A KALMAN-LIKE FIR ESTIMATOR IGNORING NOISE AND INITIAL CONDITIONS

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ABSTRACT

A p-shift finite impulse response (FIR) unbiased estimator (UE) is addressed for linear discrete time-varying filtering ($p = 0$), p-step prediction ($p > 0$), and p-lag smoothing ($p < 0$) of signal models in state space with no requirements for initial conditions and zero mean noise. A solution is found in a batch form and represented in a computationally efficient iterative Kalman-like one. It is shown that the Kalman-like FIR UE is able to overperform the Kalman filter if the noise covariances and initial conditions are not known exactly, noise is not white, and both the system and measurement noise components need to be filtered out. Otherwise, the errors are similar.

1. INTRODUCTION

For such unsuited applications of the Kalman filter (KF) [1] as estimation of nonlinear models, under unknown initial conditions, and in the presence of nonwhite or multiplicative noise sources, the Kalman-like one is often designed to save the recursive structure, while connecting the algorithm components with the model in different ways. Because there can be found an infinity of Kalman-like solutions depending on applications, we meet a number of propositions suggesting some new qualities while saving (or not deteriorating substantially) the advantages of KF: fast computation and accuracy.

Cox in [2] and others have derived the extended KF (EKF) for nonlinear models by a linearization of the state-space equations. Referring to the fact that EKF can give particularly poor performance when the model is highly nonlinear [3], Julier and Uhlmann employed in [4] the unscented transform and proposed the unscented KF (UKF). Both EKF and UKF have then been used extensively and the former was developed in [5] to the invariant EKF for nonlinear systems possessing symmetries (or invariances). For high-dimensional systems, the ensemble KF was proposed by Evensen in [6] and, for systems with sparse matrices, the fast KF applied by Lange in [7]. Applications has also found the robust Kalman-type filter designed by Masreliez [8] [9] for linear state-space relations with non-Gaussian noise referred to as heavy tailed noise or Gaussian one mixed with outliers. Useful Kalman-like algorithms can also be found in works by Nahi [10], Basseville *et al*. [11], Baccarelli and Cusani [12], Ait-El-Fquih and Desbouvries [13], Carmi *et al*. [14], Stefanatos and Katsaggelos [15], and the list can be extended.

In spite of great efforts in extending the applications and improving the performance of KF, its structure still remains recursive thus with the infinite impulse response (IIR). Investigating in [3] both the IIR and finite impulse response (FIR) filters, Jazwinski resumed that the limited memory filter (having FIR) appears to be more robust against the unbounded perturbation in the system. Referring to [3], optimal FIR filtering has been developed by W. H. Kwon *et al*. in [16]. There were also proposed several Kalman-like FIR estimators by Kwon *et al*. in [17], Han *et al*. in [18], and Shmaliy in [19]. A distinctive feature of such algorithms is that white Gaussian noise in the convolution-based estimate obtained over N past measured points is reduced as a reciprocal of N [20] disregarding the model [19]. Moreover, the unbiased and optimal FIR estimates typically become strongly consistent if N occurs to be large [21] or the mean square initial state function dominates the noise covariance functions in the order of magnitudes [19]. It is also known that the optimal horizon N_{opt} makes the FIR estimate (optimal or unbiased) similar or even better than the Kalman one [16, 19–23].

Owing to the exciting engineering features of the Kalman-like FIR algorithms uniting advantages of KF and inherent properties of FIR structures such as the bounded input/bounded output (BIBO) stability as well as better robustness against temporary model uncertainties, non Gaussian noise, and round-off errors, it may be expected that the FIR unbiased estimator (UE) ignoring noise and initial conditions will serve efficiently instead of optimal filters in many applications.

2. SIGNAL MODEL

Consider a class of discrete time-varying (TV) linear state-space models represented with the state and observation equations, respectively,

$$\mathbf{x}_n = \mathbf{A}_n\mathbf{x}_{n-1} + \mathbf{B}_n\mathbf{w}_n\,, \quad (1)$$

$$\mathbf{y}_n = \mathbf{C}_n\mathbf{x}_n + \mathbf{D}_n\mathbf{v}_n\,, \quad (2)$$

where $\mathbf{x}_n \in \Re^K$ and $\mathbf{y}_n \in \Re^M$ are the state and observation vectors, respectively. Here, $\mathbf{A}_n \in \Re^{K\times K}$, $\mathbf{B}_n \in \Re^{K\times P}$, $\mathbf{C}_n \in \Re^{M\times K}$, and $\mathbf{D}_n \in \Re^{M\times M}$. The vectors $\mathbf{w}_n \in \Re^P$ and $\mathbf{v}_n \in \Re^M$ are zero mean, $E\{\mathbf{w}_n\} = \mathbf{0}$ and $E\{\mathbf{v}_n\} = \mathbf{0}$. It is implied that $\mathbf{w}_n$ and $\mathbf{v}_n$ are mutually uncorrelated and independent processes, $E\{\mathbf{w}_i\mathbf{v}_j^T\} = \mathbf{0}$, having arbitrary distributions and known covariances

