ACHIEVABLE SUM-RATES IN THE TWO-USER GAUSSIAN MULTIPLE-ACCESS CHANNEL WITH A MIMO-AF-RELAY

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ABSTRACT

We consider a two-user multiple-access channel (MAC), where an amplify-and-forward (AF) full-duplex relay is used to increase the achievable sum rate. While the transmitters and the receiver are assumed to have a single antenna, the relay is equipped with multiple antennas. Thus, spatial processing can be applied at the relay, which is subject to optimization for achieving higher data rates. This optimization problem is non-convex and hard to solve in general. However, for the two special cases in which either the direct links between transmitter and receiver are not present or the relay operates in another frequency band, we are able to derive upper and lower bounds that are performing reasonably well for a large set of parameters. Using the insights from the special cases, we also obtain upper and lower bounds for the general case.

1. INTRODUCTION

In today's wireless communication systems, the demand for higher data rates and wide-range coverage is steadily growing. To meet this requirements, a high density of base stations is necessary, which entails high costs for installation and maintenance. Another possibility to increase coverage and range is the use of relay nodes, which have much lower costs. Relay channels were considered in [1] first, and have drawn more and more research attention in the last decades.

Depending on how the signals are processed at the relay, we distinguish between different types of relaying schemes. The most common ones are amplify-and-forward (AF, also called non-regenerative relaying) and decode-and-forward (DF, also called regenerative relaying). While in AF, the relay simply amplifies the received signals subject to a power constraint, a complete decoding and re-encoding of the signal is necessary when using DF which yields higher costs and larger delays. For these reasons, we will restrict ourselves to AF relaying schemes in this paper.

In addition to relays, the deployment of nodes with multiple antennas helps to further increase the possible data rates. The combination of these two paradigms, especially in multiuser systems, is an enormous challenge and has been considered in numerous publications, such as [2] and the references therein. For instance, [3] introduces different power allocation algorithms for multiple-input multiple-output (MIMO) systems with multiple users and an AF-relay for both up- and downlink. However, this work assumes there are no direct connections between transmitter(s) and receiver(s) such that all communication takes place over the relay. In [4] the direct links are considered but only for a single-user system. Moreover, in contrast to [3] and our work, [4] investigates a system with a half-duplex relay.

In this work, we are going to consider a two-user multiple-access channel (MAC) with a MIMO full-duplex AF relay. Contrary to [3], we also consider scenarios with direct links between transmitters and receiver and show that up to a certain available relay power, their influence can not be neglected. In detail, we consider three different scenarios. For the first scenario, we assume that no direct links between the users and the receiver are present. In the second scenario, direct links are present, but the relay-to-receiver communication takes place in a different frequency band. The last (and most involved) scenario uses both direct links and communication of all stations in the same band.

This work is structured as follows: In section 2 we introduce the notation and the underlying channel model. The channel model is given in its most general case first, and then modified to the specific cases mentioned above. Achievable rates and upper bounds for those scenarios are derived in section 3. Subsequently those rates are evaluated and plotted (section 4). Section 5 finally concludes the paper.

2. PROBLEM FORMULATION

2.1 Notation

We denote all column vectors in bold lower case and matrices in bold upper case. The trace and the determinant of a matrix $\mathbf{A}$ are identified by $\mathrm{tr}(\mathbf{A})$ and $|\mathbf{A}|$, respectively. We use $\|\mathbf{x}\|$ to denote the euclidean norm of a vector $\mathbf{x}$ and $\mathbf{I}$ to describe the identity matrix. Furthermore, $\lambda_{\max}(\mathbf{A})$ and $\mathbf{v}_{\max}(\mathbf{A})$ indicate the largest eigenvalue of a matrix $\mathbf{A}$ and its corresponding eigenvector. The signals that the relay receives and transmits in time slot j are named $\mathbf{y}_r(j)$ and $\mathbf{x}_r(j)$, respectively. However, we omit the time-slot whenever it is superfluous or can be grasped from the context.

