TENSOR CODING FOR CDMA-MIMO WIRELESS COMMUNICATION SYSTEMS

Gérard Favier[a], Michele N. da Costa[a,b], André L.F. de Almeida[c], João Marcos T. Romano[b]

[a]I3S Laboratory, University of Nice-Sophia Antipolis, CNRS, Sophia Antipolis, France
[b]DSPCom Laboratory, University of Campinas (UNICAMP), Campinas, Brazil
[c]Department of Teleinformatics Engineering, Federal University of Ceará, Fortaleza, Brazil
favier@i3s.unice.fr (G. Favier), nazareth@i3s.unice.fr (M.N. da Costa), andre@gtel.ufc.br (A.L.F. de Almeida), romano@dmo.fee.unicamp.br (J.M.T. Romano)

ABSTRACT

In this paper, we propose a tridimensional tensor coding for multiple-input multiple-output (MIMO) communication systems. This coding allows spreading and multiplexing the transmitted symbols in both space and time domains, owing the use of two allocation matrices. Assuming flat Rayleigh fading channels, the signals received by K receive antennas during P time blocks, composed of N symbol periods each, with J chips per symbol, form a fourth-order tensor that satisfies a new constrained tensor model. A two-step alternating least squares (ALS) algorithm is proposed for blindly and jointly estimating the channel and the transmitted symbols. The performance of the proposed blind receiver is evaluated by means of computer simulations.

1. INTRODUCTION

The key idea for improving the error performance in wireless communication systems is to jointly exploit several diversities, which means redundancy into the information-bearing signals available at the receiver. This redundancy can be obtained through spreading operations at the transmitter, in space, time and/or frequency domains.

Generally speaking, space diversity results from the use of multiple antennas at both transmitter and receiver ends, which leads to multiple-input multiple-output (MIMO) channels. As now well known, the deployment of multiple antennas in wireless systems allows improving the transmission rate and reliability over single-transmit antenna systems, while keeping the same transmission bandwidth and power.

Space spreading results from the use of several transmit antennas for transmitting the same symbol or data stream, whereas time spreading consists in repeating the same symbol multiplied by spreading codes, during several chip periods associated with each symbol. Time spreading can also be obtained by transmitting the same symbols or data streams over multiple blocks, each symbol period corresponding to a single channel use.

On the other hand, space multiplexing that consists in transmitting independent data streams in parallel on multiple antennas, allows to increase the transmission rate.

Space-time (ST) coding is one of the most popular approaches relying on multi-antenna transmissions for achieving the fundamental tradeoff between error performance (in terms of bit error rate, abbreviated as BER) and data rate (in bits per channel use) [10].

Since the pioneering work of [9], several tensorial approaches have been developed for space-time MIMO wireless communication systems with matrix ST coding and blind receivers [1, 2, 3, 4, 5, 6, 8].

In this paper, we propose a new tensor space-time (TST) coding. This TST coding allows spreading and multiplexing the transmitted symbols in both space (transmit antennas) and time (chips and blocks) domains, through the use of a third-order code tensor admitting transmit antenna, data stream and chip as modes, and two allocation matrices that allocate transmit antennas and data streams to each block. Assuming flat Rayleigh fading propagation channels, the signals received by K receive antennas during P time blocks, composed of N symbol periods each, with J chips per symbol, form a fourth-order tensor that satisfies a new constrained tensor model, called a PARATUCK-(2,4) model. The proposed transmission system can be viewed as an extension of the ST transmission system of [4] that relies on a PARATUCK-2 tensor model for the received signals. This extension results from the introduction of a time-spreading code. Then, a blind TST-based receiver is derived for joint channel and symbol estimation using a two-step alternating least squares (ALS) algorithm.

The rest of the paper is organized as follows. Section 2 presents the proposed MIMO transmission system using a TST coding. The tensor of received signals is then derived assuming flat Rayleigh fading propagation channels. Section 3 discusses the identifiability and uniqueness conditions for the PARATUCK-(2,4) model of the received signals, and a blind TST-ALS based receiver is proposed for joint channel and symbol estimation. In Section 4, some simulation results are provided to illustrate the performance of this receiver, before concluding the paper in Section 5.

