BLIND CHANNEL SHORTENING OF ADSL CHANNELS WITH A SINGLE-CHANNEL LINEAR PREDICTOR

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ABSTRACT

Equalization of non-stationary communication channels is preferably blind, to minimise overheads that reduce data throughput, and adaptive so as to track the channel characteristic. In this paper we consider the use of a single-channel linear predictor as an equalizer for multi-carrier modulated (MCM) signals. The MCM method with its cyclic-prefix has a level of tolerance for inter-symbol interference. We show that despite an intrinsic minimum-phase characteristic, the predictor can equalize mixed-phase ADSL test channels satisfactorily. We also show that by increasing the predictor delay, channel-shortening may be explicitly introduced, and that use of this version of the predictor improves equalization performance at higher channel noise levels. Minimising computation loads of blind equalization algorithms is also desirable. The predictor has significantly lower computational load than other blind equalization schemes, though at a cost of lower convergence speed. There is potential for faster adaptation while retaining a lower computational load than other schemes.

1. INTRODUCTION

Blind channel equalization refers to equalization of a communications channel without knowledge of the transmitted data content. The ability to equalize without knowing the data removes the requirement for training sequences within the data. This increases the efficiency of the data transfer, particularly of non-stationary channels where the equalization must be periodically or adaptively updated.

Instead of knowledge of the data, blind equalization techniques use knowledge of some general characteristic or property of the expected signal, such as the signal format or a statistical parameter, to equalize the channel. The term "property-restoral" is often used to describe this action. For example, the constant modulus algorithm (CMA) expects a constant modulus in a QPSK-modulated signal, and adjusts an equalizer to restore this property to a received signal.

Multi-Carrier Modulation (MCM) is a scheme that simultaneously transmits multiple "sub-carriers" of orthogonal frequencies within the transmitted signal. It is in wide operational use, being the modulation scheme used by, for example, Digital Subscriber Line (DSL), Digital Audio Broadcasting (DAB) and IEEE802.11 "Wi-Fi". MCM is capable of tolerating a limited degree of inter-symbol interference (ISI) due to its inclusion of a Cyclic Prefix (CP) between symbols, though if the channel impulse response length exceeds the CP length, ISI will begin to degrade the transmission. This has an important implication for MCM signal equalization. The channel impulse response need only be reduced in length —i.e. "shortened"— to the length of the CP.

Relevant examples of blind channel-shortening methods for MCM are Sum-squared Auto-correlation Minimization (SAM) [1], Partial Update SAM [2] and Lag-hopping SAM [3]. These explicitly restore the low-autocorrelation property of MCM signals. The basic SAM algorithm [1] has high computational complexity requiring typically several thousand multiply-accumulate operations per signal sample; the two latter methods aim to reduce its complexity.

A Linear Predictor may be used for equalization, as it operates by minimising autocorrelation. For example, it is used as part of an acoustic equalization scheme in [4]. It has the important drawback however that, when in forward predictor form, it always produces minimum-phase zeros [5]; this makes it incapable of equalizing maximum-phase poles, and thus of fully equalizing mixed-phase channels.

A more complex oversampling version of the linear predictor for equalization, exploiting cyclostationarity, has been proposed in [6]. The intention is to allow phase information to be retained by the equalizer and thus allow the predictor to equalize mixed-phase channels. (It is interesting though to note in [8] that an oversampling linear-predictor exhibits inferior performance for a non-minimum phase channel.) In [7] a hybrid form using both predictor and Subspace methods is proposed. However, the cyclostationarity methods are computationally complex, and are not included here.

In this paper we propose a novel, simple, adjustment to the single channel linear predictor that makes it operate as a channel-shortener. We examine the performance of the predictor in equalizer and shortener forms for high and low signal-to-noise ratio scenarios. Mixed-phase ADSL test channels, chosen to allow comparison with earlier research such as in [1].

In sections 2 and 3 we describe the linear predictor as an equalizer and as a channel-shortener, then in section 4 the system model and test model. In section 5 we show the modelled results and in section 6, conclusions.

2. THE LINEAR PREDICTOR

The Linear Predictor is an algorithm that predicts the value of a sample of a sequence from a linear combination of the other samples. Typically, a Forward Linear Predictor (FLP), as shown in Figure 1, will predict the most recent sample of the sequence from older samples. The coefficients of the combination are obtained by comparison of the actual value and the predicted value.

