LOW COMPLEXITY FREQUENCY-RESPONSE MASKING FILTERS USING MODIFIED STRUCTURE BASED ON SERIAL MASKING

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ABSTRACT

This paper presents a technique to synthesize low-complexity high-selectivity arbitrary-bandwidth finite impulse response (FIR) digital filters using the frequency-response masking (FRM) technique. A serial masking scheme is used to perform the masking task in two stages lowering the complexity of the masking filters. The band-edge shaping filter is implemented as a prefilter-equalizer cascade lowering the complexity of the band-edge shaping filter. The design of the FRM filter is formulated as a nonlinear nonconvex semi-infinite programming optimization problem whereby the maximum frequency response magnitude error of the overall filter is minimized. A constraint transcription method and a smoothing technique are employed to transform the continuous inequality constraints into equality constraints. By using the concept of penalty function, the transformed constraints are incorporated into the cost function to form a new cost function. The nonlinear optimization problem with inequality constraints is approximated by a sequence of unconstraint optimization problems which can be solved efficiently by a limited-memory BFGS (L-BFGS) method [1]. An example taken from the literature demonstrates that this technique yields sharp filter having reduced magnitude error and total amount of nonzero coefficients.

1. INTRODUCTION

One of the computationally most efficient methods to reduce the complexity of sharp arbitrary-bandwidth FIR filters is the frequency-response masking (FRM) technique [2]. Over the years, the FRM technique has been extended to the synthesis of various types of filters such as half-band filters [3, 4], 2-D filters [5, 6], IIR filters [7–11], filter banks [12–15], decimators and interpolators [16, 17], and hilbert transformers [18, 19]. The basic structure of a filter synthesized using the FRM technique is shown in Fig. 1. The idea behind the FRM technique is to synthesize a sharp filter using several subfilters with gentle transition band. The design process consists of two parts. In the first part, sharp transition band is achieved by using a pair of periodic frequency response complementary band-edge shaping filters obtained by replacing each delay of a prototype filter by M delays. In the second part, the undesired periodic passbands of the band-edge shaping filters are removed by using two masking filters. The outputs of the masking filters are summed to form the overall filter's output. There have been many developments and improvements to the FRM technique in two areas, i.e. the filter structure and the optimization method.

In [20], the two masking filters themselves are synthesized by using the FRM technique. This results in significant saving in the complexity of the two masking filters. It alleviates the problem that arises from overly lengthy masking filters when M is large. This structure is best suited to the design of narrowband filters. In [21], the masking filters are realized as a cascade of a pair of subfilters and a common filter by exploiting the property that the frequency responses of the masking filters are similar except that at frequencies near the band edges. This realization reduces the complexity of the masking filters. In [22], a narrowband FRM [23] filter is used for the band-edge shaping filter. The narrowband FRM band-edge shaping filter is further improved by employing cyclotomic polynomial prefilters [24]. This improvement can only be achieved by restricting M to some integer such that the band-edge shaping filter becomes a narrowband filter. This forces one of the masking filters to be a sharp filter and creates an unbalanced pair of masking filters. This structure is not the optimum solution because the minimum complexity is achieved only when the transition widths of the two masking filters are equal according to [25] and [26]. Another approach for reducing the complexity is adopted in [27]. In [27], a prefilter-equalizer filter with a very low complexity prefilter is inserted into the band-edge shaping stage. The number of masking filters are reduced from two to one.

The application of nonlinear optimization technique further reduces the arithmetic complexity in the FRM filters by optimizing the coefficients of all subfilters simultaneously [28–36].

FRM filter structures based on serial masking schemes are proposed in [37]. The structures have lower complexity than the original FRM structure [2] since the transition width of one of the masking filters can be a multiple of that in the original FRM structure. In this paper, we shall make further improvements to these structures. First, the band-edge shaping filters are designed by adopting the prefilter-equalizer technique in [38] and [39] to reduce their complexity. Second, the overall filter is optimized jointly by solving a nonlinear nonconvex semi-infinite optimization problem.

