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MAXIMIN ROBUST RADAR WAVEFORM DESIGN FOR DETECTION OF WEAPONS

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ABSTRACT

We consider the problem of designing transmit waveforms based on target signature exploitation for detection of weapons under single-antenna monostatic radar operation. The target impulse response changes with the target orientation relative to the radar, which may not always be available or accurately determined in practical situations. We assume that the true target impulse response belongs to some uncertainty class of impulse response functions, which encompasses the impulse responses corresponding to the various target orientations. A transmit waveform-receiver filter combination is then designed to achieve the best lower bound on performance within this class, assessed by signal-to-clutter-and-noise ratio. Supporting design examples using electromagnetic modeled data are provided.

1. INTRODUCTION

The ultimate objective of an urban sensing system is to detect, locate, and classify animate and inanimate targets, such as humans and weapons, behind walls and barriers [1]. Radio frequency (RF) is the modality of choice since RF signals have the ability to penetrate optically opaque media. Design of appropriate waveforms with desirable characteristics is important for improved detection of targets of interest which is an important part of the performance of urban sensing radar systems applied to point and spatially extended targets.

Sufficient information about the properties and characteristics of the targets, such as shape, size, and composition, is available *a priori through EM modeling and experimentations*. Waveform design techniques based on target signature exploitation make use of the a priori information to achieve higher probability of target detection [2]-[4]. Design of transmission waveforms using target signatures has been recently investigated in the context of urban sensing and imaging targets behind walls [5], [6].

In this paper, we do not address urban sensing problems of imaging based on multiple antennas or arrays [7], [8], but rather consider the problem of designing transmit waveforms and receivers for detection of weapons under single-antenna monostatic radar operation. The optimal approach to this problem is based on the matched illumination signature exploitation concept in which the transmit waveform and the receiver filter are designed such that the signal-to-clutter-and-noise-ratio (SCNR) at the output of the receive filter is maximized. This approach assumes perfect knowledge of the target impulse response. In practice, however, the target impulse response changes with target orientation relative to the radar, which may not always be available especially when the target is behind walls or in enclosed structures. Thus, it may be more appropriate to assume that the true target impulse response belongs to some uncertainty class of impulse response functions which encompasses the impulse responses corresponding to the various target orientations. A waveform-filter combination can then be designed to achieve the best lower bound on performance over this class. That is, we design transmit waveforms and receive filters which are max-min solutions for the SCNR over the class of allowable target impulse response functions.

We consider an AK-47 assault rifle as the target of interest and design both aspect dependent matched illumination and orientation independent maximin waveforms using electromagnetically modeled AK-47 impulse responses. Detection performance of designed waveforms is evaluated in the presence of noise only and is compared to that of the commonly used chirp signal of the same duration and energy.

The paper is organized as follows. Section 2 summarizes the signature exploitation waveform design approaches for known and uncertain target orientations. Supporting waveform design examples for the AK-47 rifle are provided in Section 3. Performance comparison with a chirp waveform is also provided in this Section. Section 4 contains concluding remarks.

2. WAVEFORM DESIGN

We model the stationary or slow-moving extended targets of interest as linear time-invariant systems over the observation period. Let the finite-energy $N_z \times 1$ transmitted signal vector be defined as $\boldsymbol{z} = [z_0, z_1, \ldots, z_{(N_z-1)}]^T$. Let the target impulse response apparent to the radar be given by an $N_q \times 1$ vector $\boldsymbol{q} = [q_0, q_1, \ldots, q_{(N_q-1)}]^T$. Then, the received target return, free of noise and clutter, is represented by $\boldsymbol{s} = [s_0, s_1, \ldots, s_{(N_s-1)}]^T$, where $N_s = N_z + N_q - 1$, and can be expressed as

$$\boldsymbol{s} = \boldsymbol{Q}\boldsymbol{z}. \tag{1}$$

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Here, $\boldsymbol{Q}$ is the $N_s \times N_z$ target convolution matrix, given by

$$\boldsymbol{Q} = \begin{bmatrix} q_0 & 0 & \cdots & 0 \\ q_1 & q_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ q_{(N_q-1)} & q_{(N_q-2)} & \cdots & q_{(N_q-N_z-1)} \\ 0 & q_{(N_q-1)} & \cdots & q_{(N_q-N_z)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{(N_q-2)} \end{bmatrix}. \qquad (2)$$