2.2 Basic channel model

In this paper, we will consider three relaying scenarios:

(I) Relaying without direct links
(II) Out-of-band relaying with direct links
(III) In-band relaying with direct links

For all scenarios, we assume that the channels in between the nodes are flat fading channels and that the nodes know them perfectly. We consider the achievable sum of the rates of user 1 and user 2 as well as its upper bound for the above scenarios. In this subsection, we will describe what is common for all scenarios, while in the next subsection we describe their differences.

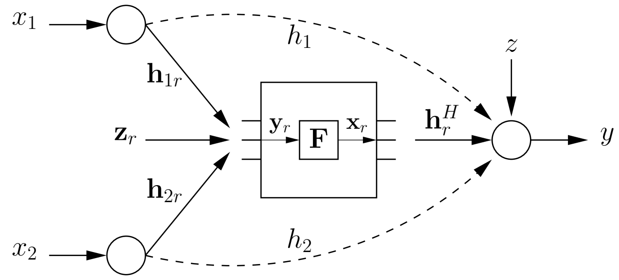

Figure 1: Multiple access channel with a MIMO relay

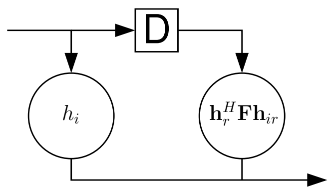

Figure 2: ISI channel of user i in scenario III

We consider a multiple-access relay channel (MARC) with two transmitters, one receiver and an amplify-and-forward (AF) full-duplex relay as shown in Figure 1. It is assumed that both transmitters and the receiver have only one antenna, while the relay has $M_r \geq 1$ antennas. This is a valid assumption, given the users are mobile stations with stronger constraints on cost and size than the relay.

The channel gains between the transmitters and the receiver are given by h_1 and h_2, while the vectors $\mathbf{h}_{1r}$ and $\mathbf{h}_{2r}$ denote the channel gains from transmitter one and two to the relay, respectively. The vector channel gain from the relay to the destination is described by ${\mathbf{h}_r}^H$.

Thus, the relay's received signal y_r can be written as

$$\mathbf{y}_r = \mathbf{h}_{1r} x_1 + \mathbf{h}_{2r} x_2 + \mathbf{z}_r,$$

where z_r is a white Gaussian noise vector of unit variance. The received symbol is amplified by the matrix $\mathbf{F} = \gamma \widehat{\mathbf{F}}$ and sent in the next time slot. Thus, the transmit vector of the relay in time slot j is $\mathbf{x}_r(j) = \mathbf{F}\mathbf{y}_r(j-1)$. Its average power is

$$\mathbb{E}(\mathrm{tr}(\mathbf{x}_r \mathbf{x}_r^H)) = \gamma^2 \widehat{\mathbf{F}}(\mathbf{h}_{1r} P_1 \mathbf{h}_{1r}^H + \mathbf{h}_{2r} P_2 \mathbf{h}_{2r}^H + I)\widehat{\mathbf{F}}^H, \qquad (1)$$

where $\gamma > 0$ is a factor, which ensures that the average transmit power of the relay is limited by

$$\mathbb{E}(\mathrm{tr}(\mathbf{x}_r \mathbf{x}_r^H)) \leq P_r. \qquad (2)$$

Moreover, the maximum power of the transmitted signals is

$$\mathbb{E}(|x_i|^2) \leq P_i \quad (i = 1, 2)$$

and the white Gaussian noise z at the receiver is assumed to have unit variance as well.