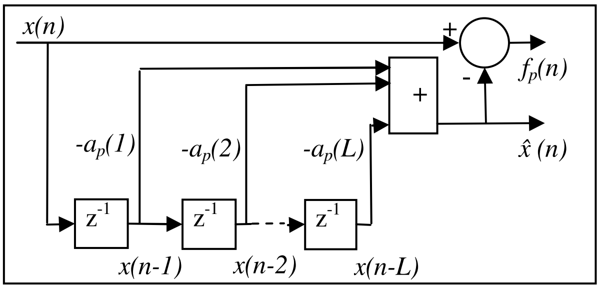

Figure 1 Forward Linear Predictor

The predicted value of $x(n)$, $\hat{x}(n)$, is given by:

$$\hat{x}(n) = -\sum_{k=1}^{L} a_p(k)x(n-k)$$

The prediction error is then given by:

$$f_p(n) = x(n) - \hat{x}(n)$$
$$= x(n) + \sum_{k=1}^{L} a_p(k)x(n-k)$$

(The above syntax is taken from [5].)

A static solution to the coefficients may be obtained by minimising the mean-square of $f_p(n)$; a set of normal equations is obtained from sample data, and solved using a method such as the Levinson-Durbin algorithm.

An adaptive solution, more appropriate to a non-stationary channel, may be obtained using the steepest-descent method or the least-mean-squares (LMS) method. Both methods minimise the mean-square of $f_p(n)$. The LMS method is used here.

Minimising the mean-square of $f_p(n)$ implies minimising the correlation between samples of the $f_p(n)$ sequence. The predicted sequence $\hat{x}(n)$ may be considered as that part of the signal $x(n)$ that is correlated with the earlier samples, and hence is predictable. Thus, $f_p(n)$ is $x(n)$ with correlated terms removed.

This then is applicable to channel equalization. If an independent and identically distributed (i.i.d) data sequence $d(n)$ is passed through a channel **h**, its output $x(n)$ becomes autocorrelated according to the impulse response of **h**. The linear prediction function, where the output is $f_p(n)$, will remove the autocorrelation of $x(n)$; with an important qualification, this may be seen as an equalizer of **h**. In this mode, where the prediction error signal $f_p(n)$ is also the system output, the predictor is called a Prediction-Error Filter.

The characteristics of the linear predictor are well-understood, [5]. Two will be mentioned here. First, the adaptation algorithm relies on minimising autocorrelation within $x(n)$. It is thus prone to adapt the filter to false zeros in the z-plane, that minimise autocorrelation but do not equalize the channel. (This is the "zero-flipping" phenomenon described in [9].) Second, the zeros of a forward linear predictor will be minimum-phase; similarly the zeros of backward linear predictor (BLP) will be maximum-phase [5]. The consequence of these two characteristics is that a prediction-error filter will be unable to fully equalize a mixed-phase channel; an FLP will introduce minimum-phase zeros where there should be maximum-phase zeros, and a BLP will introduce maximum-phase zeros where there should be minimum-phase zeros.

Nonetheless, it is worthwhile examining the equalization behaviour of a prediction-error filter, particularly since it has a low computational cost when the LMS adaptation is used. In the case of the ADSL test channels—mixed-phase but in the main minimum-phase—it is plausible that most of the channel will be equalized, leaving an acceptable residual.

3. CHANNEL-SHORTENING LINEAR PREDICTOR

Modifying the linear predictor so that it predicts D samples (rather than 1 sample) into the future has the effect of changing the associated prediction-error filter from a channel equalizer to a channel-shortener. Consider the predicted value of $x(n)$, $\hat{x}(n)$, given by:

$$\hat{x}(n) = -\sum_{k=D}^{L} a_p(k)x(n-k), \qquad D > 1$$

and the prediction error $f_p(n)$:

$$f_p(n) = x(n) + \sum_{k=D}^{L} a_p(k)x(n-k)$$

$$\text{or,} \quad f_p(z) = 1 + \sum_{k=D}^{L} a_p(k)z^{-k}$$

The prediction error only takes into account the autocorrelation of $x(n)$ with samples at least D intervals older, and filter coefficients will be obtained accordingly. As a result, samples of the error sequence $f_p(n)$ less than D intervals apart will remain correlated. Effectively, where $x(n)$ is derived from an i.i.d signal $d(n)$ and a channel **h**, the channel impulse response has been shortened to a length D.

