COMMON ERROR HIERARCHICAL NLMS ALGORITHM

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ABSTRACT

Two-stage common error hierarchical normalized least-mean-square (NLMS) algorithm is presented in the context of network echo cancellers and sparse systems. The suggested adaptive filter structure is generic, uses a common error feedback for both stages, and is applicable with any type of error minimization technique. The simulation results show that the two-stage method exploits the sparseness of the system better than the proportionate NLMS (PNLMS) while keeping the initial convergence rate intact and improving the steady state convergence time significantly.

1. INTRODUCTION

Sparse systems are systems in which only a small percentage of the whole impulse response has significant components, and the other components are close to zero [1]. Although the two-stage adaptive structure presented in this paper can be used in many system identification problems, we have limited our discussion to sparse adaptive systems where it is possible to observe significant benefits of the proposed method. In addition, we have chosen network echo cancellers (NEC) adopting NLMS type adaptive filtering as the leading application since network echo path is sparse in nature and NLMS based algorithms are widely used in the context of NEC.

Classic NLMS algorithms, [2] which do not take into account the sparseness of the echo path, suffer from longer convergence time and higher estimation mismatch while adapting all coefficients at each iteration with uniformly distributed weight [3]. To cope with these problems, various algorithms that harness the sparseness model of the echo path have been developed (see [1] and the references therein). In one approach, the PNLMS algorithm [3] sets the adaptation step size of each filter coefficient to be proportional to the current magnitude of the filter coefficient. In this case, adaptation of the more significant coefficients is emphasized significantly, resulting in fast initial convergence. However, the convergence rate of the PNLMS algorithm becomes slower than the classic NLMS after this initial convergence, since smaller coefficients receive small adaptation weight. Several variations of the PNLMS algorithm have been proposed to reduce these side effects by combining PNLMS with the classic NLMS algorithm [1]. Other, more advanced versions of the PNLMS algorithms are presented in [4],[5], [6] and [7]. Although the advanced PNLMS algorithms mentioned above produce significant improvement over the classic PNLMS algorithm, they require higher computational load and therefore may prove less attractive for real-time NEC applications.

Another research direction suggests that the adaptive combination of different adaptive filters improves the performance of each individual filter. An extended topology of this idea is studied in [7], [8], and [9], in the framework of multi-stage (MS) adaptive algorithms [8]. In the first stage, independent multiple adaptive filters, using their own error signals for adaptation, are operated to generate separate estimated signals. In the second stage, these estimated signals are further combined adaptively to generate the final estimated signal. In [9], the hierarchical least-mean-squares (HLMS) algorithm is presented where the full-length adaptive filter is first partitioned into equal-length independent adaptive filters with independent error signals. Then, the output of each filter is combined for use in the second-stage adaptive filter. However [10] analyzed this scheme and showed that the individual error adaptation approach results in biased estimation and may worsen the convergence speed and limit of the adaptive filter when compared to the classic NLMS algorithm.

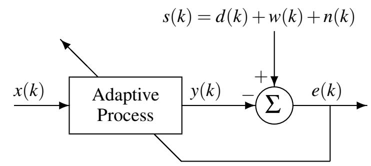

Figure 1: Generic echo canceller.

In this work, we propose a two-stage common error hierarchical NLMS (CEH-NLMS) structure that better exploits the sparseness of the echo filter than the PNLMS filter with comparable complexity to the NLMS algorithm. We also show that the CEH-NLMS algorithm outperforms both NLMS and PNLMS algorithms with the existence of a disturbing signal such as in double-talk situation. In Section 2, we provide some background on NLMS and PNLMS filters. The proposed CEH-NLMS algorithm is then presented in Section 3. Simulation results are shown in Section 4 and conclusions are discussed in Section 5.