Finally, in scenarios I and II, we can write the received vector $\mathbf{y}$ as

$$\mathbf{y} = \widetilde{\mathbf{h}}_1 x_1 + \widetilde{\mathbf{h}}_2 x_2 + \widetilde{\mathbf{z}}, \qquad (3)$$

i.e., as the output of an equivalent MAC (without relay). The differences among the two scenarios are the equivalent channel vectors $\widetilde{\mathbf{h}}_1$, $\widetilde{\mathbf{h}}_2$, and the equivalent noise covariance matrix $\widetilde{\mathbf{Z}} = \mathbb{E}\left(\widetilde{\mathbf{z}}\widetilde{\mathbf{z}}^H\right)$. Note, that we generalized $\mathbf{y}$ as a vector because this is required to model the out-of-band relaying scheme. The achievable sum rates for scenarios I and II can be obtained using successive interference cancellation and are given by

$$R = \log_2 |\widetilde{\mathbf{Z}} + \widetilde{\mathbf{h}}_1 P_1 \widetilde{\mathbf{h}}_1^H + \widetilde{\mathbf{h}}_2 P_2 \widetilde{\mathbf{h}}_2^H| - \log_2 |\widetilde{\mathbf{Z}}|. \qquad (4)$$

For scenario III we can also model the MARC as an equivalent MAC. However, this MAC is now frequency selective, which requires more involved schemes for finding the achievable sum rates. Thus, (4) is no longer valid. A formula for calculating the achievable rate in scenario III is given in the next section.

2.3 Relaying scenarios

2.3.1 *Relaying without direct links*

For the first scenario, we have $h_1 = h_2 = 0$. Thus, all communication between the transmitters and receiver goes through the relay and we can write the channel and noise matrices of the equivalent MAC as

$$\widetilde{\mathbf{h}}_i = {\mathbf{h}_r}^H \mathbf{F} \mathbf{h}_{ir} \quad (i = 1, 2)$$
$$\widetilde{\mathbf{Z}} = {\mathbf{h}_r}^H \mathbf{F}\mathbf{F}^H \mathbf{h}_r + 1$$

2.3.2 *Out-of-band relaying*

The second scenario that we consider has direct links between transmitters and receivers. But, in contrast to the first scenario, the relay transmits in another frequency band than the transmitters. This can be modeled by extending the receive symbol y to a vector $\mathbf{y} = [y_i \ \ y_o]^T$, where y_i is the in-band signal from the transmitters and y_o is the out-of-band signal from the relay. Due to the separation, the time shift between the transmitted signals and their amplified versions due to the processing at the relay is irrelevant, as they can be stored and jointly processed. Thus, we can write the parameters of the equivalent MAC as

$$\widetilde{\mathbf{h}}_i = [{\mathbf{h}_r}^H \mathbf{F} \mathbf{h}_{ir} \ \ h_i]^T \quad (i = 1, 2) \qquad (5)$$

$$\widetilde{\mathbf{Z}} = \begin{bmatrix} {\mathbf{h}_r}^H \mathbf{F}\mathbf{F}^H \mathbf{h}_r + 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (6)$$

2.3.3 *In-band relaying with direct links*

Finally, in the third scenario, we assume that the direct links are present and all stations transmit in the same frequency band. As the amplified signals from the relay arrive at the destination with a delay of one time slot, the channel can be seen as a MAC with inter-symbol interference (ISI), which is discussed in [5]. In a time-discrete model, the channel of user i (without noise) can be described by the linear filter in Figure 2, where the element "D" indicates the delay of one time slot.

Thus, in contrast to the previous scenarios, both users' channels are now frequency selective and no longer flat-fading channels. Their spectrum can be obtained by the Fourier transform of the filter's impulse response. Its squared magnitude normalized by the noise-power ${\mathbf{h}_r}^H\mathbf{F}\mathbf{F}^H\mathbf{h}_r+1$ at the receiver is given by

$$T_i(w)=\frac{|h_i+{\mathbf{h}_r}^H\mathbf{F}\mathbf{h}_{ir}\cdot e^{jw}|^2}{{\mathbf{h}_r}^H\mathbf{F}\mathbf{F}^H\mathbf{h}_r+1}\quad(i=1,2),$$

where w denotes the frequency. As (3) does not subsume frequency selective channels, (4) can not be used to calculate the achievable sum rate for scenario III. Instead, the achievable sum rate is given by [5] [1]

$$R\leq\frac{1}{2\pi}\int_0^{2\pi}\log_2(1+S_1(w)T_1(w)+S_1(w)T_1(w))dw,\quad(7)$$

where $S_i(w)$ $(i=1,2)$ are the power spectral densities (PSDs) of the transmitted signals. The PSDs that maximize (7) can be found by multi-user water-filling [5].