This may be illustrated by considering a channel $\boldsymbol{h_1}$ with a single pole, equalized by a prediction-error filter with a delay D. The channel is:

$$h_1(z) = \frac{1}{1-p_1z^{-1}} = \sum_{k=0}^{\infty} {p_1}^k z^{-k}, \qquad |p_1| < 1,$$

The prediction-error filter, with one non-zero coefficient, is: $w_1(z) = 1 - p_1^D z^{-D}$. Then the effective channel:

$$c_1(z) = h_1(z)w_1(z)$$
$$= \frac{1-p_1^D z^{-D}}{1-p_1z^{-1}}$$
$$= (1-p_1^D z^{-D})(1 + p_1z^{-1} + p_1^2z^{-2} + p_1^3z^{-3} + \ \ldots)$$
$$= 1 + p_1z^{-1} + p_1^2z^{-2} + \ \ldots + p_1^{D-1}z^{-(D-1)}$$

So the effective channel $\boldsymbol{c_1}$ impulse response— $\boldsymbol{h_1}$ shortened by the prediction error filter $\boldsymbol{w_1}$— is equal to the first D terms of the impulse response of $\boldsymbol{h_1}$, and zero thereafter; that is, $\boldsymbol{c_1}$ is a shortened version of $\boldsymbol{h_1}$, of impulse response length D.

An adaptive version of this Delayed Forward Linear Predictor (DFLP), using LMS adaptation, was simulated to ascertain its behaviour with modelled mixed-phase ADSL channels.

4. SYSTEM MODEL AND SIMULATION

The system model is shown in Figure 2. The transmission channel **h** is represented as a linear finite impulse-response (FIR) filter of length L_h +1, and the channel-shortener **w** is an FIR filter of length L_w+1. The input signal $d(n)$ represents the MCM-modulated signal. The added noise $v(n)$ is uncorrelated with the channel output, is zero-mean and independent and identically distributed (i.i.d). All signals are modelled here as real.

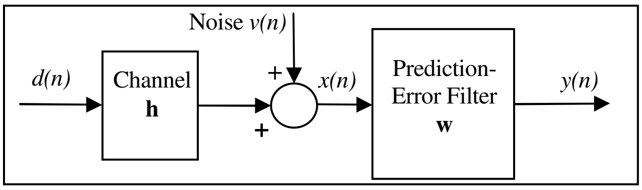

Figure 2 Transmission System Model

The input data to the receiver, *x(n)*, and *y(n),* the output of the channel-shortener **w**, are given by:

$$x(n) = \sum_{k=0}^{L_h} h(k)d(n-k) + v(n)$$

$$y(n) = \sum_{k=0}^{L_w} w(k)x(n-k)$$

The effective channel **c** is obtained by the discrete convolution of **h** and **w**, i.e. $\mathbf{c} = \mathbf{h} * \mathbf{w}$; of length $L_c + 1$, where $L_c = L_w + L_h$.

Since **w** is a prediction-error filter, then:

$$y(n) = x(n) + \sum_{k=1}^{L_w} w(k)x(n-k)$$

Note that *w(0)* =1, a fixed value. The remaining variable elements of **w**, the predictor coefficients, are here designated as the vector $\mathbf{w}_p$.

Adapting and executing the filter using the steepest-descent method has a computational load of the order of $2.L_w^2$ multiply-accumulate (MAC) operations per sample, whereas the LMS method requires about $3.L_w$ MAC operations per sample. This vector is adaptively updated in the LMS manner to stochastically minimise the mean-square of *y(n)*, as:

$$\mathbf{w}_{p,n+1} = \mathbf{w}_{p,n} + 2.\mu.y(n).\mathbf{x}(n-1)$$

where *y(n)* is both the predictor error and the filter output, μ is the adaptation coefficient, and **x**(n) is the input vector.

The delayed prediction error filter is updated in the same way, with the addition that the initial D-1 elements of $\mathbf{w}_p$ are always 0.

