TRANSCEIVER OPTIMIZATION AND POWER CONTROL IN WIRELESS DATA NETWORKS WITH FEMTOCELLS: A POTENTIAL GAME-THEORETIC APPROACH

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ABSTRACT

The problem of non-cooperative spreading code allocation, linear receiver design, and transmit power control for wireless networks employing femtocells is considered in this paper. Several utility functions to be maximized are considered, and, among them, we cite the received SINR, and the transmitter energy efficiency, which is measured in bit/Joule, and represents the number of successfully delivered bits for each energy unit used for transmission. Resorting to the theory of potential games, non-cooperative games admitting Nash equilibria in multi-cell networks regardless of the channel coefficient realizations are designed. Computer simulations confirm that the considered games are convergent, and permit to assess the benefic impact that femtocells have on the network performance.

1. INTRODUCTION AND WORK MOTIVATION

The demand for coverage and for higher data rates in wireless networks is becoming more and more urgent nowadays. In order to support cell-edge users with high-data rate services under agile frequency reuse one very promising option is the use of smaller cells. A recent development in this continuous micronization of cellular networks is given by femtocells, or home base stations, which are short-range, low-cost, low-power, indoor base stations which have been shown to achieve better performance than macrocells [2, 4, 1]. Given the short range of femtocells, a lower transmit power can be used by nearby mobile devices, which results in longer battery life. Moreover, since they operate in an indoor environment, femtocells' users suffer little interference from outdoor users and viceversa, which implies larger Signal-to-Interference-plus-Noise-Ratios (SINRs) and a remarkable indoor coverage.

As in every wireless networks, the problem of resource allocation is a crucial one also in femtocells-based networks. In [5] power allocation has been studied, wherein femtocell users adjust their maximum transmit power using an open-loop and a closed-loop technique, while in [3] an utility-based, non-cooperative SINR adaptation for transmit power allocation is studied. There, the authors considered the peculiar case wherein a single macrocell user is active with multiple cochannel femtocells users transmitting in each slot.

A customary trend in the design and analysis of resource allocation procedures is the use of game-theoretic tools. Indeed, game theory, a branch of mathematics studying the interactions among several autonomous subjects with contrasting interests, is well suited to model the interactions between selfish active users in wireless networks, who are indeed in mutual competition for the available network resources [6]. Motivated by this background, this paper is concerned with the problem of resource allocation in a femtocell-based wireless data network using a non-orthogonal multiple access strategy such as code division multiple access (CDMA). Since femtocells are employed, each macrocell in the network is composed of a number of smaller cells, and therefore can be seen as a multipoint-to-multipoint communication system. Indeed, the uplink of such a system can no longer be modeled as multiple access channel, but an interference channel model is to be used. As a consequence, while several studies and abundance of results are available on non-cooperative resource allocation procedures for single-cell data networks (see [7, 8, 9, 10] and references therein for a non-exhaustive list), the case in which multipoint-to-multipoint communication takes place is much more challenging, and several non-cooperative resource allocation games conceived for single-cell systems appear to be no longer convergent (i.e., to have no equilibrium). As notable exceptions, we cite here the work [11], wherein a non-cooperative power control game for energy-efficiency maximization is proposed, and the recent paper [12], wherein, resorting to the theory of potential games [13], a non-cooperative spreading code allocation algorithm has been proposed, under the assumption that a simple matched filter is used at the receiver. Roughly speaking, in a potential game each change in the utility enjoyed by a given player due to an *unilateral* change of strategy by that player is paired by a similar change in a global function called the potential function. In a potential game, the best response strategy always leads to a Nash equilibrium (NE). Using [12] as our departure point, in this paper we make the following contributions:

- We propose and analyze several non-cooperative games for joint transmitter and receiver optimization, aimed at maximizing utility functions strictly related to the signal-to-interference plus noise ratio (SINR).
- We propose a non-cooperative joint transceiver optimization and transmit power control game aimed at maximization of the energy efficiency of each active user. Energy efficiency, measured in bit/Joule, represents the number of bits that are *successfully* delivered at the receiver for each energy unit taken from the battery and used for transmission. Unfortunately, for such a game the existence of an NE is shown only through numerical evidence, since we were not able to obtain an analytical

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proof.

We give extensive numerical results, and show the merits of the proposed non-cooperative resource allocation algorithms with respect to the case in which femtocells are not employed.