3. ACHIEVABLE SCHEMES AND UPPER BOUNDS

For all the scenarios introduced in the preceding section, we assume the powers P_1 and P_2 at the transmitters are fixed. As the transmitters have only one antenna, the parameter that needs to be optimized is the relaying matrix $\mathbf{F}$. Unfortunately, the problem of finding the matrix $\mathbf{F}$ that maximizes the sum-rate of the MARC under a given power constraint P_r at the relay is non-convex and thus the global optimum is not straightforward to achieve.

To simplify this problem, we will first derive two upper-bounds on the sum-rate for the first two scenarios. They will be denoted by $\overline{R}_j^i$, where i is the number of the scenario (given in roman numbers) and j is the number of the bound. The first bound is obtained by assuming a noise-free relay, i.e., $\mathbf{z}_r=0$. For the second bound, we ignore the power-constraint at the relay, i.e., we allow $P_r\to\infty$. It turns out that these bounds can be maximized by choosing $\mathbf{F}$ either according to an eigenvector-based scheme (first bound) or according to the ANOMAX scheme [6] (second bound), which is briefly described in the next subsection.

In a second step, we will use the obtained matrices in the actual MARC with non-zero noise and finite transmit power at the relay to calculate achievable rates. Therefore, we set $\widehat{\mathbf{F}}$ to the matrices obtained from the upper bounds and choose γ such that (2) is fulfilled. Of course, these matrices are not generally optimal without the relaxations, but we will see in section 4, that they yield rates that achieve the upper bounds asymptotically.

For the third scenario, the procedure is a bit different as only one upper bound $\overline{R}^{\mathrm{III}}$ is derived. Moreover, this bound does not suggest any relaying matrix $\mathbf{F}$. Instead, we simply use the amplifying matrices that we obtained from the other two scenarios to get an achievable sum rate.

3.1 Algebraic norm-maximizing (ANOMAX) scheme

Originally, the ANOMAX scheme was used for the two-way relaying channel, but it can also be used in our case. The scheme finds a matrix that maximizes a weighted sum of Frobenius norms of two matrix expressions given as follows:

$$\mathbf{F}_\beta=\arg\max_\beta J_\beta(\mathbf{F})\text{ s.t. }\|\mathbf{F}\|_F=1,\text{ where}$$
$$J_\beta(\mathbf{F})=\beta^2\cdot\|\mathbf{AFB}\|_F^2+(1-\beta)^2\cdot\|\mathbf{CFD}\|_F^2,$$

[1]Note that, as we use complex channel inputs, the range of the integral changes from $[0,\pi]$ to $[0,2\pi]$

where $\beta\in[0,1]$ is a weighting factor and $||\mathbf{A}||_F$ denotes the Frobenius norm of a matrix $\mathbf{A}$. To apply ANOMAX in the MARC we set $\mathbf{A}=\mathbf{C}={\mathbf{h}_r}^H$, $\mathbf{B}=\mathbf{h}_{1r}$, $\mathbf{C}=\mathbf{h}_{2r}$, and β according to the scenario, where we want to use the ANOMAX scheme. Note that the values ${\mathbf{h}_r}^H\mathbf{F}\mathbf{h}_{ir}$ $(i=1,2)$ are scalars here, such that their Frobenius norm is simply their absolute value. For further details about the scheme, see [6].

3.2 Relaying without direct links

As announced, we derive two upper bounds for scenario I. The first one is obtained by assuming a noise-free relay, while for the second bound we ignore the power constraint at the relay.