The linear predictor algorithms were simulated according to the above model to evaluate their performance; the model code available at [10] was used as a starting point. As with previous work, the modelled MCM signal symbol FFT size is 512 samples and the CP-length 32. The 8 ADSL model test channels CSA 1-8 were used, as available from [11] and described in [12]. These channels are mixed-phase but with mainly minimum-phase components, and selected to allow comparison with earlier work such as [1]. Near-end Crosstalk (NEXT) noise was selected to be the additive noise source, and a range of Signal-to-Noise (SNR) values used.

The main measure of performance is Achievable Bit-Rate (ABR) rather than Bit-Error Rate, since MCM signals typically adapt the bit-rate to that which is achievable given the SNR of the current channel. ABR is evaluated as in [1].

5. SIMULATION TESTS AND RESULTS

The effectiveness of the Forward Linear Predictor (in Prediction-Error Filter form) as an equalizer was tested on the 8 test channels. The filter length was 16 taps (including the initial fixed tap), and the adaptation coefficient μ was set to 0.01; these parameters being empirically chosen to provide effective performance over the SNR range.

The delayed prediction-error filter was also tested on all the test channels. The results shown here are for D = 17 taps, that is, 16 taps are forced to 0. The number of active filter taps (including the initial fixed tap) is 24. The adaptation coefficient μ was set to 0.003. It was observed that a delay (D) of 32—the length of the signal CP—produced very poor results, as the minimum-phase channel components were shortened to 32 samples, and the additional maximum-phase components then lengthened the effective channel to greater than 32 samples, i.e. longer than the guard interval.

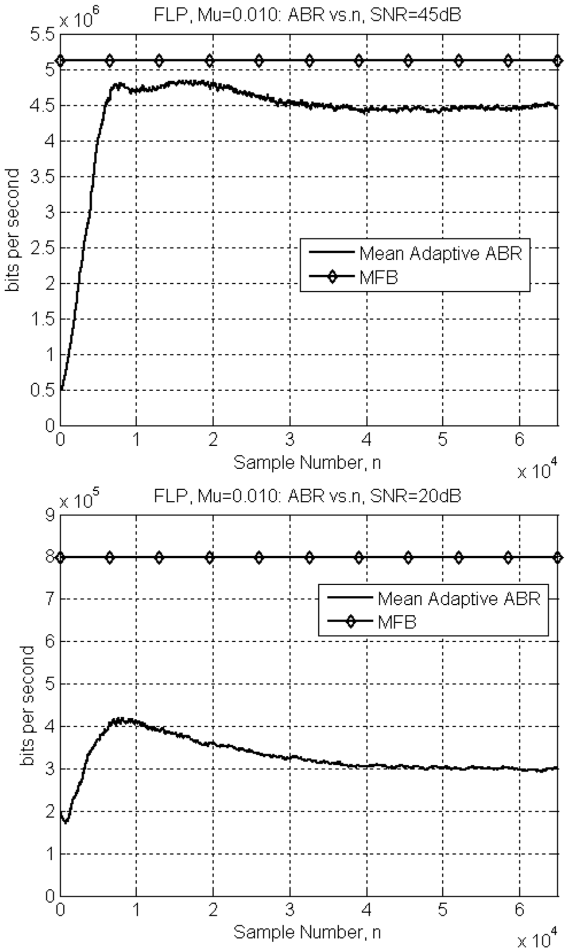

Figure 3 Ch. 3 Equalization by FLP, 45dB and 20dB

The tests were executed for SNR values of 20dB and 45dB, for scenario lengths of 120 symbols. Achievable Bit-Rate and convergence time are the main outcomes monitored.

Results are compared against the SAM algorithm [1]. The SAM shortening filter was initialised to a spike in the middle filter tap. The autoregressive adaptation mode was implemented, with per-sample updates, and the adaptation constant was 20.

The algorithms were executed on all 8 test channels; results are shown here for Channel 3. Figure 3 shows the convergence of the ABR for Channel 3 being equalized by the basic FLP, for scenario SNRs of 45dB and 20dB. The Matched-Filter Bound (MFB) for the channel is included for comparison. Equivalent results for the SAM algorithm are shown in Figure 5.