2. SYSTEM MODEL

Let us consider the uplink of a direct-sequence CDMA wireless data network in which each macro-cell contains B access points (AP)[1], and let $h_{i,j}$ be the real channel gain between the i-th user and the j-th AP; moreover, denote by $a(i)$ the index of the AP assigned to the i-th user[2]. After chip-matched filtering and chip-rate sampling, the N-dimensional received data vector at the ℓ-th AP, say $\mathbf{r}_\ell$, can be written as

$$\mathbf{r}_\ell = \sum_{k=1}^{K} \sqrt{p_k} h_{k,\ell} b_k \mathbf{s}_k + \mathbf{n}_\ell \ , \quad \ell = 1, \ldots, B \ . \tag{1}$$

Assuming that a linear detector is used at the receiver, so that the symbol b_k is detected according to the rule $\widehat{b}_k = \text{sign}\left\{\mathbf{d}_k^T \mathbf{r}_{a(k)}\right\}$, the SINR for the k-th user is expressed as

$$\gamma_k = \frac{p_k h_{k,a(k)}^2 (\mathbf{d}_k^T \mathbf{s}_k)^2}{\mathbf{d}_k^T \left(\sigma_n^2 \mathbf{I} + \sum_{j \neq k} p_j h_{j,a(k)}^2 \mathbf{s}_j \mathbf{s}_j^T \right) \mathbf{d}_k} \tag{2}$$

3. SPREADING CODE ALLOCATION

We consider now the problem of spreading code allocation for multi-cell system, thus reviewing some of the existing non-cooperative approaches, and proposing two new procedures.

3.1 Greedy spreading code allocation with LMMSE reception [10]

Consider the case that an linear minimum mean square error (LMMSE) filter is used at the receiver. In this case the k-th user SINR can be expressed as

$$\gamma_k = p_k h_{k,a(k)}^2 \mathbf{s}_k^T \left(\sigma_n^2 \mathbf{I} + \sum_{j \neq k} p_j h_{j,a(k)}^2 \mathbf{s}_j \mathbf{s}_j^T \right)^{-1} \mathbf{s}_k \ . \tag{3}$$

Given the above expression, it is trivially shown that the SINR-maximizing spreading code for the k-th user is the eigenvector associated to the minimum eigenvalue of the matrix

$$\left(\sigma_n^2 \mathbf{I} + \sum_{j \neq k} p_j h_{j,a(k)}^2 \mathbf{s}_j \mathbf{s}_j^T \right) \ ,$$

which is indeed the covariance matrix of the overall interference suffered by the k-th user. The non-cooperative game wherein users cyclically update their spreading code in order to maximize the SINR in Eq. (3) is widely known as greedy interference avoidance procedure [10]. Such a procedure, while being always convergent in single-cell systems, does not always converge in multi-cell systems, and is thus not suited to our scenario; for comparison purposes, however, in the following we will include performance results for this technique as well.

3.2 Minimization of the individual MSE [9]

As an alternative optimization criterion, we can consider minimization of the individual MSE. The MSE for the k-th user, say ε_k^2, is expressed as

$$\begin{aligned} \varepsilon_k^2 = E\left\{ (b_k - \mathbf{d}_k^T \mathbf{r}_{a(k)})^2 \right\} = 1 - 2\sqrt{p_k} h_{k,a(k)} \mathbf{d}_k^T \mathbf{s}_k - \\ - \frac{\mathcal{N}_0}{2} \|\mathbf{d}_k\|^2 + \mathbf{d}_k^T \left(\sum_{j=1}^{K} p_j h_{j,a(k)}^2 \mathbf{s}_j \mathbf{s}_j^T \right) \mathbf{d}_k \ . \end{aligned} \tag{4}$$

Following [9], it is easily seen that the minimizer of ε_k^2 can be obtained as the unique stable fixed point of the following iterations:

$$\begin{cases} \mathbf{d}_k = \sqrt{p_k} h_{k,a(k)} \mathbf{M}_{\mathbf{r}_{a(k)}}^{-1} \mathbf{s}_k \ , \\ \mathbf{s}_k = \mathbf{d}_k / \|\mathbf{d}_k\| \ , \end{cases} \tag{5}$$

for any $k = 1, \ldots, K$. In the above equation $\mathbf{M}_{\mathbf{r}_{a(k)}} = E\left\{ \mathbf{r}_{a(k)} \mathbf{r}_{a(k)}^T \right\}$ is the covariance matrix of the data vector received at the $a(k)$-th AP. Now, convergence of iterations (5) in a multi-cell system is not always guaranteed.