3.2.1 No noise at the relay

In the case, where the relay is noise-free ($\mathbf{z}_r=0$), the sum rate is given by

$$\overline{R}_1^{\mathrm{I}}=\log_2\left(1+P_1\cdot|{\mathbf{h}_r}^H\mathbf{F}\mathbf{h}_{1r}|^2+P_2\cdot|{\mathbf{h}_r}^H\mathbf{F}\mathbf{h}_{2r}|^2\right).$$

Interestingly, it turns out that this expression can be maximized by the ANOMAX scheme and setting $\beta=(P_2/P_1+1)^{-1/2}$, which delivers the optimal $\widehat{\mathbf{F}}$ for this upper bound.

3.2.2 Infinite relay power

Ignoring the power constraint (2) at the relay is equivalent to letting $\gamma\to\infty$. By doing this, we can bound the sum-rate as

$$\begin{aligned}R&=\log_2\left(1+\frac{P_1\cdot|{\mathbf{h}_r}^H\mathbf{F}\mathbf{h}_{1r}|^2+P_2\cdot|{\mathbf{h}_r}^H\mathbf{F}\mathbf{h}_{2r}|^2}{{\mathbf{h}_r}^H\mathbf{F}\mathbf{F}^H\mathbf{h}_r+1}\right)\\&\leq\log_2\left(1+\frac{{\mathbf{h}_r}^H\mathbf{F}(\mathbf{h}_{1r}\mathbf{h}_{1r}^H\cdot P_1+\mathbf{h}_{2r}\mathbf{h}_{2r}^H\cdot P_2)\mathbf{F}^H\mathbf{h}_r}{{\mathbf{h}_r}^H\mathbf{F}\mathbf{F}^H\mathbf{h}_r}\right)\\&=\log_2\left(1+\frac{{\mathbf{h}_r}^H\mathbf{F}\mathbf{G}\mathbf{F}^H\mathbf{h}_r}{{\mathbf{h}_r}^H\mathbf{F}\mathbf{F}^H\mathbf{h}_r}\right)\leq\log_2\left(1+\lambda_{\max}(\mathbf{G})\right)\\&\triangleq\overline{R}_2^{\mathrm{I}},\end{aligned}$$

where we introduced the new matrix

$$\mathbf{G}\triangleq\mathbf{h}_{1r}\mathbf{h}_{1r}^H\cdot P_1+\mathbf{h}_{2r}\mathbf{h}_{2r}^H\cdot P_2.$$

The last inequality is obtained from the upper bound of the Rayleigh quotient [7, p. 176f.]. It can be achieved by choosing $\widehat{\mathbf{F}}$ as the dyadic product of $\mathbf{h}_r$ and the eigenvector corresponding to the largest eigenvalue of $\mathbf{G}$:

$$\widehat{\mathbf{F}}=\mathbf{h}_r\mathbf{v}_{\max}(\mathbf{G})^H.$$

Choosing $\widehat{\mathbf{F}}$ as a dyadic product of $\mathbf{h}_r$ and the eigenvector of a matrix will be referred to as "eigenvector scheme" in the remainder of this paper.

3.3 Out-of-band relaying

If the link from the relay to the destination is in another band, we obtain

$$R_{\Sigma,II}=\log_2(1+S-T+P_1|h_1|^2+P_2|h_2|^2)$$

by combining (4), (5), (6), and some manipulations, where

$$S = |G_1|^2\widetilde{P}_1 + |G_2|^2\widetilde{P}_2$$
$$T = 2P_1P_2 \cdot \mathrm{Re}\{h_1h_2^H G_1^H G_2\}$$
$$G_i = \frac{\mathbf{h}_r{}^H\mathbf{F}\mathbf{h}_{ir}}{\sqrt{\mathbf{h}_r{}^H\mathbf{F}\mathbf{F}^H\mathbf{h}_r + 1}}$$
$$\widetilde{P}_i = (P_i + P_1P_2|h_j|^2) \quad (i,j = 1,2;\ j \neq i),$$

and $\mathrm{Re}\{.\}$ denotes the real part of an expression. As before, we will derive two upper bounds by the same relaxations.