(Note that there are 544 samples in a Symbol, and 10^4 samples is approximately 18 Symbols.)

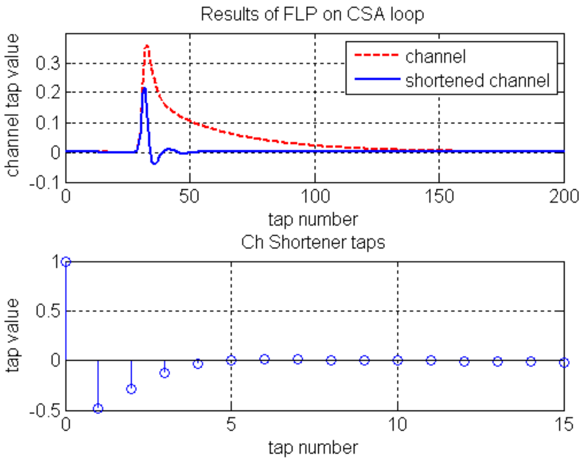

Figure 4 Channel 3, FLP and Effective Channel IRs

The impulse responses (IRs) of the channel, the equalized channel and the equalizer are shown in Figure 4 for the 45dB scenario. The equalized channel IR has not reduced to a single spike, but to the minimum-length impulse response possible where the mixed-phase channel is equalized by a minimum-phase filter.