3.3 Minimization of the sum of inverse SINR [12]

As previously discussed, non-cooperative maximum SINR game with respect to the spreading code and uplink receiver [10] is not always convergent. In [12], instead, based on the theory of potential games, a modification to the utility function to be considered has been introduced, so as to have a guaranteed convergence for any channel realizations. Let us thus assume that a matched filter (MF) is used at the receiver and consider the sum of the inverse SINR, i.e.:

$$V = \sum_{k=1}^{K} \frac{1}{\gamma_k} \ . \tag{6}$$

Pointing out the dependence on the k-th spreading code $\mathbf{s}_k$, V can be expressed as

$$\begin{aligned} V = \ & \mathbf{s}_k^T \left[\frac{\sigma_n^2}{p_k h_{k,a(k)}^2} \mathbf{I} + \right. \\ & \left. + \sum_{j \neq k} \left(\frac{p_j h_{j,a(k)}^2}{p_k h_{k,a(k)}^2} + \frac{p_k h_{k,a(j)}^2}{p_j h_{j,a(j)}^2} \right) \mathbf{s}_j \mathbf{s}_j^T \right] \mathbf{s}_k + D \ , \end{aligned} \tag{7}$$

with D an additive term independent of $\mathbf{s}_k$. It is thus clear that a non-cooperative game wherein the utility function for the k-th user is

$$\begin{aligned} u_k = \ & \mathbf{s}_k^T \left[\frac{\sigma_n^2}{p_k h_{k,a(k)}^2} \mathbf{I} + \right. \\ & \left. + \sum_{j \neq k} \left(\frac{p_j h_{j,a(k)}^2}{p_k h_{k,a(k)}^2} + \frac{p_k h_{k,a(j)}^2}{p_j h_{j,a(j)}^2} \right) \mathbf{s}_j \mathbf{s}_j^T \right] \mathbf{s}_k \ , \end{aligned} \tag{8}$$

is a potential game with potential function V and thus admits an NE.

[1] Some of these AP may be femtocells.

[2] Note that we are assuming here that each user is assigned to a certain AP, i.e. AP assignments have already taken place.

$$Q=\sum_{m=1}^{K}\rho_m=\underbrace{\mathbf{s}_k^T\left[p_k h_{k,a(k)}^2\sigma_n^2\mathbf{I}+\sum_{j\neq k}p_k p_j h_{k,a(k)}^2 h_{j,a(k)}^2\mathbf{s}_j\mathbf{s}_j^T+\sum_{j\neq k}p_k p_j h_{k,a(j)}^2 h_{j,a(j)}^2\mathbf{s}_j\mathbf{s}_j^T\right]\mathbf{s}_k}_{\text{depends on }\mathbf{s}_k}+$$
$$\underbrace{\sum_{j=1,j\neq k}^{K}p_j h_{j,a(j)}^2\mathbf{s}_j^T\left(\sigma_n^2\mathbf{I}+\sum_{\ell\neq k,j}p_l h_{\ell,a(j)}^2\mathbf{s}_\ell\mathbf{s}_\ell^T\right)\mathbf{s}_j}_{\text{does not depend on }\mathbf{s}_k}\ . \tag{10}$$

$$\sum_{m=1}^{K}\varepsilon_m^2=\underbrace{1-2\sqrt{p_k}h_{k,a(k)}\mathbf{d}_k^T\mathbf{s}_k+\frac{\mathcal{N}_0}{2}\|\mathbf{d}_k\|^2+\mathbf{d}_k^T\left(\sum_{j=1}^{K}p_j h_{j,a(k)}^2\mathbf{s}_j\mathbf{s}_j^T\right)\mathbf{d}_k+\sum_{\ell\neq k}\mathbf{d}_\ell^T\left(p_k h_{k,a(l)}^2\mathbf{s}_k\mathbf{s}_k^T\right)\mathbf{d}_\ell}_{\text{depends on }\mathbf{s}_k}+$$
$$\underbrace{(K-1)+\sum_{\ell\neq k}\mathbf{d}_\ell^T\left(\sum_{j\neq k}p_j h_{j,a(\ell)}^2\mathbf{s}_j\mathbf{s}_j^T\right)\mathbf{d}_\ell-2\sum_{\ell\neq k}\sqrt{p_\ell}h_{\ell,a(\ell)}\mathbf{d}_\ell^T\mathbf{s}_\ell+\sum_{\ell\neq k}\frac{\mathcal{N}_0}{2}\mathbf{d}_\ell}_{\text{does not depend on }\mathbf{s}_k}\ . \tag{12}$$

3.4 Greedy interference avoidance revisited

We now propose a new non-cooperative game which will be shown to achieve much superior performance levels than the previously discussed solutions.