Notations: Scalars, column vectors, matrices and higher-order tensors are written as lower-case (a, b,...), boldface lower-case ($\mathbf{a}$, $\mathbf{b}$,...), boldface upper-case ($\mathbf{A}$, $\mathbf{B}$,...), and blackboard ($\mathbb{A}$, $\mathbb{B}$,...) letters, respectively. $\mathbf{A}^{\mathrm{T}}$, $\mathbf{A}^{\mathrm{H}}$, $\mathbf{A}^*$, and $\mathbf{A}^\dagger$ stand for transpose, transconjugate (Hermitian transpose), complex conjugate, and Moore-Penrose pseudo-inverse of $\mathbf{A}$, respectively. The vector $\mathbf{A}_{i\cdot}$ (resp. $\mathbf{A}_{\cdot j}$) represents the i^{th} row (resp. j^{th} column) of $\mathbf{A}$. The scalar $a_{i_1,\cdots,i_N}$ denotes the $(i_1,\cdots,i_N)$-th entry of $\mathbb{A}$. $\mathbf{D}_i(\mathbf{A})$ is the diagonal matrix formed with the i^{th} row of $\mathbf{A}$; $\mathbf{I}_N$ is the identity matrix of order N, $\mathbf{1}_N$ is the all-one column vector of dimension $(N,1)$, and $\|\cdot\|_F$ is the Frobenius norm. The operator $\mathrm{vec}(\cdot)$ forms a vector by stacking the columns of its matrix argument, whereas $\mathrm{diag}(\cdot)$ forms a diagonal matrix from its vector argument. The Kronecker and Khatri-Rao (column-wise Kronecker) products are denoted by $\otimes$ and $\diamondsuit$, respectively. We have the following property:

$$\mathrm{vec}(\mathbf{B}\mathbf{C}\mathbf{A}^{\mathrm{T}}) = (\mathbf{A}\otimes\mathbf{B})\,\mathrm{vec}(\mathbf{C}) \tag{1}$$

This work is supported by the CAPES-COFECUB project No. Ma 544/07. André L. F. de Almeida is partially supported by CNPq/Brazil (Grant No. 303238/2010-0).

for $\mathbf{A} \in \mathscr{C}^{I\times R}$, $\mathbf{B} \in \mathscr{C}^{J\times S}$ and $\mathbf{C} \in \mathscr{C}^{S\times R}$.

2. TENSOR CODING AND TENSOR MODELING OF RECEIVED SIGNALS

2.1 Proposed TST coding

We consider a MIMO wireless communication system with M transmit antennas and K receive antennas, and we denote by $s_{n,r}$ the n^{th} symbol of the r^{th} data stream, each data stream being composed of N information symbols.

The transmission is assumed to be decomposed into P data blocks, each block being formed of N time slots. At each time slot n of the p^{th} block, the transceiver transmits a linear combination of the n^{th} symbols of the data streams determined by the stream-to-block allocation matrix $\mathbf{\Psi} \in \mathscr{R}^{P\times R}$, across a set of transmit antennas fixed by the antenna-to-block allocation matrix $\mathbf{\Phi} \in \mathscr{R}^{P\times M}$.

Each symbol $s_{n,r}$ is replicated several times after multiplication by a three-dimensional spreading code $w_{m,r,j}$, in such a way that the signal transmitted from the m^{th} transmit antenna during the n^{th} time slot of the p^{th} block, and associated with the j^{th} chip, is given by:

$$u_{m,n,p,j} = \sum_{r=1}^{R} w_{m,r,j}\, s_{n,r}\, \phi_{p,m}\, \psi_{p,r} = \sum_{r=1}^{R} g_{m,r,p,j}\, s_{n,r} \qquad (2)$$

with

$$g_{m,r,p,j} = w_{m,r,j}\, \phi_{p,m}\, \psi_{p,r} \qquad (3)$$

Remark: $\psi_{p,r} = 1$ means that the r^{th} data stream is allocated to the p^{th} block, whereas $\psi_{p,r} = 0$ means that the r^{th} data stream is not allocated to the p^{th} block.