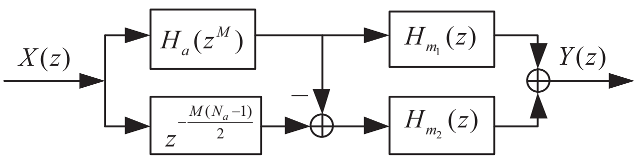

Figure 1: Basic Structure used in the FRM techniuqe

2. THE MODIFIED STRUCTURE

The original FRM structure uses a pair of masking filters to remove the unwanted periodic passbands of the band-edge shaping filter and those of its complement, respectively. In our newly proposed structure in this paper, the desired frequency response is achieved by a two-step masking approach as shown in Fig. 2. The first masking filter $H_{m1}(z^K)$ masks out the periodic passband of the complementary band-edge shaping filter whose transition band has the same frequency range as the overall filter. Meanwhile, it keeps the periodic passbands of the band-edge shaping filter within the passband of the overall filter. The second masking filter completes the masking task by removing all the unwanted passbands. Such an arrangement lowers the complexity of the first masking filter.

We shall classify our proposed structure into two categories, namely, Case A and Case B. In a Case A design, the band-edge shaping filter provides the desired transition band while, in a Case B design, the complement of the band-edge shaping filter provides the desired transition band. The proposed structure contains several alternatives based on the location of the band edges. We shall use Case A to illustrate this point in the following subsections.

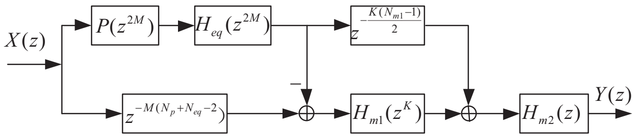

Figure 2: The propose structure for Case A with $\omega_p < \pi/2$.

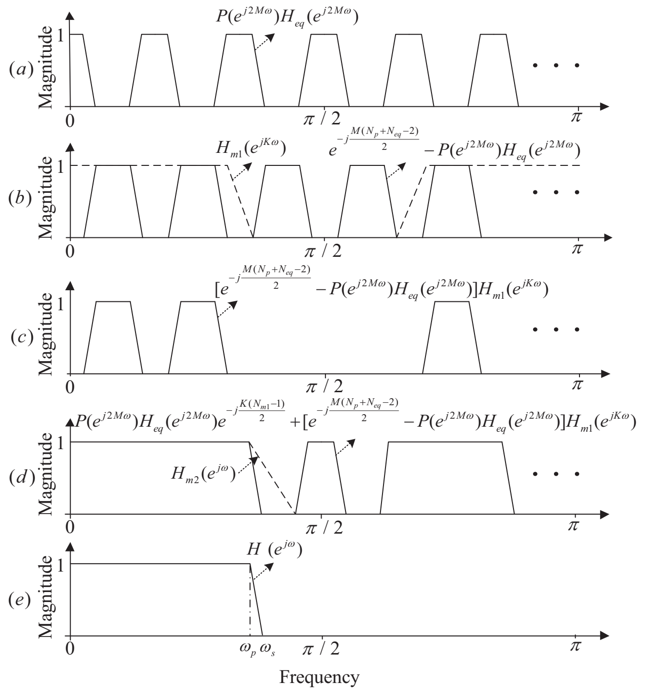

Figure 3: Frequency Responses of subfilters for Case A with $\omega_p < \pi/2$.

2.1 The Serial Masking Scheme

2.1.1 *For Designs with Passband Edge Less than $\pi/2$*

When the passband edge is less than $\pi/2$ in a Case A design, the synthesis structure is shown in Fig. 2. $P(z^2)H_{eq}(z^2)$ forms the band-edge shaping prototype filter designed using prefilter equalizer technique. N_p and N_{eq} are the length of prefilter $P(z)$ and equalizer $H_{eq}(z)$, respectively. The first and second masking filters are denoted by $H_{m1}(z^K)$ and $H_{m2}(z)$, respectively. N_{m1} and K are the length and impulse response interpolation factor of $H_{m1}(z)$, respectively. The frequency responses of the various subfilters for this structure are shown in Fig. 3. The z-transform transfer function of the overall filter can be written as

$$H(z) = [P(z^{2M})H_{eq}(z^{2M})z^{-\frac{K(N_{m1}-1)}{2}} + (z^{-M(N_p+N_{eq}-2)} \\ -P(z^{2M})H_{eq}(z^{2M}))H_{m1}(z^K)]H_{m2}(z). \quad (1)$$