2. REVIEW OF NLMS AND PNLMS ALGORITHMS

The basic structure of a generic adaptive filter for echo canceller can be found in Fig. 1 where $x(k)$ is the reference signal that excites the echo path (k will be used as time index in the sequel). The signal $s(k)$ is the superposition of $d(k)$ which is the echo of $x(k)$, the near-end signal $w(k)$ (also called double-talk signal) and any additional noise $n(k)$. At each adaptation iteration k, the echo canceller estimates N coefficients of the adaptive filter h, where $h(k,n)$ refers to tap number n of the filter at time index k and $n = 0,...,(N-1)$. The filter coefficients of h are then used together with the reference signal $x(k)$ to generate the estimated echo signal $y(k)$, which is subtracted from the input signal $s(k)$ to obtain the residual output signal of the filter $e(k)$. The error signal is finally fed back to

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the adaptive filter to calculate the filter coefficients at the next iteration. The main equations that characterize the PNLMS algorithm are defined originally in [3] as:

$$e(k) = s(k) - \sum_{n=0}^{N-1} h(k,n)x(k-n) \tag{1}$$

$$\sigma_x^2(k) = \frac{1}{N}\sum_{n=0}^{N-1} x^2(k-n) \tag{2}$$

$$h(k+1,n) = h(k,n) + \frac{g(k,n)}{\bar{g}(k)} \frac{\mu_x e(k)x(k-n)}{N\sigma_x^2 + \beta_x} \tag{3}$$

where $g(k,n) \sim |h(k,n)|$ and $\bar{g}(k)$ is the average of all N weighting coefficients. If the term $g(k,n)/\bar{g}(k)$ was eliminated from (3), or equivalently equal weights were used for all filter taps, the PNLMS update equation would be identical to NLMS update. Eq. (3) suggests that when the current estimate of $h(k,n)$ is significantly high, it receives a large adaptation weight in the next iteration step. Although this behavior is desired at the beginning of the process in order to achieve a high convergence rate, at a later stage it results in a slower convergence rate for the smaller taps, as stated in Section 1.

3. THE PROPOSED METHOD: CEH-NLMS

The common error hierarchical adaptive structure proposed in this paper is depicted in Fig. 2. In the first stage, the reference signal delay line with elements $x(k-n)$ and filter coefficients $h(k,n)$ are divided into M blocks with length L, where $N = ML$ and $n = 0,...,N-1$. Using this partition approach, M segments generate M distinct partial estimated echo signals $u(k,m)$ for $m = 0,...,M-1$. In the standard NLMS scheme, the direct addition of the signals $u(k,m)$ with unit gain results in the generation of the regular overall estimated signal $y(k)$. In our CEH-NLMS scheme, we introduce a second-stage filter which combines the partial estimated signals $u(k,m)$ with weights $a(k,m)$ in order to generate a final estimated echo signal $y(k)$. Finally $y(k)$ is subtracted from the input signal $s(k)$ in order to calculate the common error $e(k)$. In the CEH-NLMS algorithm, both first-stage filter coefficients and second-stage adaptation weights are derived adaptively by using distinct NLMS tap update equations. For the first stage, filter coefficient $h(k,n)$ adaptation is done in the same way described in (3) where all N filter coefficients are calculated using $x(k)$ as the regression signal and the common error signal $e(k)$ is used as the error signal. In the second stage, the same NLMS adaptation equation (3) is duplicated to update all M adaptation weights, where $u(k,m)$ is used as the regression signal and the common residual signal $e(k)$ is used as the error signal.

The equations of the proposed CEH-NLMS algorithm can be summarized as follows:

$$u(k,m) = \sum_{l=0}^{L-1} h(k,mL+l)x(k-(mL+l)) \tag{4}$$

$$e(k) = s(k) - \sum_{m=0}^{M-1} a(k,m)u(k,m) \tag{5}$$

$$\sigma_x^2(k) = \frac{1}{N}\sum_{n=0}^{N-1} x^2(k-n) \tag{6}$$

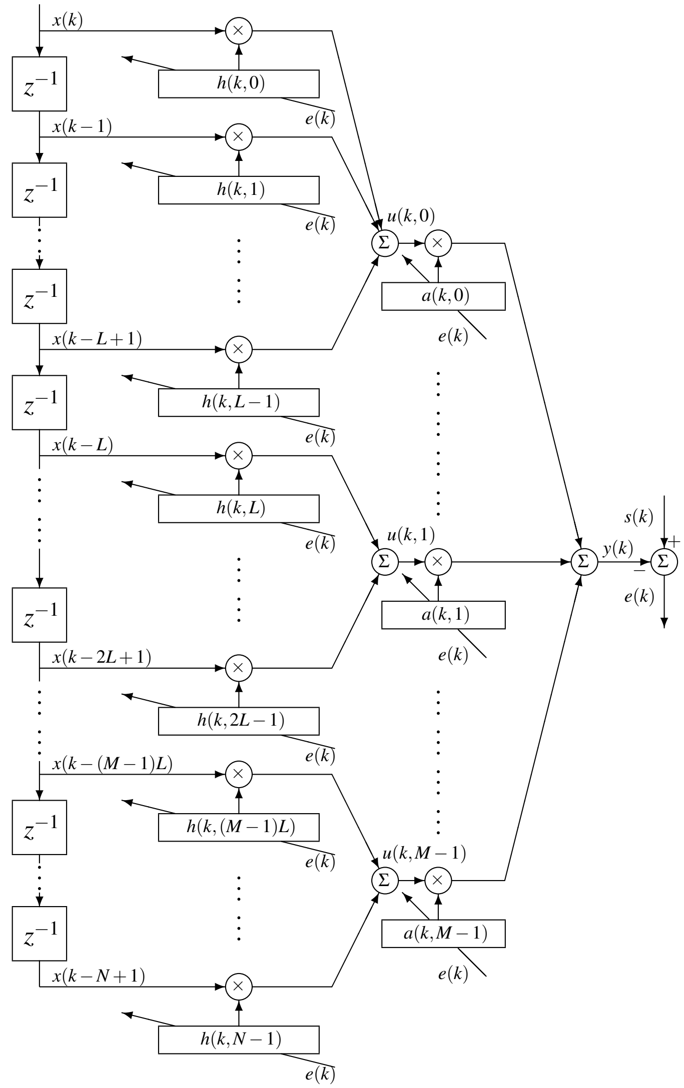


Figure 2: CEH-NLMS algorithm structure

$$\sigma_u^2(k) = \frac{1}{M}\sum_{m=0}^{M-1} u^2(k,m) \tag{7}$$

$$h(k+1,n) = h(k,n) + \frac{\mu_x e(k)x(k-n)}{N\sigma_x^2 + \beta_x} \tag{8}$$

$$a(k+1,m) = a(k,m) + \frac{\mu_u e(k)u(k,m)}{M\sigma_u^2 + \beta_u} \tag{9}$$

where μ_x, μ_u are adaptation step sizes and β_x, β_u are the regularization parameters for the two stages. In addition, the weight coefficients $a(k,m)$ are bounded such that $\xi \leq a(k,m) \leq 1/\xi$ where $0 < \xi < 1$ in order to prevent the weights from being too high or too low. It is possible to conclude that the computational complexity load of this approach is comparable to the original NLMS method, since the weights adaptation filter length M is relatively small when compared to the first-stage filter length N.

Considering the second-stage adaptive filter, the common error signal (5) can be written as:

$$e(k) = s(k) - \underline{a}^T(k) \cdot \underline{u}(k) \tag{10}$$

where $\underline{a}(k) = [a(k,0),\ldots,a(k,M-1)]^T$ and $\underline{u}(k) = [u(k,0),\ldots,u(k,M-1)]^T$ and $(\bullet)^T$ denotes transpose operation. In the minimum mean square error (MMSE) sense, it is known that the solution to (10) is given by [2]:

$$\underline{a}(k) = \mathbf{R}_{uu}^{-1}\underline{p}_{us} \tag{11}$$

where $\mathbf{R}_{uu}$ is the autocorrelation matrix of $\underline{u}(k)$ and $\underline{p}_{us}$ is the cross-correlation vector of $\underline{u}(k)$ and $s(k)$ defined as:

$$\mathbf{R}_{uu} = E\{\underline{u}(k)\cdot\underline{u}^T(k)\} \tag{12}$$

$$\underline{p}_{us} = E\{\underline{u}(k)\cdot s(k)\} \tag{13}$$

$E\{\bullet\}$ denotes statistical expectation operator. Turning now to the the first-stage adaptive filter output $u(k,m)$ (4), for $m = 0,\ldots,M-1$, we have:

$$u(k,m) = \underline{h}^T(k,m)\cdot\underline{x}(k,m) \tag{14}$$

$$\underline{h}(k,m) = [h(k,mL),\ldots,h(k,(m+1)L-1)]^T \tag{15}$$

$$\underline{x}(k,m) = [x(k-mL),\ldots,x(k-(m+1)L-1)]^T \tag{16}$$

Using (12), (14)-(16) it can be shown that the elements of second-stage autocorrelation matrix $r_{uu}(i,j)$ can be represented as:

$$r_{uu}(i,j) = \underline{h}^T(k,i)\cdot\mathbf{R}_{xx}^{i,j}\cdot\underline{h}(k,j) \tag{17}$$

where $\mathbf{R}_{xx}^{i,j}$ is the sub-autocorrelation matrix of the input signal defined as:

$$\mathbf{R}_{xx}^{i,j} = E\{\underline{x}(k,i)\cdot\underline{x}^T(k,j)\} \tag{18}$$