2.2 Tensor modeling of received signals

In the noiseless case and assuming flat Rayleigh fading propagation channels, the discrete-time baseband-equivalent model for the signal received at the k^{th} receive antenna during the j^{th} chip period of the n^{th} symbol period of the p^{th} block, is given by:

$$x_{k,n,p,j} = \sum_{m=1}^{M} h_{k,m}\, u_{m,n,p,j} = \sum_{m=1}^{M}\sum_{r=1}^{R} g_{m,r,p,j}\, h_{k,m}\, s_{n,r} \qquad (4)$$

The fading coefficients $h_{k,m}$ between transmit antennas (m) and receive antennas (k) are assumed to be independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables. They are also assumed to be constant during at least P blocks.

The fourth-order tensor $\mathbb{X} \in \mathscr{C}^{K\times N\times P\times J}$ of received signals satisfies the constrained tensor model (3)-(4) that we will call a PARATUCK-(2,4) model.

Remark: If we set $J = 1$, (2) becomes independent of j:

$$u_{m,n,p} = \sum_{r=1}^{R} w_{m,r}\, s_{n,r}\, \phi_{p,m}\, \psi_{p,r} \qquad (5)$$

which corresponds to the ST transmission system proposed in [4] that leads to a PARATUCK-2 model for the received signals.

Comparing (5) with (2), we can conclude that the proposed TST coding allows to take a supplementary time diversity into account. This diversity is associated with the third mode (j) of the code tensor that induces an extra time-spreading of the symbols. Note that the transmission rate for both transceivers is given by $V = \frac{R}{P}\log_2(\mu)$ bits per channel use, where μ is the cardinality of the information symbol constellation.

3. BLIND TST-BASED RECEIVER

3.1 Matrix representations of the received signal tensor

Let us define $\mathbf{X}_{\cdot\cdot p,j} \in \mathscr{C}^{K\times N}$ as the matrix slice of the received signal tensor $\mathbb{X} \in \mathscr{C}^{K\times N\times P\times J}$, obtained by slicing it along the plane (p, j), i.e. by fixing the two last indices. Using (4) leads to the following factorization

$$\mathbf{X}_{\cdot\cdot p,j} = \mathbf{H}\,\mathbf{G}_{\cdot\cdot p,j}\,\mathbf{S}^{\mathrm{T}} \qquad (6)$$

where $\mathbf{G}_{\cdot\cdot p,j} \in \mathscr{C}^{M\times R}$ can be deduced from (3)

$$\mathbf{G}_{\cdot\cdot p,j} = \mathbf{D}_p(\mathbf{\Phi})\,\mathbf{W}_{\cdot\cdot j}\,\mathbf{D}_p(\mathbf{\Psi}) \qquad (7)$$

Applying property (1) to (6) and (7) gives

$$\operatorname{vec}\left(\mathbf{X}_{\cdot\cdot \mathrm{p,j}}\right) = (\mathbf{S}\otimes\mathbf{H})\operatorname{vec}\left(\mathbf{G}_{\cdot\cdot \mathrm{p,j}}\right) \qquad (8)$$

and

$$\begin{aligned}\operatorname{vec}\left(\mathbf{G}_{\cdot\cdot \mathrm{p,j}}\right) &= \left(\mathbf{D}_p(\mathbf{\Psi})\otimes\mathbf{D}_p(\mathbf{\Phi})\right)\operatorname{vec}\left(\mathbf{W}_{\cdot\cdot \mathrm{j}}\right) \qquad (9)\\ &= \operatorname{diag}\left(\operatorname{vec}\left(\mathbf{W}_{\cdot\cdot \mathrm{j}}\right)\right)\left(\mathbf{\Psi}^{\mathrm{T}}_{\mathrm{p}\cdot}\otimes\mathbf{\Phi}^{\mathrm{T}}_{\mathrm{p}\cdot}\right) \qquad (10)\end{aligned}$$

which gives

$$\operatorname{vec}\left(\mathbf{X}_{\cdot\cdot \mathrm{p,j}}\right) = (\mathbf{S}\otimes\mathbf{H})\operatorname{diag}\left(\operatorname{vec}\left(\mathbf{W}_{\cdot\cdot \mathrm{j}}\right)\right)\left(\mathbf{\Psi}^{\mathrm{T}}_{\mathrm{p}\cdot}\otimes\mathbf{\Phi}^{\mathrm{T}}_{\mathrm{p}\cdot}\right) \quad (11)$$