2.1.2 *For Designs with Stopband Edge Greater than $\pi/2$*

When the stopband edge is greater than $\pi/2$, the desired filter has a wide passband. Such a filter may be designed by first designing its complementary highpass filter which has a passband edge greater than π/2. The desired lowpass filter is then obtained by subtracting from unit the gain of the highpass filter. For a Case A design, the proposed structure is shown in Fig. 4. The overall process is easily understood by inspecting the frequency response magnitudes of the subfilters, shown in Fig. 5. The z-transform transfer function of the overall filter can be written as

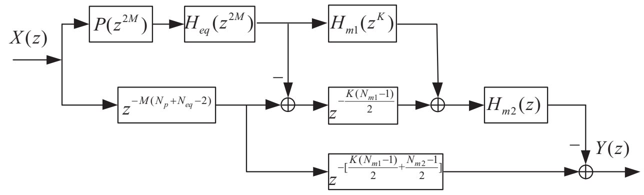

Figure 4: The propose structure for Case A with $\omega_s > \pi/2$.

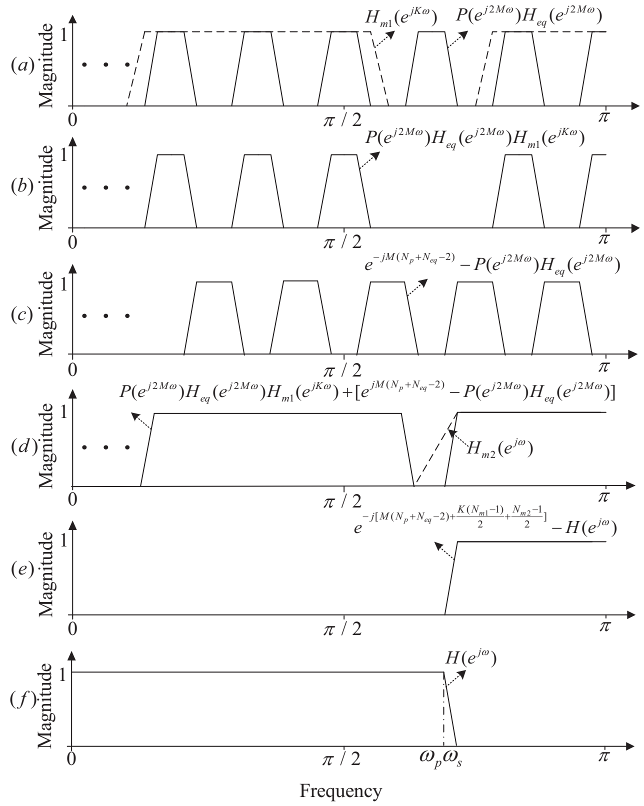

Figure 5: Frequency Responses of subfilters for Case A with $\omega_s > \pi/2$.

$$H(z) = z^{-[M(N_p+N_{eq}-2)+\frac{K(N_{m1}-1)}{2}+\frac{N_{m2}-1}{2}]} - [P(z^{2M})H_{eq}(z^{2M})H_{m1}(z^K) \\ +(z^{-M(N_p+N_{eq}-2)} - P(z^{2M})H_{eq}(z^{2M}))z^{-\frac{K(N_{m1}-1)}{2}}]H_{m2}(z). \quad (2)$$

2.2 Using Prefilter Equalizer as the Band-edge Shaping Filter

In general, the band-edge shaping filter contributes more than half of the total number of arithmetic operations [40]. As in [41], we decompose the design of the band-edge shaping filter into two parts, i,e., the realization of an efficient prefilter [42] and the design of the corresponding amplitude equalizer. Many options exist for the prefilter equalizer design. A good candidate is the one proposed in [39]. Consider a first-order, even-length filter $P_e(z)$ with z-transform transfer function given by