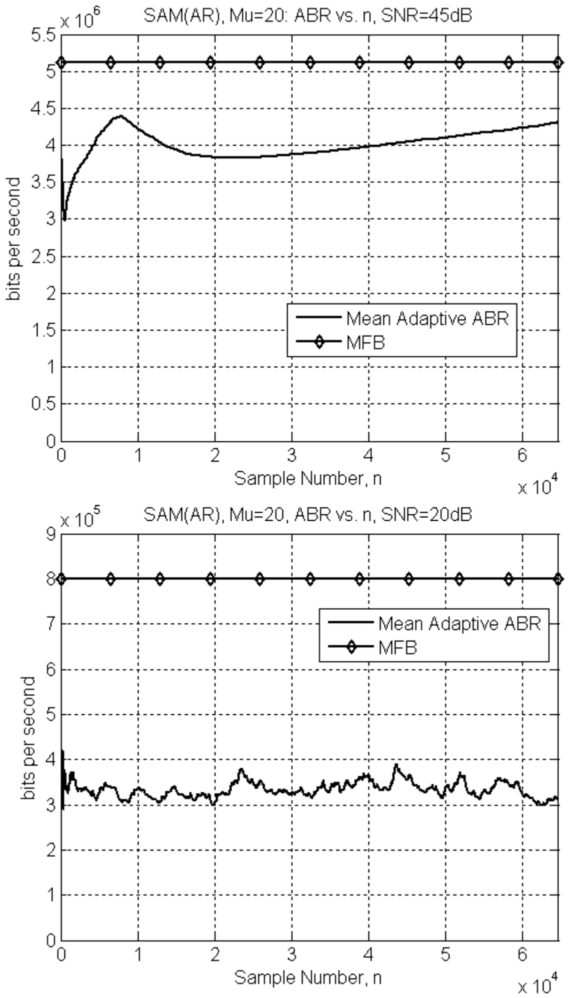

Figure 5 Ch. 3 Equalization by SAM, 45dB and 20dB

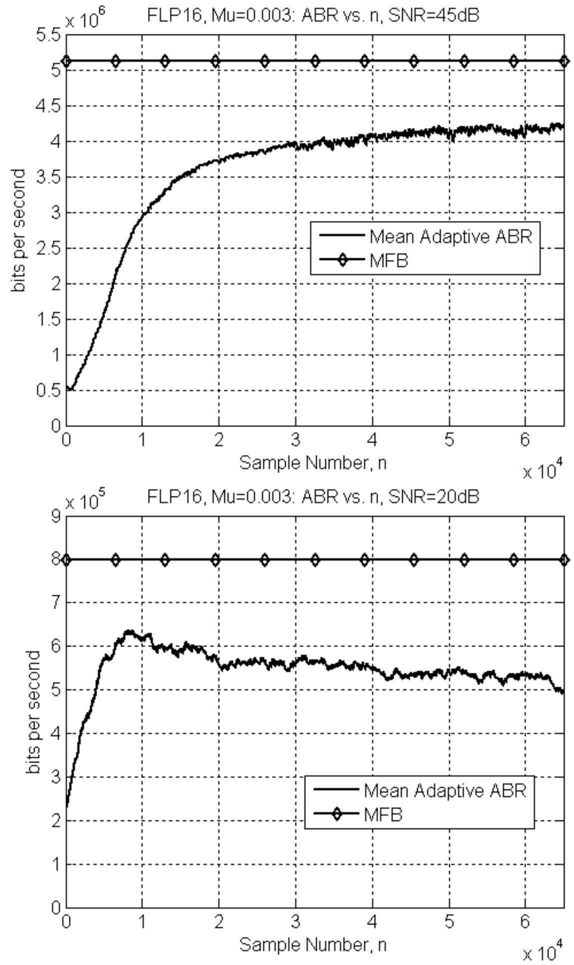

Figure 6 Ch. 3 Equalization by DFLP, 45dB and 20dB

Figure 6 shows the ABR convergence and Figure 7 the IRs for the Delayed FLP (DFLP), the channel-shortening version of the FLP. It can be seen that the shortened channel IR and the channel IR are coincident for the first 17 taps, after which the shortened channel IR energy falls to near-zero.

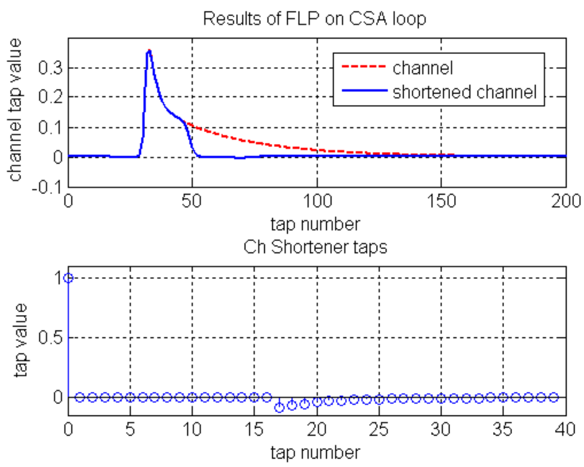

Figure 7 Channel 3, DFLP and Effective Channel IRs

Comparison of the Achievable Bit-Rate for the different equalizers or shorteners shows:

1. At 45dB, the SAM algorithm is significantly faster than the FLP, and the DFLP is slowest. For the particular adaptation constants used here, and measuring the time taken to converge to 80% of the final value, SAM is about 25 times faster than the FLP, and 80 times faster than the DFLP.
2. The FLP achieves the highest level of ABR over this length of scenario. Longer scenarios show that the SAM algorithm ABR wanders, and the FLP delivers consistently higher ABR than SAM. The DFLP performance is lower than the FLP, but is higher than SAM.
3. For the 20dB scenario the SAM algorithm remains fastest, but the DFLP delivers higher ABR than SAM or the FLP. This is the case for each of the 8 channels, with the DFLP being better than SAM and the FLP by a factor between 2 and 5. Lowering the adaptation step size for SAM by a factor of 16 did not improve its ABR.

It is assumed here that this improved behaviour is due to lower noise of the DFLP filter, given the lower level of the tap coefficients.

4. Inspection of the z-plane zeros of the original channel and the effective channel after equalization by the linear predictors shows that channel maximum-phase zeros (those outside the unit-circle) remain after equalization. This is as expected, since the FLP and DFLP are constrained to be minimum-phase. The energy of the maximum-phase zeros remains in the effective IR, so the FLP by attempting equalization has effectively shortened the channel to the length of the maximum-phase channel zeros.

The significant observation is that the linear predictor, equalizing only with minimum-phase zeros, provides an effective shortening algorithm for the 8 mixed-phase ADSL test channels.

The retention of the maximum-phase channels may also be seen in the behaviour of the DFLP. When the predictor delay D was extended beyond 24, the performance dropped markedly, since the effective channel then included the maximum-phase zeros and a longer minimum-phase IR.