From (6), we deduce the following two matrix unfoldings of the received signal tensor $\mathbb{X}$:

$$\mathbf{X}_2 = \begin{bmatrix}\mathbf{X}_{\cdot\cdot 1,1}\\ \vdots\\ \mathbf{X}_{\cdot\cdot P,1}\\ \vdots\\ \mathbf{X}_{\cdot\cdot 1,J}\\ \vdots\\ \mathbf{X}_{\cdot\cdot P,J}\end{bmatrix} \in \mathscr{C}^{PJK\times N}, \quad \mathbf{X}_3 = \begin{bmatrix}\mathbf{X}^{\mathrm{T}}_{\cdot\cdot 1,1}\\ \vdots\\ \mathbf{X}^{\mathrm{T}}_{\cdot\cdot P,1}\\ \vdots\\ \mathbf{X}^{\mathrm{T}}_{\cdot\cdot 1,J}\\ \vdots\\ \mathbf{X}^{\mathrm{T}}_{\cdot\cdot P,J}\end{bmatrix} \in \mathscr{C}^{PJN\times K},$$

$$= (\mathbf{I}_{PJ}\otimes\mathbf{H})\,\mathbf{G}_2\mathbf{S}^{\mathrm{T}} \qquad\qquad = (\mathbf{I}_{PJ}\otimes\mathbf{S})\,\mathbf{G}_3\mathbf{H}^{\mathrm{T}} \qquad (12)$$

with

$$\mathbf{G}_2 = \begin{bmatrix}\mathbf{G}_{\cdot\cdot 1,1}\\ \vdots\\ \mathbf{G}_{\cdot\cdot P,1}\\ \vdots\\ \mathbf{G}_{\cdot\cdot 1,J}\\ \vdots\\ \mathbf{G}_{\cdot\cdot P,J}\end{bmatrix} \in \mathbb{C}^{PJM\times R}, \quad \mathbf{G}_3 = \begin{bmatrix}\mathbf{G}^{\mathrm{T}}_{\cdot\cdot 1,1}\\ \vdots\\ \mathbf{G}^{\mathrm{T}}_{\cdot\cdot P,1}\\ \vdots\\ \mathbf{G}^{\mathrm{T}}_{\cdot\cdot 1,J}\\ \vdots\\ \mathbf{G}^{\mathrm{T}}_{\cdot\cdot P,J}\end{bmatrix} \in \mathbb{C}^{PJR\times M}. \qquad (13)$$

Using (11), we can build a third matrix unfolding of $\mathbb{X}$ as:

$$\begin{aligned}\mathbf{X}_1 &= [\ \mathrm{vec}(\mathbf{X}_{..1,1}) \ \cdots \ \mathrm{vec}(\mathbf{X}_{..P,1}) \ \cdots \ \mathrm{vec}(\mathbf{X}_{..P,J})\ ] \\ &= (\mathbf{S}\otimes\mathbf{H})\,\mathbf{G}_1 \in \mathscr{C}^{NK\times JP}\end{aligned} \tag{14}$$

where

$$\begin{aligned}\mathbf{G}_1 =&[\mathrm{diag}\left(\mathrm{vec}\left(\mathbf{W}_{..1}\right)\right)\left(\mathbf{\Psi}^{\mathrm{T}}\diamondsuit\mathbf{\Phi}^{\mathrm{T}}\right)\ \cdots \\ &\cdots\ \mathrm{diag}\left(\mathrm{vec}\left(\mathbf{W}_{..J}\right)\right)\left(\mathbf{\Psi}^{\mathrm{T}}\diamondsuit\mathbf{\Phi}^{\mathrm{T}}\right)] \in \mathscr{C}^{\mathrm{RM}\times\mathrm{JP}}\end{aligned} \tag{15}$$

3.2 Identifiability and uniqueness issues

Structure of the third-order code tensor $\mathbb{W} \in \mathscr{C}^{M\times R\times J}$

For the code tensor, we choose a third-order Vandermonde tensor defined as:

$$w_{m,r,j} = e^{i2\pi mrj/MRJ}, \tag{16}$$

where $i^2 = -1$. An important reason behind this choice for the code tensor is that this Vandermonde structure guarantees the existence of a minimum value of the spreading length J ensuring the identifiability in the LS sense of the channel ($\mathbf{H}$) and symbol ($\mathbf{S}$) matrices when $M \neq R$. Due to a lack of space, this result can not be developed in this paper.