$$P_e(z) = 1 + z^{-1} \quad (3)$$

The frequency response of $P_e(z^2)$ is

$$P_e(e^{j2\omega}) = e^{-j\omega}\tilde{P}_e(2\omega) \quad (4)$$

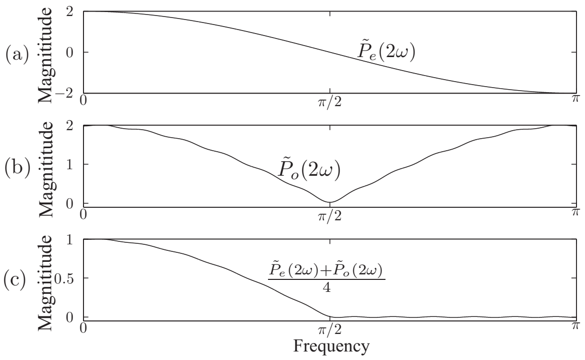

Figure 6: The zero-phase frequency response of (a) $P_e(z^2)$, (b) $P_o(z^2)$, and (c) $P(z^2)$.

where $\widetilde{P}_e(2\omega)$ is a real function. The frequency response $\tilde{P}_e(2\omega)$ is positive in the interval $[0,\pi/2]$ and negative in the interval $[\pi/2,\pi]$, as shown in Fig 6a. Next, an even-symmetric linear phase FIR digital filter $P_o(z)$ is designed. The frequency response of $P_o(z^2)$ is given by

$$P_o(e^{j2\omega}) = e^{-j(N_o-1)\omega}\tilde{P}_o(2\omega) \tag{5}$$

where $\widetilde{P}_o(2\omega)$ is real. $P_o(z)$ is designed such that $\tilde{P}_o(2\omega)$ approximates $\tilde{P}_e(2\omega)$ in the interval $[0,\pi/2]$ and approximates $-\widetilde{P}_e(2\omega)$ in the interval $[\pi/2,\pi]$, as shown in Fig 6b. By connecting $\widetilde{P}_e(2\omega)$ and $\widetilde{P}_o(2\omega)$ in parallel, a stopband can be created in the interval $[\pi/2,\pi]$, as shown in Fig 6c. Note that each delay element in $P_e(z)$ and $P_o(z)$ should be replaced by two delay elements when connecting them in parallel to avoid a half-sample delay. Let the length of $P_o(z)$ be N_o. The z-transform transfer function of the prefilter $P(z^2)$ is given by

$$P(z^2) = \frac{P_e(z^2)z^{-(N_o-2)} + P_o(z^2)}{4}. \tag{6}$$

As the prefilter $P(z^2)$ has a passband similar to that of $(1+z^{-2})/2$, it is always possible to find an equalizer $H_{eq}(z^2)$ such that the prefilter-equalizer pair meets the specifications of any lowpass filters with stopband edge less than or equal to $\pi/2$. Let the z-transform transfer function of the prefilter-equalizer pair be $H_{pe}(z^2)$. $H_{pe}(z^2)$ is given by

$$H_{pe}(z^2) = P(z^2)H_{eq}(z^2). \tag{7}$$

For a lowpass filter with passband edge greater than or equal to $\pi/2$, $P_e(z)$ and $P_o(z)$ can be connected in a different manner to form a new filter $H_{pew}(z)$ as follows:

$$H_{pew}(z^2) = z^{-(N_{pe}-1)} + \frac{[P_e(z^2)z^{-(N_o-2)} - P_o(z^2)]}{2}H_{eq}(z^2) \tag{8}$$

where N_{pe} is the sum of the filter lengths of $P_o(z)$ and $H_{eq}(z)$.

3. PROBLEM FORMULATION

In order to minimize the maximum frequency response magnitude error, the design of the FRM filter can be formulated as a nonlinear semi-infinite programming optimization problem as follows:

$$\text{minimize: } \delta^2 \tag{9a}$$
$$\text{subject to: } W^2(\omega)|H(\boldsymbol{x},w) - M_d(\omega)|^2 \le \delta^2,\ \forall\omega \in [0,\omega_p] \tag{9b}$$
$$W^2(\omega)|H(\boldsymbol{x},w) - M_d(\omega)|^2 \le \delta^2,\ \forall\omega \in [\omega_s,\pi] \tag{9c}$$