$$\mathbf{Q}_w(i,j) = E\{\mathbf{w}_i\mathbf{w}_j^T\}\,, \quad (3)$$

$$\mathbf{Q}_v(i,j) = E\{\mathbf{v}_i\mathbf{v}_j^T\}\,, \quad (4)$$

for all i and j, to mean that $\mathbf{w}_n$ and $\mathbf{v}_n$ should not obligatorily be Gaussian and delta-correlated.

Following the strategies of the recursive KF [1] and iterative Kalman-like FIR unbiased filter (UF) [19], the TV

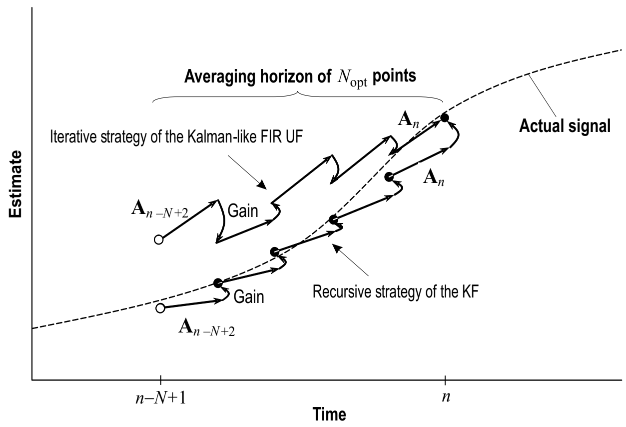

Figure 1: Strategies of the recursive KF and iterative Kalman-like FIR UF algorithms.

estimates of $\mathbf{x}_n$ can be obtained as shown in Fig. 1. Here, KF starts at some initial point $n-N+1$, where $N \geqslant 2$, with known initial conditions and recursively produces estimates at each subsequent point up to n. The estimate is formed in two steps. First, the system matrix $\mathbf{A}_{n-N+2}$ makes a projection from $n-N+1$ to $n-N+2$ and then the Kalman gain adjusts the result to be the estimate at $n-N+2$. The procedure repeats recursively, provided the covariances of white noise sequences.

The Kalman-like FIR UF does not require the noise performance and initial conditions [24], but needs an optimal averaging horizon $N_{\rm opt}$ [25] in order for the estimate to have the minimum mean square error (MSE) and be consistent to the Kalman one [19, 22]. This filter starts with any unknown value at $n-N+1$ and iteratively produces estimates at subsequent points in two steps similarly to KF, although the true value is taken only at n when $N = N_{\rm opt}$. The algorithm operates in any noise environment that makes it highly attractive for engineering applications.

3. TIME-VARYING BATCH UNBIASED FIR ESTIMATOR

In order to find the FIR UE, (1) and (2) can be extended on a horizon of N points from $m = n-N+1$ to n following [26] and similarly to [21] as, respectively,

$$\mathbf{X}_{n,m} = \mathbf{A}_{n,m}\mathbf{x}_m + \mathbf{B}_{n,m}\mathbf{W}_{n,m}\,, \quad (5)$$

$$\mathbf{Y}_{n,m} = \mathbf{C}_{n,m}\mathbf{x}_m + \mathbf{G}_{n,m}\mathbf{W}_{n,m} + \mathbf{D}_{n,m}\mathbf{V}_{n,m}\,, \quad (6)$$

where $\mathbf{X}_{n,m} \in \Re^{KN}$, $\mathbf{Y}_{n,m} \in \Re^{MN}$, $\mathbf{W}_{n,m} \in \Re^{PN}$, and $\mathbf{V}_{n,m} \in \Re^{MN}$ are specified by, respectively,

$$\mathbf{X}_{n,m} = \left[\mathbf{x}_n^T\ \mathbf{x}_{n-1}^T\ \ldots\ \mathbf{x}_m^T\right]^T, \quad (7)$$

$$\mathbf{Y}_{n,m} = \left[\mathbf{y}_n^T\ \mathbf{y}_{n-1}^T\ \ldots\ \mathbf{y}_m^T\right]^T, \quad (8)$$

$$\mathbf{W}_{n,m} = \left[\mathbf{w}_n^T\ \mathbf{w}_{n-1}^T\ \ldots\ \mathbf{w}_m^T\right]^T, \quad (9)$$