Since the greedy interference avoidance procedure is not always convergent in multi-cell systems, we resort to the theory of potential games in order to come up with a modified utility function whose non-cooperative maximization leads to an NE. Assume that an LMMSE detector is user at the receiver, so that the k-th user SINR can be shown to be written as

$$\gamma_k=p_k h_{k,a(k)}^2\mathbf{s}_k^T\left(\sigma_n^2\mathbf{I}+\sum_{j\neq k}p_j h_{j,a(k)}^2\mathbf{s}_j\mathbf{s}_j^T\right)^{-1}\mathbf{s}_k\ .$$

Considering the minimization of the sum of the inverse SINRs (as done in [12] for the case of a matched filter receiver) reveals to be a complicated task in this case, and, also, maximization of the sum of the SINRs turns out to be complicated as well. We consider instead the following quantity

$$Q=\sum_{k=1}^{K}\rho_k=\sum_{k=1}^{K}p_k h_{k,a(k)}^2\mathbf{s}_k^T\left(\sigma_n^2\mathbf{I}+\sum_{j\neq k}p_j h_{j,a(k)}^2\mathbf{s}_j\mathbf{s}_j^T\right)\mathbf{s}_k\ . \tag{9}$$

Note that the above quantities is directly tied to the SINRs enjoyed by the active users in the network, since it is easy to show that Q is a decreasing function of the SINR of each user. Upon straightforward algebraic manipulation, we find Eq. (10) shown at the top of the page. Accordingly, a non-cooperative game wherein each user aims at maximizing the utility

$$u_k=-\mathbf{s}_k^T\left[\sigma_n^2\mathbf{I}+\sum_{j\neq k}p_j h_{j,a(k)}^2\mathbf{s}_j\mathbf{s}_j^T+\sum_{j\neq k}p_j\frac{h_{k,a(j)}^2}{h_{k,a(k)}^2}h_{j,a(j)}^2\mathbf{s}_j\mathbf{s}_j^T\right]\mathbf{s}_k\ , \tag{11}$$

is a potential game whose potential function is $-Q$. Accordingly, such a non-cooperative game always admits an NE.

3.5 Non-cooperative minimization of the TMSE

Since non-cooperative minimization of the individual MSE is not always convergent in a multi-cell scenario, we can again resort to the theory of potential games to obtain a convergent non-cooperative game in this case too. Let us thus consider the total MSE, defined as $\sum_{k=1}^{K}\varepsilon_k^2$. Upon some straightforward algebraic manipulations, we have Eq. (12), shown at the top of this page. It is easy to realize that the part dependent on $\mathbf{s}_k$, say $L(\mathbf{s}_k)$, may be written as

$$L(\mathbf{s}_k)=\varepsilon_k^2+\sum_{\ell\neq k}\mathbf{d}_\ell^T\left(p_k h_{k,a(l)}^2\mathbf{s}_k\mathbf{s}_k^T\right)\mathbf{d}_\ell\ , \tag{13}$$

thus implying that the latter summand in the right-hand-side of the above equation is the correcting term that needs to be added to the MSE for the k-th user to make the non-cooperative game convergent. Summing up, we thus consider the following game:

$$\min_{\mathbf{s}_k,\mathbf{d}_k}L(\mathbf{s}_k)\ \ ,\ \text{subject to: }\|\mathbf{s}_k\|=1\ . \tag{14}$$

Using standard Lagrangian optimization techniques, we have that the solution to (13) is written as

$$\mathbf{s}_k=\sqrt{p_k}h_{k,a(k)}\left(\lambda\mathbf{I}+\sum_{\ell=1}^{K}p_k h_{k,a(\ell)}^2\mathbf{d}_\ell\mathbf{d}_\ell^T\right)^{-1}\mathbf{d}_k\ , \tag{15}$$

where λ, the Lagrange multiplier, is such that $\|\mathbf{s}_k\|=1$.