Likewise, by representing the noise by $\boldsymbol{n} = [n_0, n_1, \ldots, n_{(N_s-1)}]^T$ and the clutter impulse response matrix as

$$\boldsymbol{C} = \begin{bmatrix} c_0 & c_{-1} & \cdots & c_{(N_z-1)} \\ c_1 & c_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & c_0 \\ \vdots & \ddots & \ddots & c_1 \\ \vdots & \ddots & \ddots & \vdots \\ c_{(N_s-1)} & 0 & \cdots & c_{(N_q-2)} \end{bmatrix}, \qquad (3)$$

the received signal can be expressed as

$$\boldsymbol{r} = \boldsymbol{s} + \boldsymbol{C}\boldsymbol{z} + \boldsymbol{n}, \;\; \boldsymbol{r} = [r_0, r_1, \ldots, r_{(N_s-1)}]^T \qquad (4)$$

Accordingly, the system output, after receiver filter, is given by

$$y = \boldsymbol{b}^T\boldsymbol{r} = \boldsymbol{b}^T\boldsymbol{s} + \boldsymbol{b}^T\boldsymbol{C}\boldsymbol{z} + \boldsymbol{b}^T\boldsymbol{n} \qquad (5)$$

where $\boldsymbol{b} = [b_0, b_1, \ldots, b_{(N_s-1)}]^T$ is the receive filter impulse response.

We assume that the target's impulse response is deterministic, whereas the clutter and the noise are assumed to be independent wide sense stationary zero-mean real stochastic processes with known covariance matrices. SCNR at the output of the matched filter detector is given by [6]

$$\gamma = \frac{\left|\boldsymbol{b}^T\boldsymbol{Q}\boldsymbol{z}\right|^2}{E\{\left|\boldsymbol{b}^T\boldsymbol{C}\boldsymbol{z}\right|^2\} + E\{\left|\boldsymbol{b}^T\boldsymbol{n}\right|^2\}} \qquad (6)$$

Under the assumption of exact knowledge of the target orientation and, hence, known impulse response, matched illumination approach is used for optimal target detection in which the transmit waveform and the receiver filter are designed such that the SCNR at the output of the receive filter is maximized. In this case, the optimum matched filter can be written as the Weiner-Hopf equation [3]

$$\boldsymbol{b} = (\boldsymbol{R}_c + \boldsymbol{R}_n)^{-1}\boldsymbol{Q}\boldsymbol{z} \qquad (7)$$

where

$$\boldsymbol{R}_c = E\{\boldsymbol{C}\boldsymbol{z}\boldsymbol{z}^T\boldsymbol{C}^T\}, \; \boldsymbol{R}_n = E\{\boldsymbol{n}\boldsymbol{n}^T\} \qquad (8)$$

and $E\{\cdot\}$ is the expected value operator. Further optimization of the SCNR with respect to $\boldsymbol{z}$ yields the matched illumination waveform. In case of zero clutter, the optimal waveform is proportional to the eigenvector of the matrix $\boldsymbol{\Omega} = \boldsymbol{Q}^T\boldsymbol{R}_n^{-1}\boldsymbol{Q}$ corresponding to the largest eigenvalue. When both noise and clutter are present, the optimal waveform can be obtained by using an iterative procedure [3], [6].

For the case of unknown target orientation and thus unknown impulse response, we assume that the true target impulse response belongs to a convex uncertainty class of impulse response functions which encompasses the impulse responses corresponding to the various target orientations. That is, the target convolution matrix $\boldsymbol{Q}$ is a member of a class

$$S = \{\boldsymbol{Q} \mid \left\|\boldsymbol{Q} - \boldsymbol{Q}_0\right\|_F^2 \le \Delta\} \qquad (9)$$