$$\mathbf{V}_{n,m} = \left[\mathbf{v}_n^T\ \mathbf{v}_{n-1}^T\ \ldots\ \mathbf{v}_m^T\right]^T, \quad (10)$$

and $\mathbf{A}_{n,m} \in \Re^{KN\times K}$, $\mathbf{C}_{n,m} \in \Re^{MN\times K}$, $\mathbf{G}_{n,m} \in \Re^{MN\times PN}$, and $\mathbf{D}_{n,m} \in \Re^{MN\times MN}$ are given with, respectively,

$$\mathbf{A}_{n,m} = \left[\mathscr{A}_n^{m+1^T}\ \mathscr{A}_{n-1}^{m+1^T}\ \ldots\ \mathbf{A}_{m+1}^T\ \mathbf{I}\right]^T, \quad (11)$$

$$\mathbf{C}_{n,m} = \begin{bmatrix} \mathbf{C}_n\mathscr{A}_n^{m+1} \\ \mathbf{C}_{n-1}\mathscr{A}_{n-1}^{m+1} \\ \vdots \\ \mathbf{C}_{m+1}\mathbf{A}_{m+1} \\ \mathbf{C}_m \end{bmatrix}, \quad (12)$$

$$\mathbf{G}_{n,m} = \bar{\mathbf{C}}_{n,m}\mathbf{B}_{n,m}\,, \quad (13)$$

$$\mathbf{D}_{n,m} = \mathrm{diag}\big(\underbrace{\mathbf{D}_n\ \mathbf{D}_{n-1}\ \ldots\ \mathbf{D}_m}_{N}\big), \quad (14)$$

where we assigned $\mathscr{A}_{n-h}^{n-g} = \prod_{i=h}^{g}\mathbf{A}_{n-i}$, $\bar{\mathbf{C}}_{n,m} = \mathrm{diag}\big(\underbrace{\mathbf{C}_n\ \mathbf{C}_{n-1}\ \ldots\ \mathbf{C}_m}_{N}\big)$, and performed $\mathbf{B}_{n,m} \in \Re^{KN\times PN}$ as $\mathbf{B}_{n,m}$

$$= \begin{bmatrix} \mathbf{B}_n & \mathbf{A}_n\mathbf{B}_{n-1} & \ldots & \mathscr{A}_n^{m+2}\mathbf{B}_{m+1} & \mathscr{A}_n^{m+1}\mathbf{B}_m \\ \mathbf{0} & \mathbf{B}_{n-1} & \ldots & \mathscr{A}_{n-1}^{m+2}\mathbf{B}_{m+2} & \mathscr{A}_{n-1}^{m+1}\mathbf{B}_{m+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{B}_{m+1} & \mathbf{A}_{m+1}\mathbf{B}_m \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{B}_m \end{bmatrix}, \quad (15)$$

The model, (5) and (6), suggests that the state equation at the initial point m is $\mathbf{x}_m = \mathbf{x}_m + \mathbf{B}_n\mathbf{w}_m$ that for $\mathbf{B}_n$ specialized with (15) can uniquely be satisfied with $\mathbf{w}_m$ zero-valued. The initial state $\mathbf{x}_m$ must thus be known *a priori* or estimated optimally *a posteriori* as shown in [19].

By the convolution, the estimate[1] $\tilde{\mathbf{x}}_{n+p|n}$ of $\mathbf{x}_n$ can now be obtained if we assign a $K \times MN$ gain matrix $\mathbf{H}_{n,m}(p)$ and claim that

$$\tilde{\mathbf{x}}_{n+p|n} = \mathbf{H}_{n,m}(p)\mathbf{Y}_{n,m} \quad (16a)$$

$$= \mathbf{H}_{n,m}(p)(\mathbf{C}_{n,m}\mathbf{x}_m + \mathbf{G}_{n,m}\mathbf{W}_{n,m} + \mathbf{D}_{n,m}\mathbf{V}_{n,m})\,. \quad (16b)$$

The estimate (16a) will be unbiased if and only if the following unbiasedness condition is satisfied

$$E\{\tilde{\mathbf{x}}_{n+p|n}\} = E\{\mathbf{x}_{n+p}\}\,, \quad (17)$$

where E means averaging of the succeeding relation.