3.3.1 No noise at the relay

If the relay is noise-free, G_i reduces to $G_i = \mathbf{h}_r{}^H\mathbf{F}\mathbf{h}_{ir}$, i.e., S can be maximized by the ANOMAX scheme with $\beta = (\widetilde{P}_1/\widetilde{P}_2 + 1)^{-1/2}$. For T, a lower bound $T^* \leq T$ is given by

$$T^* = \begin{cases} 0 & T \geq 0 \\ 2P_1P_2\mathrm{Re}\{\gamma^2\|\mathbf{h}_r\|^4 h_1h_2^H\mathbf{h}_{1r}^H\mathbf{h}_{2r}\} & T < 0, \end{cases}$$

where the expression for the case $T < 0$ is obtained by setting $\widehat{\mathbf{F}} = \mathbf{h}_r\mathbf{v}_{\max}^H(\mathbf{h}_{2r}\mathbf{h}_{1r}^H)$ and selecting γ such that (1) is fulfilled.

3.3.2 Infinite relay power

For the other bound, we will first rewrite T as

$$T = P_1P_2\frac{\mathbf{h}_r{}^H\mathbf{F}\mathbf{X}\mathbf{F}^H\mathbf{h}_r}{\mathbf{h}_r{}^H\mathbf{F}\mathbf{F}^H\mathbf{h}_r + 1},$$

where $X = h_1h_2^H\mathbf{h}_{2r}\mathbf{h}_{1r}^H + h_1^Hh_2\mathbf{h}_{1r}\mathbf{h}_{2r}^H$. Hence, we obtain

$$S - T \rightarrow \frac{\mathbf{h}_r{}^H\mathbf{F}(\mathbf{h}_{1r}\mathbf{h}_{1r}^H\widetilde{P}_1 + \mathbf{h}_{2r}\mathbf{h}_{2r}^H\widetilde{P}_2 - P_1P_2X)\mathbf{F}^H\mathbf{h}_r}{\mathbf{h}_r{}^H\mathbf{F}\mathbf{F}^H\mathbf{h}_r}$$

if we assume infinite power at the relay. This expression is again a Rayleigh coefficient and can be maximized by the following eigenvector scheme:

$$\widehat{\mathbf{F}} = \mathbf{h}_r\mathbf{v}_{\max}(\mathbf{h}_{1r}\mathbf{h}_{1r}^H\widetilde{P}_1 + \mathbf{h}_{2r}\mathbf{h}_{2r}^H\widetilde{P}_2 - P_1P_2X).$$

Finally this upper bound can be explicitly formulated as

$$\overline{R}_2^{\mathrm{II}} = \log_2(1 + \lambda_{\max}(\mathbf{h}_{1r}\mathbf{h}_{1r}^H\widetilde{P}_1 + \mathbf{h}_{2r}\mathbf{h}_{2r}^H\widetilde{P}_2 - P_1P_2X) + P_1|h_1|^2 + P_2|h_2|^2).$$

3.4 In-band relaying with direct links

As already stated in subsection 2.3.3, the setup for scenario III is a MAC with ISI. Obtaining a matrix $\mathbf{F}$, which maximizes the expression in (7) directly is rather difficult. Even for maximizing $T_1(w)$ and $T_2(w)$ separately, it is hard to find optimal matrices $\mathbf{F}$. This difficulty is revealed by the results of [5], where it is stated that maximizing the channel gains of the second tap of the ISI-channels (in our case $\mathbf{h}_r^H\mathbf{F}\mathbf{h}_{ir}$, see Figure 2) can decreases the sum-rate if $T_1(w)$ and $T_2(w)$ both have a low-pass characteristic. On the other hand if the channels have different characteristics, increasing the second tap can also increase the sum rate. Thus, also the phase of the channel gains plays an important role in this scenario.