4. A NON-COOPERATIVE GAME FOR ENERGY EFFICIENT COMMUNICATIONS

Let us now consider the case that each transmitter is interested in maximizing its energy-efficiency, i.e. the number of data bits successfully delivered to the receiver for each energy unit taken from the battery and used for transmission.

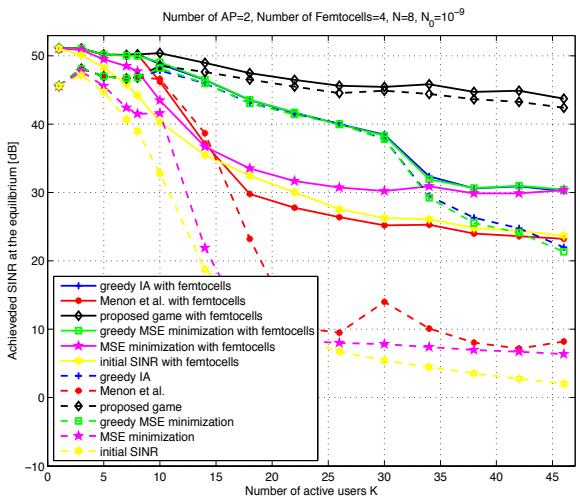

Figure 1: Achieved SINR at the NE versus the number of users.

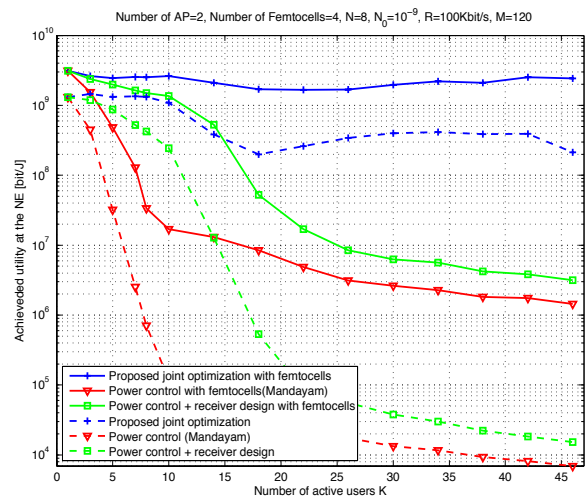

Figure 2: Achieved utility at the NE versus the number of users.

Following [11, 8], the following utility function should be considered for the k-th user

$$u_k = R\frac{L}{M}\frac{f(\gamma_k)}{p_k}\,, \tag{16}$$

with R the transmit data rate, L/M the ratio between the payload length and the total length of each data packet, and $f(\cdot)$ the efficiency-function, which is usually expressed as $f(\gamma_k) = (1-e^{-\gamma_k})^M$. We are here interested in the non-cooperative maximization of u_k with respect to p_k, $\mathbf{s}_k$ and $\mathbf{d}_k$. While things are easy in a single-cell system, and indeed results for this scenario are reported in [8], in multi-cell systems some approximations and modifications are to be considered in order to obtain a game admitting an NE.

We thus propose to consider the concatenation of two different games, namely

a) for fixed transmit powers, the non-cooperative minimization of the TMSE with respect to the users' spreading codes, assuming that an LMMSE receiver is used at the receiver; and
b) for fixed spreading codes, the maximization of the energy-efficiency (16) with respect to the transmit powers[3].

More precisely, we assume that users continuously switch between games a) and b), until convergence is reached. Unfortunately, we are not able to provide an analytical proof that the proposed alternative strategy always converges to an NE, and indeed this is the object of current investigation; however, we point out that extensive numerical simulations have shown that an NE always exists; the remarkable performance advantage that the proposed strategy brings with respect to the case in which spreading code adaptation is not carried out are discussed in the forthcoming section.

5. NUMERICAL RESULTS

We have considered a system with processing gain $N = 8$ and users randomly located in a square of 10^6 sq. meters. We compare the scenario in which there are 2 AP's, and the scenario in which we have 2 APs and 4 femtocell APs serving an area of radius 100m. The channel coefficients $h^2_{i,j}$ have been generated according to an exponential distribution with mean equal to $d^{-2}_{i,j}$, with $d_{i,j}$ the distance between the i-th user and the j-th access point. It is assumed that each user's data are decoded at the AP with the largest channel coefficient, namely $a(k) = \arg\max_{\ell=1,\ldots,B}\left(h^2_{k,\ell}\right)$. The curves here shown come from an average over 500 independent realizations of the channel coefficients, users' locations, and starting set of spreading codes.