Averaging in (16b), by (17), means removing the zero mean noise matrices that gives us $E\{\tilde{\mathbf{x}}_{n+p|n}\} = \bar{\mathbf{H}}_{n,m}(p)\mathbf{C}_{n,m}\mathbf{x}_m$, where $\bar{\mathbf{H}}_{n,m}(p)$ is the FIR UE gain. In turn, $E\{\mathbf{x}_n\}$ can be substituted with the first vector row of (5) by removing noise as $E\{\mathbf{x}_n\} = \mathscr{A}_n^{m+1}\mathbf{x}_m$. Since n can be arbitrary, one can substitute it with $n+p$ and write

$$E\{\mathbf{x}_{n+p}\} = \mathscr{A}_{n+p}^{m+1}\mathbf{x}_m\,. \quad (18)$$

Equating $E\{\tilde{\mathbf{x}}_{n+p|n}\}$ to (18) leads to the unbiasedness constraint for TV models

$$\mathscr{A}_{n+p}^{m+1} = \bar{\mathbf{H}}_{n,m}(p)\mathbf{C}_{n,m}\,. \quad (19)$$

[1] $\tilde{\mathbf{x}}_{n+p|n}$ is an estimate at $n+p$ via measurement from the past to n; $\hat{\mathbf{x}}_{n+p|n}$ mean optimal and $\bar{\mathbf{x}}_{n+p|n}$ unbiased.

If we further multiply (19) from the right hand sides with the identity matrix $(\mathbf{C}_{n,m}^T\mathbf{C}_{n,m})^{-1}\mathbf{C}_{n,m}^T\mathbf{C}_{n,m}$ and then remove $\mathbf{C}_{n,m}$ from both sides, we go to

$$\bar{\mathbf{H}}_{n,m}(p) = \mathscr{A}_{n+p}^{m+1}(\mathbf{C}_{n,m}^T\mathbf{C}_{n,m})^{-1}\mathbf{C}_{n,m}^T \tag{20}$$

representing the gain of the TV FIR UE. It can easily be verified that (20) becomes that derived in [19] for time-invariant (TI) models.

Provided (20), the TV batch FIR UE is specified by the following theorem, which proof belongs to (5)-(20).

Theorem 1 *Given (1) and (2) with zero mean mutually uncorrelated and independent $\mathbf{w}_n$ and $\mathbf{v}_n$ having arbitrary distributions and known covariance functions. Then, filtering ($p = 0$), p-lag smoothing ($p < 0$), and p-step prediction ($p > 0$) are provided at $n+p$ using data taken from $m = n-N+1$ to n by the batch FIR UE as*

$$\begin{aligned}\bar{\mathbf{x}}_{n+p|n} &= \bar{\mathbf{H}}_{n,m}(p)\mathbf{Y}_{n,m} &(21a)\\ &= \mathscr{A}_{n+p}^{m+1}(\mathbf{C}_{n,m}^T\mathbf{C}_{n,m})^{-1}\mathbf{C}_{n,m}^T\mathbf{Y}_{n,m}\,, &(21b)\end{aligned}$$

where $\mathbf{C}_{n,m}$ is given by (12) and $\mathbf{Y}_{n,m}$ is the data vector (8).

4. TIME-VARYING KALMAN-LIKE ESTIMATOR

Although theorem 1 establishes an exact convolution-based rule to estimate unbiasedly the TV state at n as shown in Fig. 1, the computational problem arises when $N \gg 1$ owing to large dimensions of all of the matrices and vectors. For fast computation, the batch FIR UE (21b) can be represented in an iterative Kalman-like form stated by the following theorem, which proof is similar to that given in [19].

Theorem 2 *Given the batch FIR UE (theorem 1). Then its iterative Kalman-like form is the following:*

$$\begin{aligned}\bar{\mathbf{x}}_{l+p|l} = {}& \mathbf{A}_{l+p}\bar{\mathbf{x}}_{l+p-1|l-1} + \mathbf{A}_{l+p}\boldsymbol{\Upsilon}_l^{-1}(p)\mathbf{F}_l\mathbf{C}_l^T\\ &\times[\mathbf{y}_l - \mathbf{C}_l\boldsymbol{\Upsilon}_l(p)\bar{\mathbf{x}}_{l+p-1|l-1}]\,, \end{aligned}\tag{22}$$

in which

$$\boldsymbol{\Upsilon}_l(p) = \begin{cases} \mathscr{A}_l^{l-|p|}, & p \leqslant -1 \quad (\textit{smoothing})\\ \mathbf{A}_l\,, & p=0 \quad (\textit{filtering})\\ \mathbf{I}, & p=1 \quad (\textit{prediction})\\ \prod_{i=1}^{p-1}\mathbf{A}_{l-i}^{-1}\,, & p>1 \quad (\textit{prediction})\end{cases},$$

$$\begin{aligned}\mathbf{F}_l &= [\mathbf{C}_l^T\mathbf{C}_l + (\mathbf{A}_l\mathbf{F}_{l-1}\mathbf{A}_l^T)^{-1}]^{-1}\,, &(23)\\ \bar{\mathbf{x}}_{s+p|s} &= \mathscr{A}_{s+p}^{m+1}\mathbf{P}\mathbf{C}_{s,m}^T\mathbf{Y}_{s,m}\,, &(24)\\ \mathbf{F}_s &= \mathscr{A}_s^{m+1}\mathbf{P}\mathscr{A}_s^{m+1^T}\,, &(25)\\ \mathbf{P} &= (\mathbf{C}_{s,m}^T\mathbf{C}_{s,m})^{-1}\,, &(26)\end{aligned}$$

where $s = m+K-1$, $m = n-N+1$, and an iterative variable l ranges from $m+K$ to n, because $\mathbf{C}_{l,m}^T\mathbf{C}_{l,m}$ is singular with $l < m+K$. The true estimate corresponds to $l = n$.