Therefore, we resort to an unconstructive upper bound for the expression in (7), which does not suggest any optimal

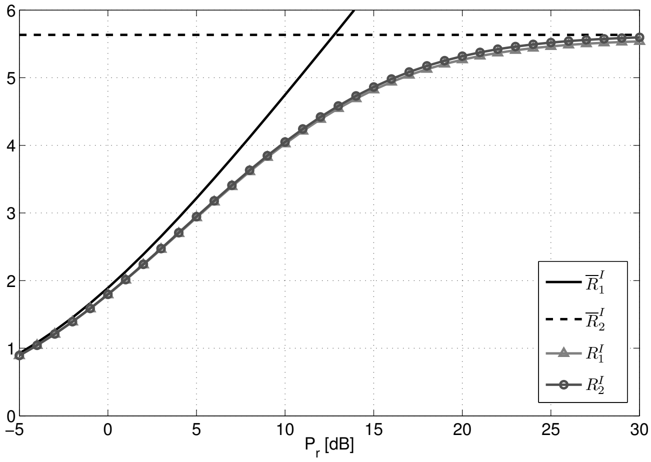

Figure 3: Achievable sum-rates and upper bound for scenario I (no direct links) and $P_1 = 12$ dB, $P_2 = 7$ dB, $M_r = 3$

$\mathbf{F}$. This bound is obtained bounding both $T_1(\omega)$ and $T_2(\omega)$ by their largest values, i.e., we have

$$T_i(\omega) \leq |h_i|^2 + ||\mathbf{h}_{ir}||^2 \quad (i = 1,2). \tag{8}$$

Unlike the previous bounds, which could be obtained from the ANOMAX or the eigenvector scheme, the derivation of this bound involves writing $\mathbf{F}$ as the sum of rank-1 matrices and a simple maximization. Due to the page constraints, we omit a detailed proof here. Using (8), we can replace $T_i(\omega)$ by their upper bound in (7) and obtain a flat fading channel again, such that

$$\overline{R}^{\mathrm{III}} = \log_2(1 + P_1 \cdot (|h_1|^2 + \|\mathbf{h}_{1r}\|^2) + P_2 \cdot (|h_2|^2 + \|\mathbf{h}_{2r}\|^2))$$

delivers an upper bound on the sum-rate for the in-band relaying scenario with direct links.

4. SIMULATION RESULTS

In this section we will discuss the achievable rates and upper bounds for all three scenarios. In our simulations, we assumed that the channel gain vectors and scalars are Rayleigh distributed and independent. All results are obtained by averaging over 1000 channel replications. As already described in the beginning of the last section, our lower bounds are achieved by using the amplifying matrices of the upper bounds. Therefore, we use R_j^i to denote the sum-rate that is achieved by the amplifying matrix $\mathbf{F}$ which maximizes the j-th upper bound of the i-th scenario.

4.1 Relaying without direct links

The lower and upper bounds for the scenario where no direct links are present are plotted in Figure 3. It can be seen that for both small and high values of the available power at the relay P_r, the achievable rates get arbitrarily close to the upper bounds. Moreover, we can observe that selecting $\mathbf{F}$ according to scheme 2 (the eigenvector scheme) leads to a slightly better performance than using the ANOMAX scheme.

4.2 Out-of-band relaying

If the links between transmitters and destination are present and the relay transmits in a different band, we obtain the