First of all we consider the waveform adaptation games discussed in Section 3: Fig. 1 shows the achieved SINR at the equilibrium for the illustrated spreading code allocation procedures versus the number of active users. A maximum of 5000 iterations has been included in the simulation program in order to have a stopping rule for the resource allocation games of section 3.1 and 3.2, which indeed are not always convergent in multicell systems. It is seen that the proposed resource allocation strategy of section 3.4 achieves the best performance. It is also seen that when femtocells are active we have a much better performance.

Fig.'s 2 - 4 refer to the system performance at the equilibrium for the case in which transmit power and spreading code are tuned so as to maximize energy efficiency. In Fig. 2 we report the achieved energy-efficiency (bit/Joule) at the equilibrium (which, we recall, has been reached in all the randomly generated scenarios) versus the number of active users for three different non-cooperative games, i.e. (a) power control with a matched filter at the receiver [11], (b) joint power control and uplink receiver design, and (c) joint power control, spreading code allocation and uplink receiver design. Fig.'s 3 and 4 report, for this considered scenario, the average transmit power and the fraction of users transmitting at the maximum power (that is indeed taken equal to 0dBW) at the NE. Again, we see that the newly proposed joint procedure greatly outperforms the competing alternatives, and that femtocells bring substantial performance improvements:

[3]This game admits a unique NE [11].

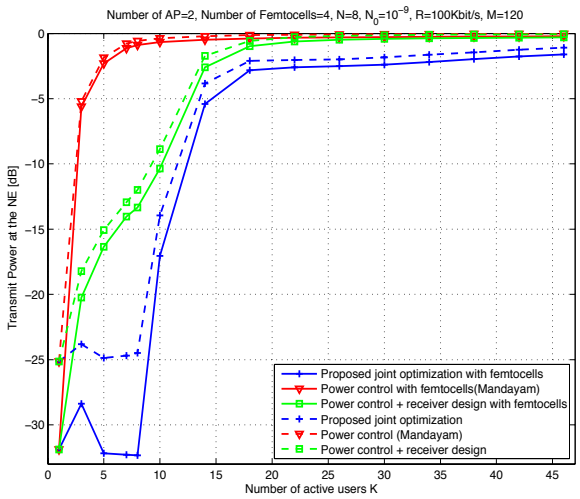

Figure 3: Average transmit power at the NE versus the number of users.

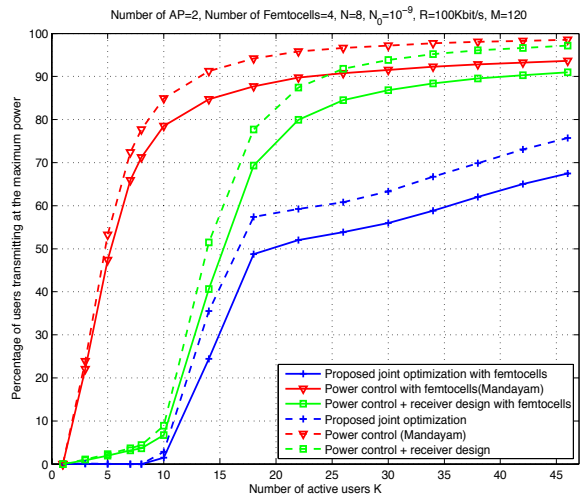

Figure 4: Fraction of users transmitting at the maximum power at the NE versus the number of users.

indeed, for a fully loaded system (i.e. $K = 46$ users), the proposed game coupled with femtocells provides at the NE an energy efficiency that is 10 times larger than that of the proposed game in a system with no femtocells, and several orders of magnitude larger that that achieved by the alternatives. Similar considerations apply when considering the average transmit power and the fraction of users transmitting at the maximum power.

6. CONCLUSIONS

This paper has considered the problem of joint transmitter waveform adaptation and power control in a multi-cell multiuser wireless data network equipped with femtocells. Leveraging on the study [12], wherein it has been revealed that the theory of potential games can be used to obtain convergent non-cooperative resource allocation games in multi-cell networks, we have proposed a new transmitter waveform adaptation game. Additionally, we also considered the issue of energy-efficiency in a multi-cell network, and a new joint power control and transmit waveform adaptation game has been proposed for its maximization. Overall results have confirmed that femtocells have a positive impact on the whole network performance.

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