Proceeding in the same way as in [4], it is easy to deduce the following results.

Identifiability

Each matrix $\mathbf{S}$ and $\mathbf{H}$ is estimated by alternately solving the two equations (12) in the LS sense with respect to one matrix conditionally to the knowledge of previously estimated value of the other matrix. Assuming that the symbol and channel matrices are full column-rank, which implies $N \geq R$ and $K \geq M$, uniqueness of their conditional LS estimates requires that $\mathbf{G}_2 \in \mathscr{C}^{PJM\times R}$ and $\mathbf{G}_3 \in \mathscr{C}^{PJR\times M}$ be also full column-rank. From this double condition, we deduce the following theorem.

Theorem 1. *Assuming that* $\mathbf{S}$ *and* $\mathbf{H}$ *are full column-rank, a necessary condition for identifiability is given by:*

$$PJ \geq \max\left(\left\lceil \frac{R}{M}, \frac{M}{R} \right\rceil\right). \tag{17}$$

where $\lceil x \rceil$ denotes the smallest integer number greater than or equal to x.

This condition (17) defines a constraint that the design parameters (P,J,M,R) must satisfy. It is interesting to notice that the supplementary diversity introduced by the time-spreading mode (j) of the code tensor allows us to get a more relaxed condition on the number P of data blocks that is necessary for LS identifiability.

Theorem 2. *Assuming that* $\mathbf{S}$ *and* $\mathbf{H}$ *are full column-rank, and choosing a Vandermonde code tensor as defined in* (16) *and the allocation matrices such that $\mathbf{\Phi}_{p\cdot} = \mathbf{1}_M^{\mathrm{T}}$ and $\mathbf{\Psi}_{p\cdot} = \mathbf{1}_R^{\mathrm{T}}$, for a given $p \in \{1,\ldots,P\}$, then* $\mathbf{S}$ *and* $\mathbf{H}$ *are identifiable in the LS sense if $M = R$, for all values $J \geq 1$.*

This sufficient condition is identical to that of Theorem 2 in [4] obtained for $J = 1$. However, unlike [4], introducing the time-spreading mode in the Vandermonde code tensor allows to derive a minimum value of the spreading length J that ensures the identifiability of $\mathbf{S}$ and $\mathbf{H}$ in the case $M \neq R$.

Uniqueness

Theorem 3. *If $\mathbf{\Psi}^{\mathrm{T}}\diamondsuit\mathbf{\Phi}^{\mathrm{T}}$ is full row-rank, which implies $P \geq RM$, then* $\mathbf{S}$ *and* $\mathbf{H}$ *are unique up to a scalar factor, i.e.*

$$\mathbf{S} = \alpha\hat{\mathbf{S}}, \qquad \mathbf{H} = \frac{1}{\alpha}\hat{\mathbf{H}}. \tag{18}$$

This theorem is identical to theorem 3 of [4], which means that the introduction of the time-spreading mode in the code tensor does not modify the uniqueness property of the tensor model. The scaling ambiguity α can be eliminated in assuming known the first transmitted symbol $s_{1,1}$.

3.3 ALS algorithm for blind joint symbol and channel estimation

Assuming that the code tensor $\mathbb{W}$ and the allocation matrices $\mathbf{\Phi}$ and $\mathbf{\Psi}$ are known at the receiver, the matrices $\mathbf{G}_2$ and $\mathbf{G}_3$ can be pre-calculated. Blind joint symbol and channel estimation can be carried out by applying the ALS technique for solving the two equations (12) with respect to $\mathbf{S}$ and $\mathbf{H}$, respectively.