As can be seen, (22) is the Kalman estimate, in which $\mathbf{A}_{l+p}\boldsymbol{\Upsilon}_l^{-1}(p)\mathbf{F}_l\mathbf{C}_l^T$ plays the role of the Kalman gain that, however, does not depend on noise and initial conditions. The algorithm has two batch forms, (24) and (25), which can be computed fast for small K.

Table 1: Full-Horizon TV Kalman-Like FIR UE Algorithm

Stage	
Given:	$K,\ p,\ n \geqslant K$
Set:	$\boldsymbol{\Upsilon}_n(p)$ by (23)
	$\mathbf{P} = (\mathbf{C}_{K-1,0}^T\mathbf{C}_{K-1,0})^{-1}$
	$\mathbf{F}_{K-1} = \mathscr{A}_{K-1}^1\mathbf{P}\mathscr{A}_{K-1}^{1^T}$
	$\bar{\mathbf{x}}_{K+p-1\vert K-1} = \mathscr{A}_{K+p-1}^1\mathbf{P}\mathbf{C}_{K-1,0}^T\mathbf{Y}_{K-1,0}$
Update:	$\mathbf{F}_n = [\mathbf{C}_n^T\mathbf{C}_n + (\mathbf{A}_n\mathbf{F}_{n-1}\mathbf{A}_n^T)^{-1}]^{-1}$
	$\bar{\mathbf{x}}_{n+p\vert n} = \mathbf{A}_{n+p}\bar{\mathbf{x}}_{n+p-1\vert n-1}$
	$+\mathbf{A}_{n+p}\boldsymbol{\Upsilon}_n^{-1}(p)\mathbf{F}_n\mathbf{C}_n^T$
	$\times[\mathbf{y}_n - \mathbf{C}_n\boldsymbol{\Upsilon}_n(p)\bar{\mathbf{x}}_{n+p-1\vert n-1}]$

4.1 Full-Horizon Time-Varying Kalman-Like Estimator

In special cases when noise is nonstationary or both the system and measurement noise components need to be filtered out, all the data available should be processed. By letting $N = n+1$ and $l = n \geqslant K$ in (22)–(26), the relevant full-horizon algorithm becomes as shown in Table 1 and, for TI models, simplifies to that proposed in [19]. As can be seen, the algorithm (Table 1) requires only K and p, thus has extremely strong engineering features.

4.2 Error Bound

Provided $\bar{\mathbf{H}}_{n,m}(p)$, the estimate error bound can be ascertained via the noise power gain (NPG) in the three-sigma sense as follows:

$$EB_{k(vg)}(n,N,p) = 3\sigma_k K_{k(vg)}^{1/2}(n,N,p) \tag{27}$$

where σ_k is the noise variance of the measurement of the kth state and $K_{k(vg)}(n,N,p)$ is the (vg)th component of the square NPG matrix $\mathbf{K}_k \triangleq \mathbf{K}_k(n,N,p)$ specialized as $\mathbf{K}_k = \mathscr{H}_k\mathscr{H}_k^T$. Here the thinned $K\times N$ gain $\mathscr{H}_k \triangleq \left(\bar{\mathbf{H}}_{n,m}(p)\right)_k$ is composed by Kth columns of $\bar{\mathbf{H}}_{n,m}(p)$ starting with the kth one.

5. EXAMPLES OF APPLICATIONS

Below, we provide filtering with $p = 0$ and prediction with $p > 0$ of the two-state polynomial model, (1) and (2), specified with $\mathbf{B}_n = \mathbf{I}$, $\mathbf{D}_n = \mathbf{I}$, $\mathbf{C}_n = [1\ 0]$, and

$$\mathbf{A}_n = \begin{bmatrix} 1 & (1+d_n)\tau\\ 0 & 1\end{bmatrix}, \tag{28}$$

where d_n temporary takes different values. Such a situation occurs in oscillators undergoing temporary frequency "jumps" or in moving vehiculars with velocity "jumps". For TI filtering, d_n represents uncertainty and, in the TV case, d_n is supposed to be known exactly. We mostly allow noise to be white Gaussian, noticing that the relevant investigations for the uniformly distributed and highly intensive sawtooth noise were provided in [20–22, 24, 29, 30].