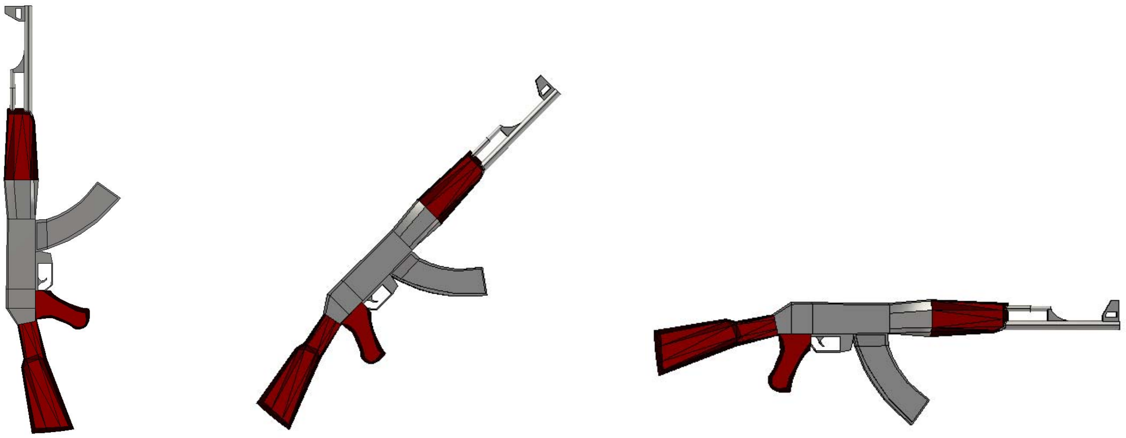
of target matrices which are within Δ of the nominal target matrix $\boldsymbol{Q}_0$ in terms of the squared of the Frobenius norm [6]. The optimization problem can then be expressed as

$$\max_{\boldsymbol{z},\boldsymbol{b}} \min_{\boldsymbol{Q}\in S} \frac{\left|\boldsymbol{b}^T\boldsymbol{Q}\boldsymbol{z}\right|^2}{E\{\left|\boldsymbol{b}^T\boldsymbol{C}\boldsymbol{z}\right|^2\} + E\{\left|\boldsymbol{b}^T\boldsymbol{n}\right|^2\}} \qquad (10)$$

An iterative algorithm, proposed in [6], can then be used to design the transmit waveform and receive filter which maximizes the worst SCNR among all of the possible target impulse responses in the uncertainty class S.

3. DESIGN EXAMPLES

Electromagnetic simulations were carried out using XFDTD®, a commercial full-wave electromagnetic simulator from Remcom Inc., for computing the impulse responses (range profiles) of the AK-47 rifle for vertical polarization corresponding to three tilt angles (0°, 45°, 90°) and azimuthal aspect angles from 0° to 359° at 1° increment. The rifle model used consists of 3mm cubical XFDTD mesh cells. In the model, the metallic parts of the rifle are assigned perfect electric conductors and the wooden components are chosen to be lossless with an assigned permittivity of 2. The simulation geometry for the three different tilt angles is shown in Fig. 1 for 0° azimuthal aspect. The azimuth angle is measured in a counter-clockwise fashion. The incident waveform was chosen to be a modulated Gaussian with frequency content ≥ 10 dB over the 0.5 GHz to 10 GHz frequency range. The scattered field due to the incident waveform was collected over 360° in azimuth under monostatic operation for each tilt angle. The corresponding target impulse responses were obtained by deconvolving the transmitted waveform from the scattered fields. In order to avoid any numerical modeling errors at both the lower and higher ends of the frequency range of interest, both the incident waveform and the target returns were bandpass filtered with a passband from 1 to 8 GHz prior to deconvolution. All of the impulse responses were resampled so that there is only one sample per range bin. The sampling period of the resultant impulse responses is 0.0625 ns.