4. SIMULATION RESULTS

The performance of the proposed TST coding and the associated ALS-based blind receiver is evaluated by means of Monte Carlo simulations, in terms of BER and normalized mean square error (NMSE) on channel estimation, defined as

$$\mathrm{NMSE}_{\mathrm{dB}} = 10\log_{10}\left(\frac{1}{L}\sum_{l=1}^{L}\frac{\|\mathbf{H}-\hat{\mathbf{H}}_{l(\infty)}\|_F^2}{\|\mathbf{H}\|_F^2}\right), \tag{19}$$

where $\hat{\mathbf{H}}_{l(\infty)}$ is the channel matrix estimated at convergence of the l^{th} run, and $L = 2000$ is the total number of Monte Carlo runs corresponding to 2000 random wireless channels, with different symbol sequences randomly drawn from a QPSK constellation, and different additive random noise sequences, for each simulated channel. A different random initialization $\hat{\mathbf{H}}_{l(0)}$ is also used for each run. The BER is calculated by averaging the results obtained for the R data streams and the L Monte Carlo runs. The signal-to-noise ratio (SNR) is determined by

$$\mathrm{SNR} = 10\log_{10}\frac{\|\mathbf{X}_2\|_F^2}{\|\mathbf{V}_2\|_F^2}, \tag{20}$$

where $\mathbf{V}_2$ is the unfolded matrix of the additive noise tensor.

The default values of the tuning parameters are chosen as follows: $R = 2$, $N = 10$, $J = 3$, $P = 4$, $K = M = 2$.

For $P = 10$, $M = 2$ and $R = 4$, the allocation matrices are chosen such as:

$$\mathbf{\Phi}_{10} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{\Psi}_{10} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}. \tag{21}$$

The matrices $\mathbf{\Phi}_4$ and $\mathbf{\Psi}_4$ for $M=2$ and $R=4$ are obtained by discarding the last six rows of $\mathbf{\Phi}_{10}$ and $\mathbf{\Psi}_{10}$ given in (21), respectively. In the same way, the matrix $\mathbf{\Psi}_{10}$ for $R=2$ is obtained by discarding the last two columns of $\mathbf{\Psi}_{10}$ for $R=4$. Use of the default values ($P=4$, $R=2$) implies a transmission rate equal to 1 bit per channel use.

In the sequel, we study the influence of the spreading code length (J), and of the numbers of blocks (P) and data streams (R). Then, we compare the proposed TST coding with the KRST coding of [8], and a comparison is also made with the zero-forcing (ZF) receiver assuming a perfect knowledge of the channel matrix.

4.1 Influence of the spreading code length

Figures 1, 2 and 3 show the channel NMSE, the BER and the number of iterations needed for convergence, versus SNR, for four values of the spreading code length ($J \in \{1,3,6,10\}$), respectively. From Figures 1 and 2, we can conclude that an increase of J induces a significant performance improvement in terms of both channel estimation and symbol recovery. Moreover, the use of $J>1$ implies a faster convergence comparatively to the one obtained with $J=1$ (see Figure 3). This improvement is due to the fact that the extra time-spreading introduced by the TST coding provides more output measurements to estimate the same number of parameters, which makes the convergence faster. It is to be recalled that the case $J=1$ corresponds to the blind receiver proposed in [4].

4.2 Influence of the block and data stream numbers

In order to emphasize the importance of time-spreading in the proposed TST coding, we analyze the BER for two values of the block number ($P \in \{4,10\}$), of the spreading code length ($J \in \{1,3\}$) and of the data stream number ($R \in \{2,4\}$) with the allocation matrices given in (21).

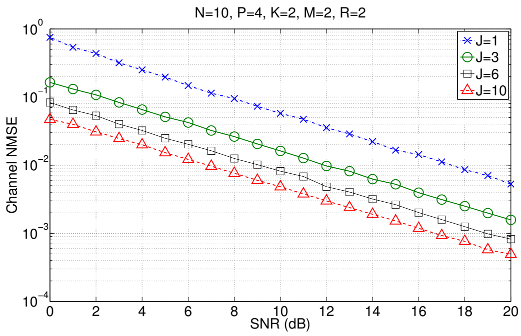

Figure 1: Influence of J: Channel NMSE versus SNR.

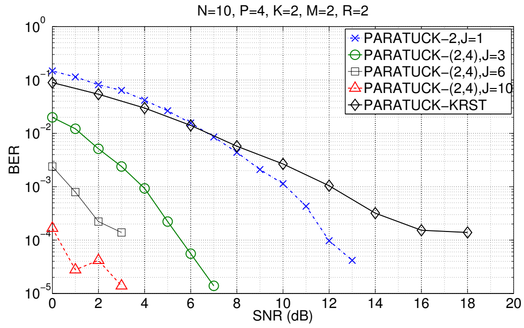

Figure 2: Influence of J: BER versus SNR.