where $H(\boldsymbol{x},w)$ is the zero-phase frequency response of the overall filter $H(z)$, $W(\omega)$ is the frequency weighting function, and $M_d(\omega)$ is the desired magnitude response of the system. In the passband, $W(\omega)$ and $M_d(\omega)$ are equal to one. In the stopband, $W(\omega)$ is equal to δ_p/δ_s (here δ_p and δ_s are the permitted passband and stopband ripples, respectively) and $M_d(\omega)$ is equal to zero. The parameter vector to be determined is

$$\boldsymbol{u} = [\delta,\ \boldsymbol{x}^T]^T \tag{10a}$$

where

$$\boldsymbol{x} = [a_o^T, a_{eq}^T, a_{m1}^T, a_{m2}^T]^T. \tag{10b}$$

Let s denote the subscript $o, eq, m1$ or $m2$

$$\boldsymbol{a}_s = [h_s(0), h_s(1), \ldots, h_s(L_s)]^T \tag{10c}$$

where $L_s = \frac{N_s-1}{2}$ if Ns is odd and $L_s = \frac{N_s}{2}$ if Ns is even. Let this optimization problem be referred to as Problem (P).

4. SOLUTION STRATEGY

Problem (P) is nonlinear and nonconvex. It is subject to continuous inequality constraints. The continuous inequality constraints (9b) and (9c) are equivalent to the following inequality constraints

$$W^2(\omega)|H(\boldsymbol{x},w) - M_d(\omega)|^2 - \delta^2 \le 0, \forall\omega \in [0,\omega_p] \tag{11a}$$

and

$$W^2(\omega)|H(\boldsymbol{x},w) - M_d(\omega)|^2 - \delta^2 \le 0, \forall\omega \in [\omega_s,\pi]. \tag{11b}$$

Let the left hand side functions of the inequality constraints (11a) and (11b) be $G_1(\boldsymbol{u},\omega)$ and $G_2(\boldsymbol{u},\omega)$, that is

$$G_1(\boldsymbol{u},\omega) = W^2(\omega)|H(\boldsymbol{x},w) - M_d(\omega)|^2 - \delta^2 \tag{12a}$$

and

$$G_2(\boldsymbol{u},\omega) = W^2(\omega)|H(\boldsymbol{x},w) - M_d(\omega)|^2 - \delta^2. \tag{12b}$$

Since $G_i(\boldsymbol{u},\omega), i = 1,2$, is continuous in $\boldsymbol{u}$ and ω, $\max\{G_i(\boldsymbol{u},\omega),\ 0\}$ is a continuous function of ω for each $\boldsymbol{u}$. Thus, the continuous inequality constraints (11a) and (11b) are equivalent to

$$\tilde{G}_i(\boldsymbol{u}) = 0,\ i = 1,2 \tag{13a}$$

where

$$\tilde{G}_1(\boldsymbol{u}) = \int_0^{\omega_p} \max\{G_1(\boldsymbol{u},\omega),\ 0\}d\omega \tag{13b}$$

and

$$\tilde{G}_2(\boldsymbol{u}) = \int_{\omega_s}^{\pi} \max\{G_2(\boldsymbol{u},\omega),\ 0\}d\omega. \tag{13c}$$

Since the function $\max\{a,0\}$ is not differentiable at $a = 0$, the function $\max\{G_i(\boldsymbol{u},\omega)0\}$ is, in general, nonsmooth. Consequently, gradient-based optimization routines would have difficulties in handling this type of equality constraint. Thus, we construct the following smoothing approximation using the smoothing technique in [43]:

Define

$$\hat{G}_{i,\varepsilon}(\boldsymbol{u},\omega) = \begin{cases} 0, & \text{if } G_i(\boldsymbol{u},\omega) < -\varepsilon \\ \frac{(G_i(\boldsymbol{u},\omega)+\varepsilon)^2}{4\varepsilon}, & \text{if } -\varepsilon \le G_i(\boldsymbol{u},\omega) \le \varepsilon \\ G_i(\boldsymbol{u},\omega), & \text{if } G_i(\boldsymbol{u},\omega) > \varepsilon \end{cases} \tag{14}$$

where ε is a small positive number. The functions $\max\{G_1(\boldsymbol{u},\omega),0\}$ and $\max\{G_2(\boldsymbol{u},\omega),0\}$ in (13b) and (13c) are now approximated by the continuous differentiable functions $\hat{G}_{i,\varepsilon}(\boldsymbol{u},\omega)$ with $i = 1,2$, respectively. Equations (13b) and (13c) are replaced by (15).