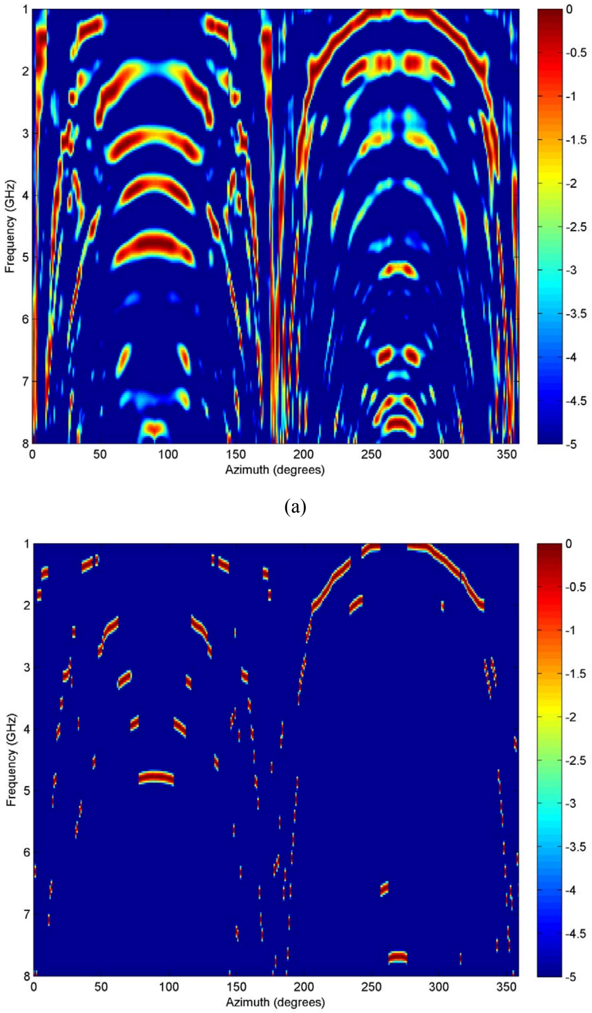


(a) (b) (c)

Fig. 1. AK-47 rifle at 0° azimuth. (a) 0° tilt, (b) 45° tilt, and (c) 90° tilt.

3.1 Known Target Response

The matched illumination concept was applied to the impulse responses covering full 360° azimuthal aspect of various tilt angles of the rifle, and the corresponding optimal waveforms were obtained for the case of white noise and zero clutter. Magnitude spectra of the AK-47 tilted at 45°, and the corresponding optimal transmission waveforms for each azimuthal aspect angle between 0° and 359° are depicted in Fig. 2. We observe that for each aspect angle, the energy in the optimal waveform is concentrated in a narrow frequency band corresponding to the frequency of the highest target response, which is characteristic of matched illumination detection waveforms in the presence of noise only [3]. The SNR as a function of azimuth aspect angle using the optimum signal and a conventionally used chirp waveform of the same energy and duration is provided in Fig. 3(a) for the rifle tilted at 45°. On average, the optimum waveform provides an improvement of 5.45 dB over the chirp signal. We note that in accordance with (7), the receive filter is matched to the expected target echo rather than the transmitted waveform for both the matched illumination and chirp waveforms.



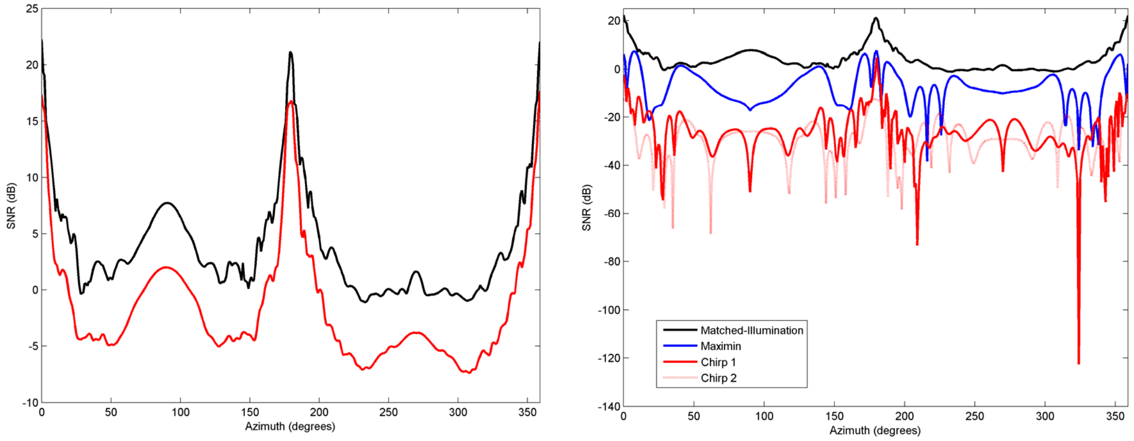
(a)

(b)

Fig. 2. Normalized magnitude of the frequency responses of (a) AK-47 rifle tilted at 45°, and (b) corresponding optimal matched illumination waveforms, over 360° in azimuth.