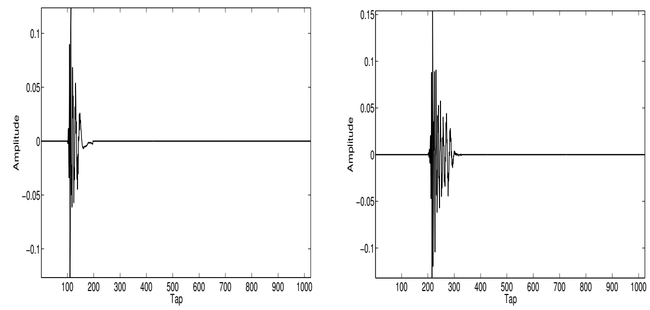
Observing (17), it can be concluded that each element of the second-stage autocorrelation matrix $\mathbf{R}_{uu}$ is a weighted average of the corresponding first-stage autocorrelation submatrix $\mathbf{R}_{xx}^{i,j}$, with weights equal to the matching vector $\underline{h}(k,i)$. This attribute derives some important benefits to the overall convergence characteristics of the CEH-NLMS algorithm. Consider the case of a white first-stage autocorrelation matrix $\mathbf{R}_{xx}$ which is not full rank, i.e., there are some zero elements on the matrix diagonal. In this case the averaging process of the $\mathbf{R}_{uu}$ matrix may remove some of the zero elements thereby transforming $\mathbf{R}_{uu}$ into a fuller rank autocorrelation matrix. Consider also the case of a correlated $x(k)$ signal, i.e., $\mathbf{R}_{xx}$ consists of some non-zero elements off the matrix diagonal. In this case, the averaging process may reduce the correlation impact in $\mathbf{R}_{uu}$. Furthermore, since both stages converge using the same error signal, they are expected to resemble one another when no correlated interference exists in $s(k)$, i.e., low weight values for inactive filter sections and high weights values for active filter sections. Resulting from this, the correlated parts of $\mathbf{R}_{uu}$ corresponding to non-active sections may be eliminated by the weighted average.

A similar analysis can be performed while evaluating $\underline{p}_{us}$ to get:

$$p_{us}(i) = \underline{h}^T(k,m)\cdot\underline{p}_{xs}(i) \tag{19}$$

where $p_{us}(i)$ is the ith element of $\underline{p}_{us}$ and $\underline{p}_{xs}(i)$ is the sub-cross-correlation vector of the input signal defined as:

$$\underline{p}_{xs}(i) = E\{\underline{x}(k,i)\cdot s(k)\} \tag{20}$$



(a) (b)

Figure 3: (a) First echo path from [11]. (b) Second echo path from [11].

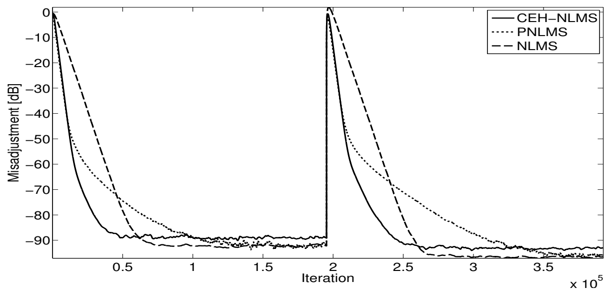
Similar conclusions can be reached as well while analyzing $\underline{p}_{us}$ expression given by (19). To conclude, it can be stated that the second-stage adaptive filter convergence characteristics are enhanced due to the weighted average process applied to $\mathbf{R}_{uu}$ and $\underline{p}_{us}(i)$, making the filter react better to correlated reference and interference signals. This results in better overall filter performance since both stage filters share the same error signal.