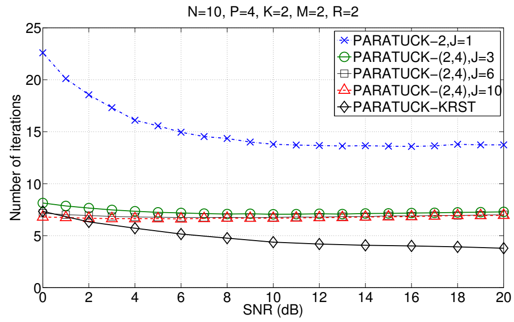

Figure 3: Influence of J: Iteration number for convergence versus SNR.

4.2.1 *Spreading code length versus block number*

In Figure 4, we note that the BER performance can be significantly improved by increasing either the number of data blocks or the spreading code length, since both actions induce an increase of time diversity. Remark that, for $R=2$, when using $J=1$, a larger number of blocks ($P=10$) is needed to obtain nearly the same BER as the one obtained with $J=3$ and $P=4$. However, for $R=2$, when the number of blocks is increased from 4 to 10, the transmission rate is reduced from 1 to 2/5 bit per channel use.

4.2.2 *Spreading code length versus data stream number*

For $P=10$, the transmission rate is equal to 2/5 and 4/5 bit per channel use when $R=2$ and $R=4$, respectively. As expected and as shown in Figure 4, the BER performance is improved when the number of data streams is reduced from 4 to 2 (for $P=10$ and $J=3$), which implies fewer symbols to be estimated with the same number of received signals. That illustrates the tradeoff to be achieved between error performance and transmission rate. It is very interesting to note that, for $P=10$, the error performance for $J=1$ and $R=2$ is close to the one obtained with $J=3$ and $R=4$, which shows that the use of TST coding allows to double the transmission rate without modifying much the BER.

4.3 Comparison with the KRST coding and the non-blind ZF receiver

The proposed blind TST-ALS based receiver is now compared with the non-blind TST-ZF receiver that estimates the symbol matrix by means of the following formula $\hat{\mathbf{S}}_{ZF}^{T} = [(\mathbf{I}_{PJ} \otimes \mathbf{H})\,\mathbf{G}_2]^{\dagger}\tilde{\mathbf{X}}_2$.

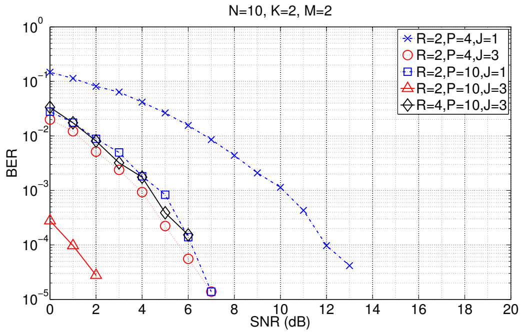

Figure 4: Influence of P and R: BER versus SNR.

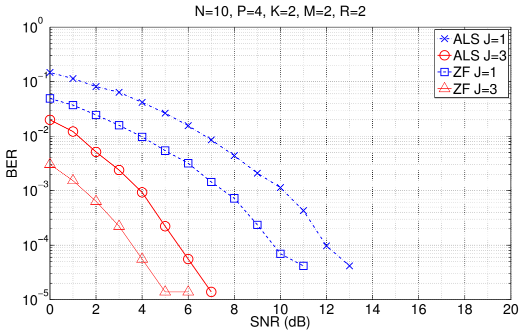

Figure 5: Comparison of TST-ALS and TST-ZF receivers.

For this comparison, the design parameters were set to their default values. The BER obtained with the two algorithms (ALS and ZF) is plotted on Figure 5 for $J \in \{1,3\}$. From these simulation results, we can conclude that, for a BER equal to 10^{-3} , the gap between the blind TST-ALS and non-blind TST-ZF receivers is around 2.5 dB in terms of SNR, for both values $J = 1$ and $J = 3$.

The simulation results are also compared with those obtained with the blind KRST-ALS based receiver of [8], with an identity precoding matrix.