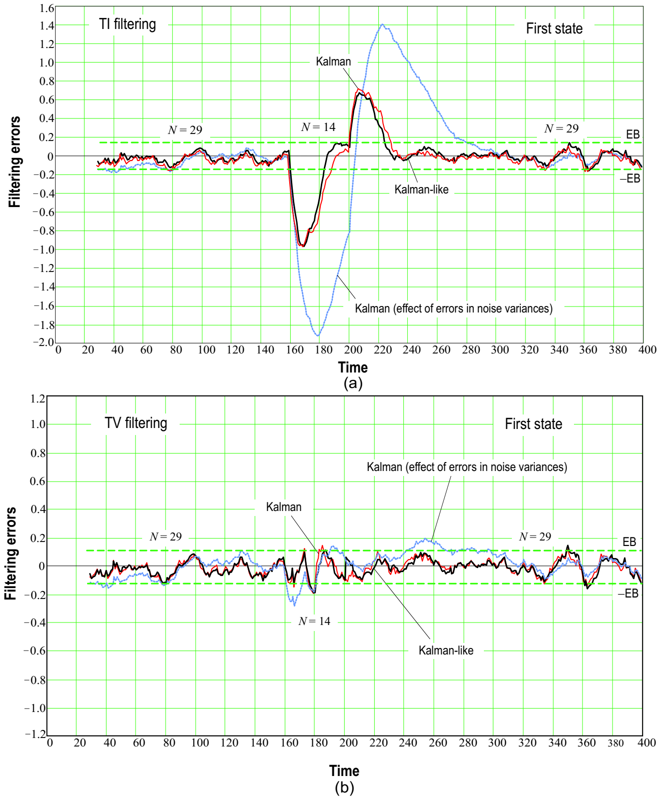

Figure 2: Errors of Kalman and Kalman-like FIR unbiased filtering of the first state of a polynomial model: (a) TI filtering and (b) TV filtering.

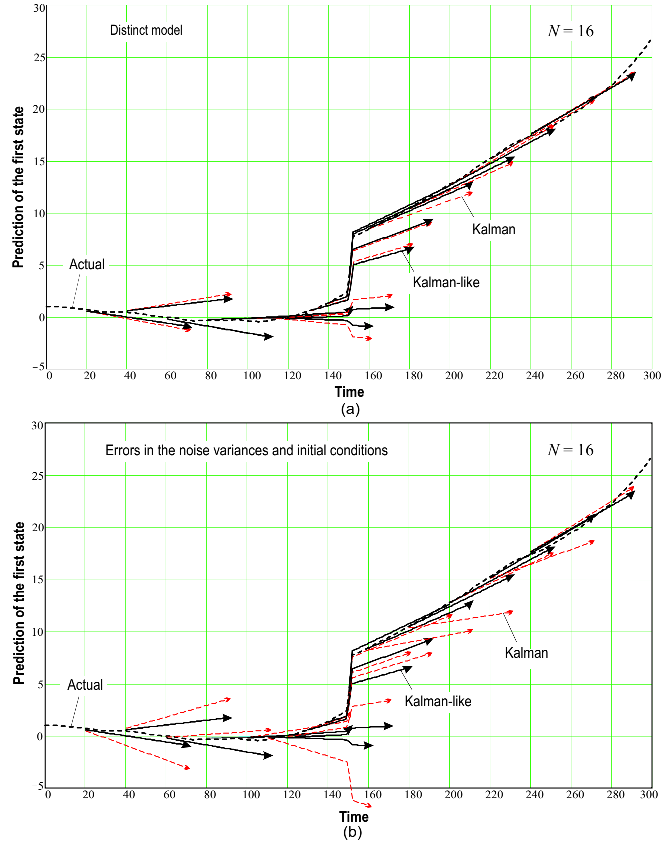

Figure 3: Typical errors of the Kalman and Kalman-like FIR unbiased prediction of the model behavior: (a) distinct model and (b) errors in the noise variances and initial conditions.

5.1 Filtering with Errors in Noise Covariances

In this experiment, we allow $x_{10} = 1$, $x_{20} = 0.01/\mathrm{s}$, $\sigma_1^2 = 10^{-4}$, $\sigma_2^2 = 4\times10^{-6}/\mathrm{s}^2$, and $\sigma_v = 0.15$. An uncertainty is induced with $d = 5$ from 160 to 200. Because the system noise is often hard to determine exactly, we reduce the standard deviations in the first and second states by the factors of 2 and 4, respectively, to have $\sigma_1^2 = 0.25\times10^{-4}$ and $\sigma_2^2 = 0.25\times10^{-6}/$ s. Figure 2 gives us a typical reproducible example of errors in the KF and Kalman-like FIR UF for this case. One notes that the Kalman-like filter outperforms the Kalman one in both the TI case (Fig. 2a) and TV case (Fig. 2b), as being independent on noise performance.