$$\tilde{G}_{i,\varepsilon}(\boldsymbol{u}) = \int_{\Omega_i} \hat{G}_{i,\varepsilon}(\boldsymbol{u},\omega)d\omega,\ i = 1,2 \tag{15}$$

where $\Omega_1 = [0,\ \omega_p]$ and $\Omega_2 = [\omega_s,\ \pi]$. We shall use the concept of penalty function to append the smooth function in (15) to the cost

Table 1: Design results and comparison

	D1 [2]	D2 [31]	D3 [37]	D4
Maximum passband ripple (dB)	0.1	0.085	0.1	0.0821
Minimum stopband attenuation (dB)	40	43.3	40	44.6
Total nonzero coefficients	91	91	66	57

function in (9a), forming a sequence of approximate optimization problems as follows:

$$\min_{\boldsymbol{u}} \; f_{\varepsilon,\gamma}(\boldsymbol{u}) \tag{16a}$$

where

$$f_{\varepsilon,\gamma}(\boldsymbol{u}) = \delta^2 + \gamma \sum_{i=1}^{2} \hat{G}_{i,\varepsilon}(\boldsymbol{u}) \tag{16b}$$

and γ is a penalty constant. Let the problem be referred to as Problem $(P_{\varepsilon,\gamma})$.

Remark 4.1: we have the following properties [43]: For every $\varepsilon > 0$, there exists a $\gamma(\varepsilon) > 0$ such that for any $\gamma > \gamma(\varepsilon)$, a solution of Problem $(P_{\varepsilon,\gamma})$ will satisfy the continuous inequality constraints of Problem (P). In addition, suppose that $(\delta^2)^*$ and $\boldsymbol{u}^*_{\varepsilon,\gamma}$ are respectively, the optimal solutions of Problem (P) and Problem $(P_{\varepsilon,\gamma})$ with $\gamma > \gamma(\varepsilon)$. Then $f_{\varepsilon,\gamma}(\boldsymbol{u}^*_{\varepsilon,\gamma})$ approaches $(\delta^2)^*$ as $\varepsilon \to 0$.

5. DESIGN PROCEDURE

We propose the following optimization algorithm using the above solution strategy to design the FRM filter.

Step 1: For a given set of ω_p, ω_s, δ_p and δ_s, an initial design for each subfilter can be obtained by using a standard method, such as linear programming [44], or Remez exchange algorithm [45]. The coefficients of the subfilters are used to form the initial vector of $\boldsymbol{x}$.

Step 2: Set $\varepsilon = 0.1$, $\gamma = 10$, $\varepsilon_0 = 10^{-8}$, $\gamma_0 = 10^8$.

Step 3: Solve Problem $(P_{\varepsilon,\gamma})$ by using a quasi-Newton Method. Let the solution obtained be $\boldsymbol{u}^*$.

Step 4: If $\boldsymbol{u}^*$ satisfies the continuous inequality constraints in (9), go to step 5. Otherwise, replace γ by 10γ. If $\gamma > \gamma_0$, stop. Otherwise, return to step 3.

Step 5: Replace ε by $\varepsilon/10$. If $\varepsilon > \varepsilon_0$, go to step 3; otherwise, stop.

In *Step 3*, we use a limited-memory BFGS (L-BFGS) algorithm with line search satisfying strong Wolfe condition [46] to solve Problem $(P_{\varepsilon,\gamma})$. The integrals in (16b) were evaluated numerically using adaptive Gauss-Kronrod quadrature rules [47]. The integrals in the gradient of (16b) were evaluated numerically using adaptive recursive Simpson's rule [47].

Remark 4.2: In *Step 4* and *Step 5* of the design procedure, it follows from Remark 4.1 that the increment of γ for each $\varepsilon > 0$ needs to be carried out only for a finite number of times.

