2_{3d}}\left(\frac{\eta}{\sigma^2}\right), \quad P_{\text{d}} = 1 - F_{\chi^2_{3d},SNR}\left(\frac{\eta}{\sigma^2}\right), \tag{11}$$

where $F_{\chi^2_{3d}}$ and $F_{\chi^2_{3d},SNR}$ denote the cumulative distribution function (cdf) of the central and noncentral chi square distributions with $3d$ degrees of freedom [17]. The same type of derivation has been made in [18] in the context of an energy detector. The expression of cdf of noncentral chi square distribution is only defined via the integral involving a modified Bessel function of the first kind, sometimes referred as incomplete Torontos functions [17, eq. 29.4], [18] or [19, eq. (100b), p. 182]. However, nowadays these cdf and their inverse (quantile) are tabulated in many mathematical softwares. Note also that some approximations have also been derived [18, and ref.].

In Neyman-Pearson strategy, that consists in fixing a false alarm probability, the optimal decision threshold can be analytically written. This is very important from an operational point of view. In the same vein, the direct expression of the probability detection versus the false alarm probability, i.e. the receiver operational characteristic (ROC) can be explicitly expressed from (11). From [17, eq. (29.25a)], one has the fact that the performance increases with the SNR, as obviously expected.

It is remarkable to note that the above developments and analytical expressions permit also (at least numerically) to determine the effect of the physical parameters on the performance. Indeed, the performance are obviously dependent on the SNR, that depends itself of the physical parameters through $\mathbf{S}$ (see [1, 3, 5]). As an example, quantifying the loss of performance induced by the decrease of the magnetic moment (and thus of the SNR) is possible. A contrario, it is possible to tune the physical parameters allowing a desired degree of performance, so that the maximal distance D of detection, or the minimal detectable magnetic momentum (e.g. to be non-detectable). Indeed, fixing a desired level of performance will determine what the SNR has to be, via (11), and thus will fix the physical quantities involved in it (i.e. in $\mathbf{S}$). The quantification of the influence of each parameter is then practically very pertinent to evaluate expected MAD system (or fixed detector) performances.

Note that if the basis is assumed known and fixed, but is erroneous, it is possible to bound the loss of performance. Let us denote $\widetilde{\mathbf{E}}$ this orthonormal erroneous basis. The estimation reads now $\widehat{\mathbf{S}} = \widetilde{\mathbf{E}}\xi + k\widetilde{\mathbf{E}}\mathbf{s} = \widetilde{\mathbf{E}}\xi + k\widetilde{\mathbf{E}}\mathbf{E}^t\mathbf{S}$ and with the same way of making as with the correct basis, one obtains performance (11), simply replacing the SNR by $\widetilde{SNR} = \frac{\|\widetilde{\mathbf{E}}\mathbf{s}^t\|^2}{\sigma^2}$ and yielding from [6], $\widetilde{SNR} \leq SNR$, with equality only if s is in the span of $\widetilde{\mathbf{E}}^t$, namely $\widetilde{\mathbf{E}}$ is an orthogonal transformation (rotation) of $\mathbf{E}$, i.e. both parameters $\widetilde{D}$ and $\widetilde{t}_0$ are the right ones.

Reminding that the performance degrades with a decreasing SNR, the performance obviously decreases when parameters $\widetilde{D}$ and $\widetilde{t}_0$ are erroneous. Furthermore, [17, eq. (29.25a)] permits to have a bound of the loss of performance, $P_{\mathrm{d}} - \widetilde{P}_{\mathrm{d}} \leq \frac{\sqrt{SNR}-\sqrt{\widetilde{SNR}}}{\sqrt{2\pi}} F_{\chi^2_{3d-1}}(F^{-1}_{\chi^2_{3d}}(1-P_{\mathrm{fa}}))$.

4.2 LO-based detectors – estimated basis

Contrary to the previous MF-based situation with known (or erroneous) basis, determining the analytical performance of the LO-based receiver is a difficult task. This is mainly due to the score function (nonlinear transformation of the data). The study of the analytical performance is still under investigation. As in the previous case, we turn out to use Monte-Carlo (MC) simulations to numerically assess the LO-based detector.

When the basis itself is unknown and thus estimated we will not yet achieve to analytically write the performance even in the case where a bank of receivers for several D and t_0 are considered instead of using an optimization algorithm. Indeed, the maximum operation leads to the study of order statistics of receivers, that are not independent. Such a task is not easy [20]. Thus, we turned out to make Monte-Carlo (MC) simulations to assess the approach in this context, both for the MF-based and the LO-based detectors. Furthermore, at this step the noise statistics will influence the performance. Indeed, contrary to the case where the basis is known, for impulsive noise, the search for a basis may lead to confusing a "spike" of noise with a magnetic signature, provoking an increase of the false alarm probability.

5. DETECTION IN ACTION

The signature and a snapshot of its noisy observation used for the following simulations are illustrated figure 1 in arbitrary units. Two kinds of corrupting noise are considered: the usual Gaussian one, and a Laplacian noise (or doubly exponential). In both cases, the noise is assumed (spatio-temporaly) white in the strict sense and the physical parameters here are so that the value of $SNR = \|\mathbf{S}\|^2/\sigma^2$ is $SNR = 9$ dB.

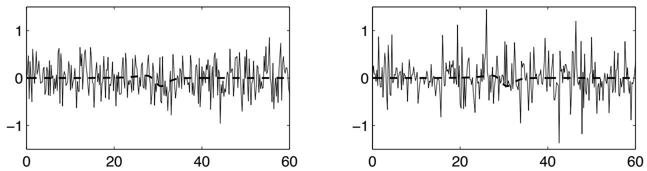

Figure 1: A snapshot of the first component of a noisy magnetic signature, where the noise is Gaussian (top) and Laplacian (bottom) respectively. The noiseless signal is depicted in dashed line.