5.2 Prediction of a Distinct Model

An objective of this study is to predict future behavior of the TV model. It follows from (22) that the Kalman-like FIR unbiased prediction can be organized at any fixed n, by increasing a step $p > 0$. In turn, the Kalman prediction can be obtained if to project the estimate from n to $n+p$ as follows:

$$\tilde{\mathbf{x}}_{n+p|n} = \mathscr{A}_{n+p}^{n+1}\hat{\mathbf{x}}_{n|n}\,. \tag{29}$$

The process was simulated with $x_{10} = 1$, $x_{20} = 0.01/\mathrm{s}$, $\sigma_1^2 = 10^{-6}$, $\sigma_2^2 = 10^{-4}/\mathrm{s}^2$. Measurement was provided with the uniformly distributed non-Gaussian noise having $\sigma_v^2 = 0.04$ and associated with sawtooth induced by the Global Positioning System timing receiver. A temporary uncertainty was organized with $d_n = 20$ from 150 to 152.

Figure 3a shows the predicted behaviors for known variances and initial conditions. Instead, Fig. 3b sketches the trends affected by not fully known ones, in which case we increase the standard deviations in the first and second states by the factors of 2 and 4, respectively, and allow for $x_{10} = 2$ and $x_{20} = 0.03/\mathrm{s}$. On the whole, the Kalman-like predictor outperforms the Kalman one here, although the estimate difference is most brightly pronounced in Fig. 3b.

6. CONCLUSION

We proposed and investigated a Kalman-like FIR UE intended for filtering ($p = 0$), p-step prediction ($p > 0$), and p-lag smoothing ($p < 0$) of discrete TV state-space models, ignoring noise and initial conditions. The most common conclusions that can be made are the following. In the ideal case of a model, initial conditions, and white noise covariances, all known exactly at each time point, the optimal Kalman filter inherently outperforms the unbiased FIR one, typically on several percents in terms of the MSE. The latter is able to overperform the former otherwise and if both the system and measurement noise components need to be filtered out. Examples considered have demonstrated better robustness of the FIR UE against the KF, although we made no efforts to improve this performance. A natural payment for these advantages is an about N time larger consumption of the computation time in the Kalman-like iterative procedure.

REFERENCES

[1] R. E. Kalman, "A new approach to linear filtering and prediction problems," *J. Basic Engineer.*, vol. 82, pp. 35–45, Mar. 1960.

[2] H. Cox, "On the estimation of state variables and parameters for noisy dynamic systems," *IEEE Trans. Autom. Contr.*, vol. 9, pp. 5–12, Jan. 1964.

[3] A. H. Jazwinski, *Stochastic Processes and Filtering Theory*, New York: Academic Press, 1970.

[4] S. J. Julier and J. K. Uhlmann, "A new extension of the Kalman filter to nonlinear systems", *Proc. of SPIE*, vol. 3068, pp. 182-193, 1997.

[5] S. Bonnabel, Ph. Martin and E. Salaün, "Invariant Extended Kalman Filter: theory and application to a velocity-aided attitude estimation problem", in *Proc. 48th IEEE Conf. Decision Contr.*, pp. 1297–1304, 2009.

[6] G. Evensen, "Sequential data assimilation with nonlinear quasi-geostrophic model using Monte Carlo methods to forecast error statistics," *Journal of Geophysical Research*, vol. 99, pp. 143–162, May 1994.

[7] A. A. Lange, "Simultaneous statistical calibration of the GPS signal delay measurements with related meteorological data", *Physics and Chemistry of the Earth, Part A: Solid Earth and Geodesy*, vol. 26, pp. 471–473, 2001.

[8] C. J. Masreliez, "Approximate non-Gaussian filtering with linear state and observation relations," *IEEE Trans. Autom. Contr.*, vol. AC-20, pp. 107–110, Feb 1975.

[9] R. D. Martin and C.J. Masreliez, "Robust estimation via stochastic approximation," *IEEE Trans. Inform. Theory*, vol. IT-21, pp. 263–271, May 1975.

[10] N. E. Nahi, "Optimal recursive estimation with uncertain observation," *IEEE Trans. Inf. Theory*, vol. IT-15, pp. 457–462. Jul. 1969.