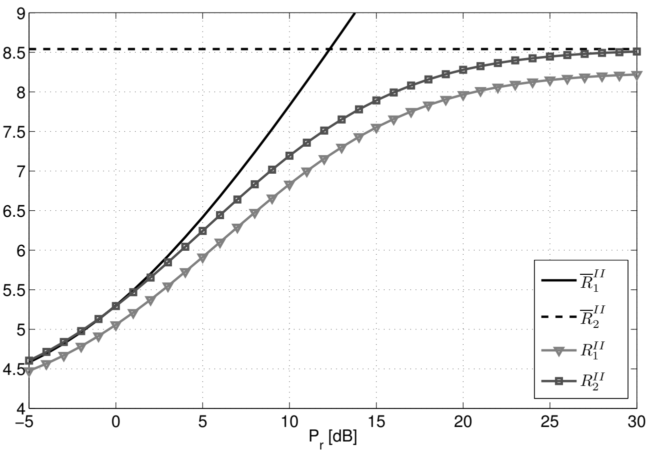

Figure 4: Achievable sum-rates and upper bound for scenario II (out-of-band-relaying with direct links) and $P_1 = 12$ dB, $P_2 = 7$ dB, $M_r = 3$

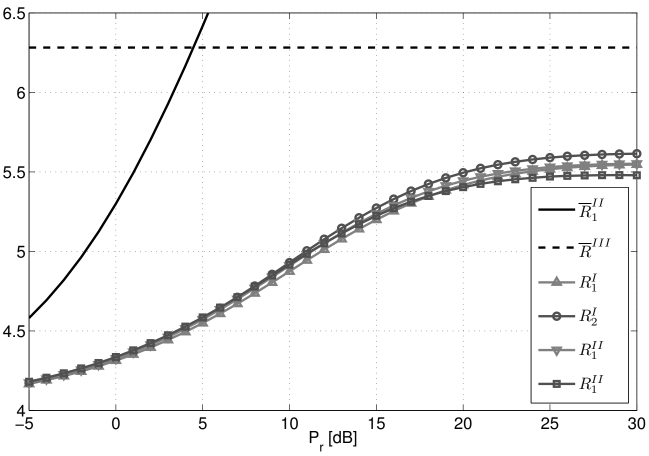

Figure 5: Achievable sum-rates and upper bound for scenario III (in-band relaying with direct links) and $P_1 = 12$ dB, $P_2 = 7$ dB, $M_r = 3$

rates plotted in Figure 4. The results are similar to those of the preceding scenario: Both for small and high values of P_r the achievable rates are close to the upper bounds. However, in comparison to the scenario without direct links, the gap between scheme 2 (eigenvector scheme) and scheme 1 (ANOMAX scheme) is larger.

4.3 In-band relaying with direct links

In the case, where all links are present and in the same band, the gap between achievable rates and upper upper bounds is quite large as it can be seen in Figure 5. This is mainly due to the upper bound for (7). To improve the existing upper bound for low P_r we also plotted the first upper bound of scenario two, which is also valid here. Another reason for the gap is the suboptimality of the chosen $\mathbf{F}$ in the plots since the optimal $\mathbf{F}$ is not straightforward to obtain.

The achievable rates are obtained by choosing $\mathbf{F}$ as for the previous scenarios, which is supposably suboptimal but still provides higher rates than heuristic choices of $\mathbf{F}$ like unity, random or DFT matrices. In a direct comparison, it can be seen that the schemes derived from the out-of-band model provide better rates for low P_r, while choosing $\mathbf{F}$ as in scenario I is good if P_r is large. This behavior is due to the fact, that for large values of P_r the path over the relay is much stronger on average, as the relay has multiple antennas and large power. Thus, the direct links can almost be neglected. On the other hand, if the relay has less power, the direct links are more important and have to be considered. This becomes clear when comparing the lower bounds of Figure 5 to the first upper bound of Figure 3, where it can be seen that at $P_r \leq 10$ dB the rate decrease is enormous if the direct links are neglected.

5. CONCLUSION

We have considered the two-user MAC with a full-duplex MIMO-AF-Relay for different three different scenarios. For the scenarios, where the direct links are not present or where the relay has an out-of-band connection to the destination, we could find upper bounds on the achievable sum rate. With the insights obtained from these bounds, achievable schemes were derived that reach the upper bounds for both small and high values of the relay power P_r. For the third scenario, where all links are present and in the same band, there remains a gap between the upper and lower bounds. However, we could show that below a considerably high value of P_r, neglecting the direct links does not fully reflect the capability of the MARC.

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