The two-stage approach of CEH-NLMS shares similar ideas to MS [8] and HLMS [9] adaptive filtering discussed in Section 1. The distinct sub-blocks of the CEH-NLMS algorithm in the first stage can be seen as distinct adaptive algorithms in the first-stage HLMS approach. In addition, the adaptive weighting approach of CEH-NLMS in the second stage is similar to the adaptive mixture algorithm of the HLMS. On the other hand, CEH-NLMS differs from HLMS in two ways. First, adaptation of the filter coefficients of the first stage is done together in CEH-NLMS via (8) which can be seen as collaboration between distinct blocks. Second, collaboration exists between the first-stage and second-stage adaptation via (8) and (9) where the same common error signal $e(k)$ is used for the adaptation of both h and a coefficients. The common error use in both stages adaptation schemes effectively removes the undesired convergence behavior described at [10].

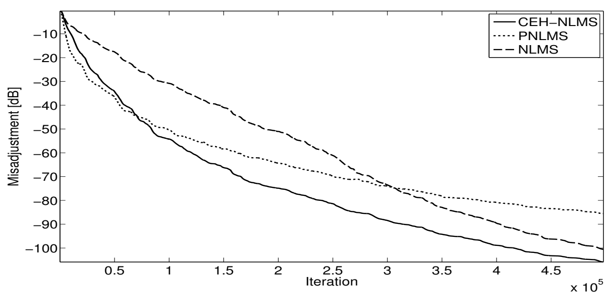
4. SIMULATION RESULTS

In the simulation, we use two different sparse impulse responses given by [11] (Fig. 3). In the first impulse response (Fig. 3a), the pure delay is set to 100 [samples] and the echo return loss (ERL) to 10 [dB]. In the second (Fig. 3b), the pure delay is equal to 200 [samples] and the ERL to 8 [dB]. The filter length of the echo canceller N is set to 1024 thereby extending both impulse responses to 1024 taps.

We choose to compare the proposed algorithm with the classic NLMS and PNLMS algorithms, both of which are widely used in NEC and possess different convergence attributes. Although there exists many extensions of PNLMS algorithms known in the literature that improve its performance significantly (e.g.[1],[4]), we decided to use PNLMS method for our simulations due to higher computational complexity burden for most of these improvements. The step size for all algorithms is set to $\mu_x = 0.1$. For CEH-NLMS algorithm, the sub-block length L is set to 64, the step size of the second stage is chosen as $\mu_u = \mu_x/(2L)$, the weighting

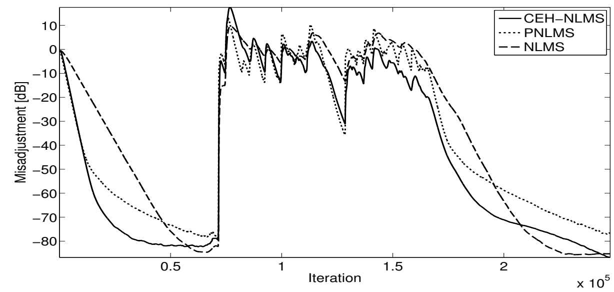


(a)

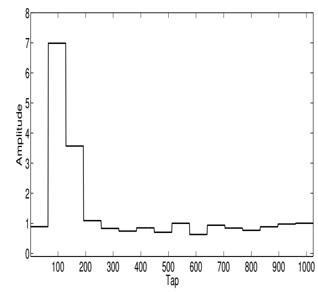


(b)

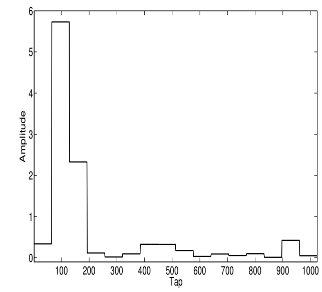
Figure 4: Misadjustment of the NLMS, PNLMS and CEH-NLMS algorithms. (a) Reference signal is a white Gaussian noise. Echo path change at iteration $2 \cdot 10^5$. (b) Reference signal is speech.