With KRST coding, only one data stream, composed of M symbols, is transmitted from M transmit antennas, during each time block p of N slots, using two coding matrices $\mathbf{W} \in \mathscr{R}^{M\times M}$ and $\mathbf{C} \in \mathscr{R}^{N\times M}$. The first one allows to combine M symbols onto each transmit antenna, for a given block p, which gives the pre-coded signal $v_{p,m} = \sum_{l=1}^{M} w_{m,l} s_{p,l}$. The second matrix is to spread such a combination transmitted by each antenna over N slots, which provides a third-order tensor for the transmitted signals defined as $u_{m,n,p} = v_{p,m} c_{n,m}$.

Observe that for KRST coding, the number of data streams is forced to be equal to the number of transmit antennas, while it can be chosen equal to $R \geq M$ with TST coding. In addition, the use of tensor coding instead of matrix-based pre- and post-coding presents the advantages of an extra time spreading on chip and not needing decoding at the receiver.

Figures 2 and 3 show that the proposed TST-ALS based receiver outperforms the KRST-ALS based receiver in terms of BER, at the cost of a slower convergence due to the greater number of parameters to be estimated (PNR symbols for the TST coding and PM symbols for the KRST coding).

5. CONCLUSION

In this paper, a new tensor space-time coding has been proposed for MIMO wireless communication systems. The associated transceiver is characterized by a third-order code tensor and two allocation matrices that allow space-time spreading-multiplexing of the transmitted symbols. The introduction of one extra time diversity via the third mode of the code tensor induces a significant performance improvement in terms of BER and channel estimation accuracy comparatively to our previous solution [4], as illustrated by means of simulation results. This extra time diversity leads to a more relaxed condition on the number of data blocks to be processed for ensuring LS identifiability of channel and symbol matrices that can be jointly and blindly estimated using a two-step ALS technique. There are several perspectives of this work that include extensions to frequency selective and/or time varying MIMO channels [7], space-time-frequency coding, code design and allocation matrices optimization, alternative receiver algorithms, and blind receiver when the code tensor is unknown at the receiver.

REFERENCES

[1] A. L. F. de Almeida, G. Favier, and J. C. M. Mota. A constrained factor decomposition with application to mimo antenna systems. *IEEE Transactions on Signal Processing*, 56(6):2429–2442, 2008.

[2] A. L. F. de Almeida, G. Favier, and J. C. M. Mota. Constrained tensor modeling approach to blind multiple-antenna cdma schemes. *IEEE Transactions on Signal Processing*, 56(6):2417–2428, 2008.

[3] A. L. F. de Almeida, G. Favier, and J. C. M. Mota. Multiuser mimo system using block space-time spreading and tensor modeling. *Signal Processing*, 88(10):2388–2402, 2008.

[4] A. L. F. de Almeida, G. Favier, and J. C. M. Mota. Space-time spreading-multiplexing for mimo wireless communication systems using the paratuck-2 tensor model. *Signal Processing*, 89(11):2103–2116, 2009.

[5] A. de Baynast, L. de Lathauwer, and B. Aazhang. Blind parafac receivers for multiple access-multiple antenna systems. In *Proc. IEEE 58th Fall Vehicular Technology Conf. (VTC 2003)*, volume 2, pages 1128–1132, 2003.

[6] L. de Lathauwer and A. de Baynast. Blind deconvolution of DS-CDMA signals by means of decomposition in rank-(1,L,L) terms. *IEEE Transactions on Signal Processing*, 56(4):1562–1571, 2008.

[7] D. Nion and N. D. Sidiropoulos. Adaptive algorithms to track the PARAFAC decomposition of a third-order tensor. *IEEE Transactions on Signal Processing*, 57(6):2299–2310, 2009.

[8] N. D. Sidiropoulos and R. S. Budampati. Khatri-rao space-time codes. *IEEE Transactions on Signal Processing*, 50(10):2396–2407, 2002.

[9] N. D. Sidiropoulos, G. B. Giannakis, and R. Bro. Blind parafac receivers for ds-cdma systems. *IEEE Transactions on Signal Processing*, 48(3):810–823, 2000.

[10] L. Zheng and D. N. C. Tse. Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels. *IEEE Transactions on Information Theory*, 49(5):1073–1096, 2003.