6. DESIGN EXAMPLE AND COMPARISON WITH OTHER FRM FILTER DESIGN APPROACHES

We shall choose the example in [37] to illustrate our technique. The specifications were: $\omega_p = 0.2\pi$, $\omega_s = 0.205\pi$, $\delta_p = 0.0116$, and $\delta_s = 0.01$. The design results of the proposed technique and the best results from literature are listed in Table 1 for comparison. D1 is the design using the single-stage FRM structure in [2]. D2 is the design of basic FRM filter using second order cone technique in [31]. D3 is the design using the method in [37]. D4 is the design using the proposed technique. It can be seen from Table 1 that the proposed technique produces a design superior in performance with fewer number of nonzero coefficients when compared to the other three techniques. The frequency response magnitudes of the subfilters as well as the overall FRM filter and its passband ripple are shown in Fig. 7.

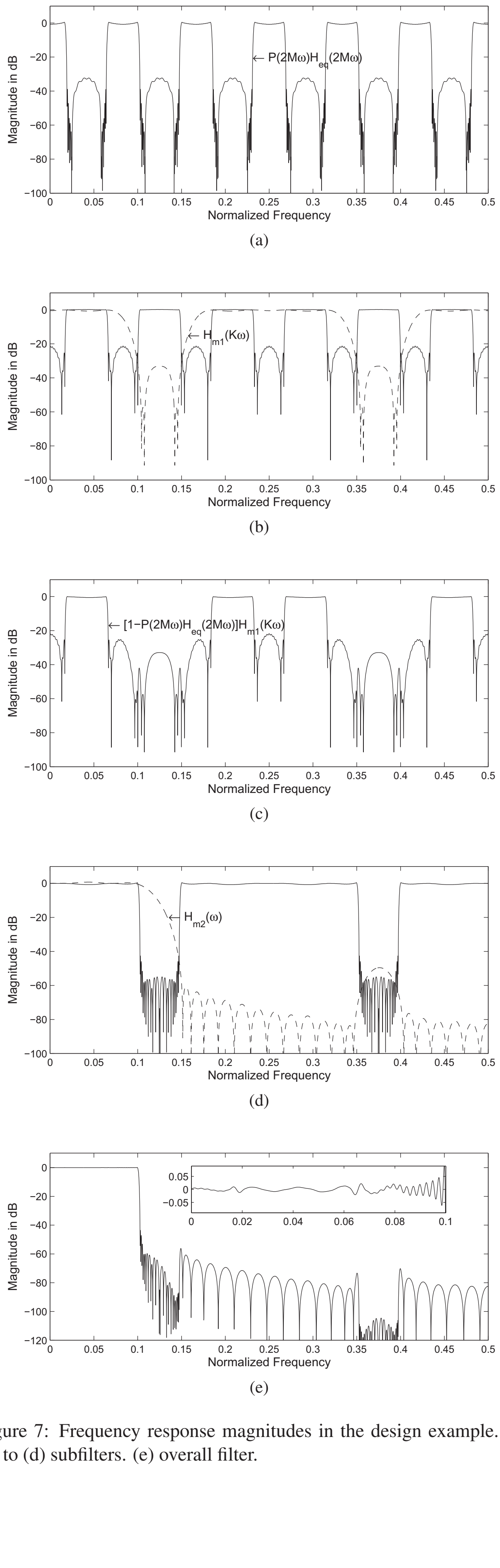

Figure 7: Frequency response magnitudes in the design example. (a) to (d) subfilters. (e) overall filter.

7. CONCLUSION

In this paper, FRM based technique is proposed for the design of high-selectivity low-complexity arbitrary-bandwidth FIR filter. The proposed filter structure reduces the complexity of the band-edge shaping filter and one of the mask filters. A nonlinear nonconvex semi-infinite optimization problem is formulated to minimize the maximum frequency response magnitude error of the overall filter. The optimization problem with inequality constraints is approximated by a sequence of unconstrained optimization problem which can be solved efficiently using a L-BFGS method. The proposed technique is superior to existing techniques as illustrated by the example in Section 6.

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