(a)



(b)



(c)

Figure 5: (a) Misadjustment of the NLMS, PNLMS and CEH-NLMS algorithms. Reference signal is a white Gaussian noise. Speech interference at iteration 0.75×10^5. (b) Behavior of adaptation weights a for a converged state. (c) Behavior of adaptation weights a for a double-talk state.

coefficients limit factor ξ is 0.01 and the values for the a weight vector is initially set to 1. Performance measure for our simulations is the normalized misadjustment defined as: $20\log_{10}(\|\mathbf{h}-\hat{\mathbf{h}}[k]\|_2/\|\mathbf{h}\|_2)$, where $\mathbf{h}$ is the desired impulse response, $\hat{\mathbf{h}}[k]$ is the estimated impulse response at iteration k and $\|\bullet\|_2$ denotes the l_2 norm.

First we evaluate the convergence performance and tracking abilities of the algorithms. The reference signal in this simulation is a white Gaussian noise. An independent white Gaussian interference is added to the input signal to achieve a SNR of 35 [dB]. At the beginning of the simulation the input signal is generated by convolving the reference signal with the first echo path displayed in Fig. 3a. At iteration 2×10^5 the echo path is instantaneously changed to the second echo path displayed in Fig. 3b. The results are presented in Fig. 4a. It can be noted that the proposed algorithm initial convergence rate is comparable to the convergence rate of the PNLMS algorithm and outperforms the initial convergence rate of the NLMS algorithm. Unlike the PNLMS algorithm, the proposed algorithm initial convergence is maintained until the steady state convergence limit is reached, thus enabling the CEH-NLMS to reach the steady state convergence limit faster than the PNLMS and NLMS algorithms. Observing the algorithms misadjustment after the echo path change at iteration 2×10^5, it can be concluded that the convergence rate behavior of the algorithm is maintained thus exhibiting good tracking ability.

Fig. 4b compares the convergence performance of the algorithms for a speech reference signal. The average misadjustment generated by seven different speech sequences of different languages from both male and female speakers sampled at 16[KHz] is displayed. The echo path of Fig. 3a is used to generate the input signal. It is possible to note that the proposed algorithm outperforms the other algorithms.

Finally the algorithm performance under double-talk interference (Fig. 5a) is compared. The reference signal used in this simulation is a white Gaussian noise convolved with the echo path of Fig. 3a. An independent white Gaussian interference is added to the input signal to achieve a SNR of 30 [dB]. An interference speech signal is added to the input signal at iteration 0.75×10^5 to generate double-talk situation, where the speech signal RMS power is -24 [dB]. It can be noticed that the proposed algorithm outperforms the NLMS and PNLMS algorithm during the double-talk interval. The performance of the CEH-NLMS algorithms during double-talk situation becomes even more perceptible during listening tests.

An explanation for the better performance of CEH-NLMS in a double-talk situation can be related to the second-stage adaptation weight values. Fig. 5b and Fig. 5c display typical second-stage adaptation weight for full convergence and double-talk situations respectively. When we compare the converged values of the second-stage filter (Fig. 5b) with the echo path response used to generate the input signal (Fig. 3a), it is clear that adaptation weight values at the active parts of the echo path get high values, emphasizing the associated first-stage filter taps values in the active region. It can also be noted that the the non-active parts of the echo path have corresponding second-stage adaptation weights whose values are lower than 1, hence the associated first-stage filter taps values in the non-active regions are attenuated. This behavior reduces the tap mismatch noise of the overall adaptive filter. When we refer to the second-stage weight values during double-talk (Fig. 5c), it can be seen that while the second-stage weights associated with the active parts of the echo path still maintain high values, the weights related to the non-active parts have close to zero magnitude. The low magnitude of these weights of the non-active part prevents the first-stage filter taps associated with the non-active parts from diverging, thus enhancing the proposed algorithm performance in double-talk situations without the use of any external double-talk detection.

5. CONCLUSIONS AND DISCUSSION

In this paper, the performance of the two-stage CEH-NLMS algorithm is introduced and compared with NLMS and PNLMS algorithms in the context of NEC employing NLMS based algorithm. It is important to note that the same two-stage structure of Fig. 2 can be used with any type of adaptive algorithms, e.g. recursive least-square (RLS) or affine projections (AP) algorithms. Simulation results state that CEH-NLMS overall convergence rate outperforms both NLMS and PNLMS with comparable complexity to the original NLMS algorithm. Our future research directions include: the extensions applying other error minimization functions, optimal sub-block partition strategies and the impact of additional hierarchical layers.

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