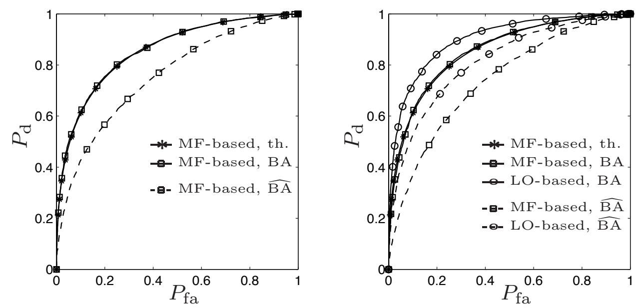

Figure 2: ROC curves for the 3D MF-based and LO-based detectors (6)-(9), for Gaussian (left) and Laplacian (right) noise. In both figures, the basis is known (solid line) or estimated via its parameters (dashed line); the stars represents the theoretical result (namely (11)), the squares correspond to MC of the MF-based receivers, and the circles to LO-based receivers (5000 snapshots have been used).

Figure 2 depicts the comparative performance for the detectors presented in the previous sections in the 3D context, for a Gaussian noise (left figure) and for a Laplacian noise (right figure). Concerning the estimation of the basis, we have considered here a genetic algorithm to achieve the maximum of (6) or of (9), with a population of 30 and at most 100 generations.

These figures and the previous analysis permit to derive the following conclusions.

- For the MF-based detector, when the basis is known, the Monte-Carlo simulations confirm the theoretical results, and validate the robustness of the performance versus the statistics of the noise. Furthermore, although not really developed here, it is possible to finely analyze the influence of each physical parameters, as previously mentioned.
- For the MF-based context and when the basis is unknown, the maximization will induce an increase of false alarm probability, and thus a loss of performance, as illustrated in the figure. In the Laplacian context, the loss of performance is more pronounced as intuitively explained in the previous section: spikes of noise can be interpreted as magnetic signatures, increasing then the false alarm probability.
- As expected, in the Laplacian case, the LOD approach provides better performance than the MF-based approach, both in the known basis and unknown basis cases. The improvement seems more pronounced in the case of unknown basis: this can be interpreted as the fact that the score (i.e. LO) function tends to smooth the spiky nature of the noise; thus the degradation due to the estimation is less pronounced than in the MF-based case. Although not illustrated, the improvement is generic (at least for all the cases among the generalized Gaussian noise we have tested). However, this remains to be shown analytically and the approach remains to be refined especially concerning the estimation of $\mathbf{S}$.

Another illustration of the improvement induced by the LO approach in the Laplacian context is given by figure 3 that depicts P_{d} as a function of the source-sensor (normalized) distance D, when P_{fa} is fixed (here at 10%). As an example for high probability of detection, a gain between 5 and 10% in terms of distance of detection is obtained (and more for smaller probability of detection).

6. DISCUSSION

In this paper, the MAD detection has been extrapolated to the case of 3D sensors, that may be used in a detection context

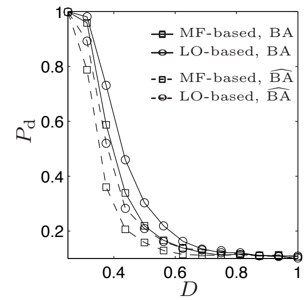

Figure 3: Probability of detection P_d as a function of the normalized distance source-sensor D, for a fixed false alarm probability $P_{fa} = 10\%$, when the noise is Laplacian. Same legend than in figure 2.

and that seems more and more accurate nowadays. While the (quite old) detection scheme is based on Gaussian noise, we have shown here how the "classical" locally optimum detector can be derived in the MAD framework. Due to the central limit theorem, the performances of the MF-based detectors are robust in the potential case of nonGaussian noise, as explained and briefly illustrated. When the noise is non-Gaussian, the LOD exhibits better performance, as illustrated in the Laplacian case. Since, in a detection context, even a slight improvement for the performance may be of great interest, the real noise statistics have to be carefully investigated and taken into account. This noise modeling represents a pertinent axis for new researches and developments for detection.

In the case of the MF-based detector using known basis, the performances have been analytically expressed. This result is of great interest for the following reasons. First, it allows to determine the decision threshold once the decision criterion is chosen (e.g. false alarm probability fixed). It also makes it possible to clearly analyze the effect of each physical parameter on performance. On the opposite, analytical expression allows the tuning of some parameters (e.g. determining the maximum distance of the sensor) to meet a given degree of performance. To generalize this possibility to the other cases presented in the paper, the analytical theoretical performances are also under investigation when the basis parameters are estimated, as well as in the two LO-based receivers. For example, concerning the MF-based receiver, the aim is to derive the statistics of the estimates of D and t_0, since conditionally to these parameters the performances of the MF-based receiver are known.

For further work, effort must be focused on the basis parameter estimation (Bayes-based scheme, comparative studies on optimization algorithms (see e.g. [21, chap. 8]), estimations/detection bounds, etc.) because estimation error may strongly affect performances. Replacing the LS estimation in the nonGaussian context, by taking into account here again the statistics of the noise, remains also to be done, although it will induce an additional computational cost. Finally, assessing the robustness versus the different assumptions (linear and constant speed motion, no rotation of the sensor along the motion, etc) has to be taken into consideration.

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