3.2 Uncertain Target Response

For this case, we assume that the target impulse response belongs to the uncertainty class S of eq. (9) wherein the nominal target convolution matrix $\boldsymbol{Q}_0$ is generated in accordance with eq. (2) using the target impulse response averaged over all tilt and azimuthal aspect angles. The corresponding target orientation independent maximin waveform was designed for the case of white noise only. The SNR as a function of azimuth aspect for the rifle tilted at 45° using the maximin and chirp waveforms of same duration and energy is shown in Fig. 3(b). Two cases of the chirp were considered. For the case labeled as ‘Chirp 1’ in the plot, the receive filter was matched to the transmitted chirp waveform, whereas for ‘Chirp 2’, the maximin receive filter is used for the chirp waveform. From Fig. 3(b), we observe that



(a) (b)

Fig. 3. SNR improvement over a chirp for the rifle tilted at 45° using (a) matched illumination waveforms, (b) maximin waveform.

the maximin waveform-filter design significantly outperforms both Chirp 1 and Chirp 2. The SNR obtained with the matched illumination waveforms is also provided and serves as an upper bound on target detection performance. On average, the maximin waveform underperforms the matched illumination waveform, which assumes perfect knowledge of the target impulse response, by about 11.6 dB. The maximin waveform and its magnitude spectrum are depicted in Fig. 4. Table I provides the average SNR improvement of the various waveforms over each other. A positive value in a cell indicates performance improvement of the design approach (column heading) over the waveform (row heading) while a degradation is indicated by a negative value.

4. CONCLUSION

In this paper, we presented target signature exploitation waveform design for detection of weapons under uncertainty in target orientation (tilt and azimuthal aspect). Performance of the designed maximin waveform was compared with that of the widely used chirp waveform of equal energy and duration for the noise only case. For the target considered and with the clutter-free target returns, the maximin waveform significantly outperformed the chirp in terms of the SNR at the output of the receive filter. The aspect dependent matched illumination approach was also provided as an upper bound on target detection performance. Future work will focus on an assessment of the maximin approach for a variety of targets of interest in cluttered scenes and waveform design with resolution constraints.

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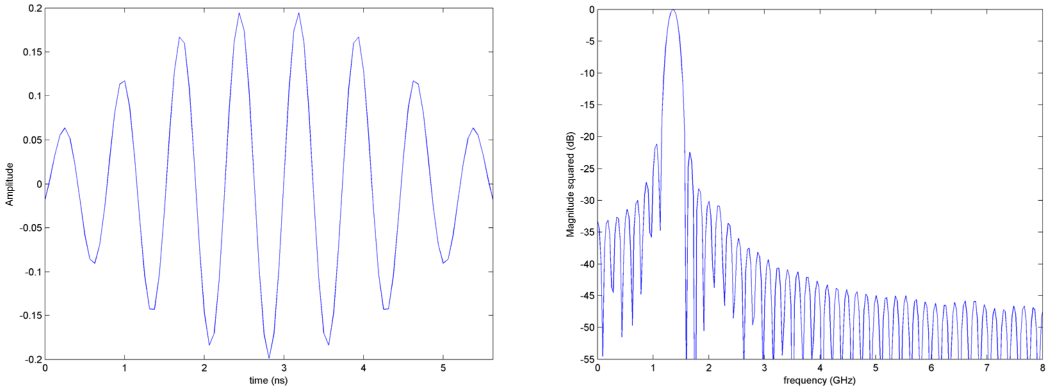

(a) (b)

Fig. 4. (a) Maximin waveform, (b) Corresponding magnitude spectrum.

Table I. Average SNR improvement of various waveform design approaches

Approach **Waveform**	**Matched Illumination Approach**	**Maximin Design Approach**
Optimal	0 dB	-11.61 dB
Maximin	11.61 dB	0 dB
Chirp 1	-	19.13 dB
Chirp 2	5.45 dB	21.87 dB