[11] M. Basseville, A. Benveniste, K. C. Chou, S. A. Golden, R. Nikoukhah, and A. S. Willsky, "Modeling and estimation of multiresolution stochastic processes," *IEEE Trans. Inf. Theory*, vol. 38, pp. 766-784, Mar. 1992.

[12] E. Baccarelli and R. Cusani, "Recursive Kalman-type optimal estimation and detection of hidden Markov chains," *Signal Process.*, vol. 51, pp. 55–64. May 1996.

[13] B. Ait-El-Fquih and F. Desbouvries, "Kalman Filtering in Triplet Markov Chains," *IEEE Trans. Signal Process.*, vol. 54, pp. 2957–2963, Aug. 2006.

[14] A. Carmi, P. Gurfil, and D. Kanevsky, "Methods for sparse signal recovery using Kalman filtering with embedded pseudo-measurement norms and quasi-norms", *IEEE Trans. on Signal Process.*, vol. 58, pp. 2405–2409, Apr. 2010.

[15] S. Stefanatos and A. K. Katsaggelos, "Joint data detection and channel tracking for OFDM systems with phase noise," *IEEE Trans. Signal Process.*, vol. 56, pp. 4230–4243, Sep. 2008.

[16] W. H. Kwon and S. Han, *Receding horizon control: model predictive control for state models*, London: Springer, 2005.

[17] W. H. Kwon, P. S. Kim, and P. Park, "A receding horizon Kalman FIR filter for discrete time-invariant systems," *IEEE Trans. Autom. Contr.* vol. 44, pp. 1787–1791, Sep. 1999.

[18] S. H. Han, W. H. Kwon, and P. S. Kim, "Quasi-deadbeat minimax filters for deterministic state-space models," *IEEE Trans. Autom. Contr.*, vol. 47, pp. 1904–1908, Nov. 2002.

[19] Y. S. Shmaliy, "Linear optimal FIR estimation of discrete time-invariant state-space models," *IEEE Trans. Signal Process.*, vol. 58, pp. 3086–3096, Jun. 2010.

[20] Y. S. Shmaliy, "An unbiased FIR filter for TIE model of a local clock in applications to GPS-based timekeeping," *IEEE Trans. on Ultrason., Ferroel. and Freq. Contr.*, vol. 53, pp. 862–870, May 2006.

[21] Y. S. Shmaliy, "Optimal gains of FIR estimators for a class of discrete-time state-space models," *IEEE Signal Process. Letters*, vol. 15, pp. 517–520, 2008.

[22] P. S. Kim, "An alternative FIR filter for state estimation in discrete-time systems," *Digital Signal Process.,* vol. 20, pp. 935–943, May 2010.

[23] W. H. Kwon, P. S. Kim, and S. H. Han, "A receding horizon unbiased FIR filter for discrete-time state space models," *Automatica*, vol. 38, pp. 545–551, Mar. 2002.

[24] Y. S. Shmaliy, "Unbiased FIR filtering of discrete-time polynomial state-space models," *IEEE Trans. Signal Process.*, vol. 57, pp. 1241–1249, Apr. 2009.

[25] Y. S. Shmaliy, J. Muñoz-Diaz, and L. Arceo-Miquel, "Optimal horizons for a one-parameter family of unbiased FIR filters," *Digital Signal Process.,* vol. 18, pp. 739–750, Sep. 2008.

[26] H. Stark and J. W. Woods, *Probability, Random Processes, and Estimation Theory for Engineers*, 2nd ed., Upper Saddle River, NJ: Prentice Hall, 1994.

[27] Y. S. Shmaliy, O. Ibarra-Manzano, L. Arceo-Miquel, and J. Muñoz-Diaz, "A thinning algorithm for GPS-based unbiased FIR estimation of a clock TIE model," *Measurement,* vol. 41, pp. 538–550, Jun. 2008.

[28] M. Blum, "On the mean square noise power of an optimum linear discrete filter operating on polynomial plus white noise input," *IRE Trans. Information Theory*, vol. 3, pp. 225–231, Dec. 1957.

[29] Y. S. Shmaliy and O. Ibarra-Manzano, "Optimal FIR filtering of the clock time errors," *Metrologia*, vol. 45, pp. 571–576, Sep. 2008.

[30] Y. S. Shmaliy, "A simple optimally unbiased MA filter for timekeeping," *IEEE Trans. Ultrason. Ferroel. Freq. Control*, vol. 49, pp. 789–797, Jun. 2002.

[31] Y. S. Shmaliy, "Linear unbiased prediction of clock errors," *IEEE Trans. Ultrason. Ferroel. Freq. Control*, vol. 56, pp. 2027–2029, Sep. 2009.

[32] G. H. Golub and G. F. van Loan, *Matrix Computations,* 3rd Ed., Baltimore: The John Hopkins Univ. Press, 1996.