6. CONCLUSION

Two primary outcomes are demonstrated here by these results. A single-channel forward linear predictor will effectively equalize a mixed-phase (though mainly minimum-phase) ADSL channel, by reducing the channel IR to its maximum-phase zeros. By including a delay in the predictor, the Delayed FLP will introduce channel-shortening.

The FLP and DFLP are much lower in computational load than another blind autocorrelation based algorithm, SAM. If a channel length of 100 is assumed (a parameter in the SAM algorithm), the load improvement is by about two orders of magnitude.

This comes at a cost in speed; the linear predictors are significantly slower than the SAM algorithm. Although not demonstrated here, introduction of the steepest-descent adaptation algorithm is likely to remove much of the speed difference, while retaining a computational load improvement over SAM by about one order of magnitude.

The Delayed FLP algorithm shows no advantage over the FLP for high SNR scenarios, however for the lower SNR scenario, it delivered a significantly better equalization performance over both SAM and the FLP. This is interesting for equalization of noisier channels.

The limitation that a forward linear predictor always has minimum-phase zeros (and conversely, maximum-phase zeros for a backward predictor) remains, so that an FLP or DFLP cannot fully equalize a mixed-phase channel. But for schemes such as MCM that can tolerate a limited amount of ISI, this work demonstrates that the predictor can act as an effective blind equalizer while only equalizing minimum-phase channel components. There is potential for the use of the predictor to be explored for other mixed-phase channels, such as wireless channels.

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7. REFERENCES

[1] J. Balakrishnan, R. K. Martin, and C. R. Johnson, Jr. “Blind, Adaptive Channel Shortening by Sum-squared Auto-correlation Minimization (SAM).” *IEEE Trans. on Signal Processing*, 51(12):3086-3093, December 2003.

[2] M. Grira, J. A. Chambers, “Random Partial Update Sum-Squared Autocorrelation Minimization Algorithm for Channel Shortening (RPUSAM)”, *Proc. 3rd Intl. Symposium on Communications, Control and Signal Processing*, St. Julians, Malta, March 2008, pp. 1400-1403

[3] K. Maatoug and J. A. Chambers, “A generalized blind lag-hopping adaptive channel shortening algorithm based upon squared auto-correlation minimization,” *The 8th IMA International Conference on Mathematics in Signal Processing*, Dec. 2008.

[4] M. Kallinger, and A. Mertins, “Room Impulse Response Shortening by Channel Shortening Concepts”, *Thirty-Ninth Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, USA, Oct-Nov 2005, pp. 898-902

[5] J. G. Proakis and D. G. Manolakis. *Digital Signal Processing, Principles, Algorithms and Applications.* Pearson Education Inc., Upper Saddle River, N.J., 2007.

[6] C. B. Papadias and D. T. M. Slock. “Fractionally Spaced Equalization of Linear Polyphase Channels and Related Blind Techniques Based on Multichannel Linear Prediction,” IEEE Trans Signal Processing, vol 47, pp 641 – 654, March 1999

[7] P. De. “Combined Linear Prediction and Subspace Based Blind Equalizers,” *IEEE Int. Symp. On Circuits and Systems, ISCAS 2000*, pp 3479 – 3482, 2007

[8] A. S. Menezes, C. M. Panazio, and J. M. T. Romano, “Blind Channel Shortening for SIMO Space-Time Channels Using Linear Prediction in OFDM Systems”, *VI International Telecommunications Symposium (ITS2006), Fortaleza-CE, Brazil*, Sept 3-6, 2006

[9] J. M. Walsh, R. K. Martin, and C. R. Johnson, “Convergence and performance issues for autocorrelation based adaptive channel shortening,” *Proc. 40th Asilomar Conf. on Signals, Systems, and Computers*, 2006

[10] R. K. Martin, Matlab Code by R. K. Martin. [Online]. Available at: http://bard.ece.cornell.edu/matlab/martin/index.html

[11] G. Arslan, M. Ding, Lu, B., Z. Shen and B. L. Evans, TEQ Design Toolbox. Univ. Texas, Austin, TX. [Online]. Avail. at: http://www.ece.utexas.edu/~bevans/projects/adsl/dmtteq/

[12] G. Arslan, B. L. Evans, and S. Kiaei. “Equalization for Discrete Multitone Receivers To Maximize Bit Rate.” *IEEE Trans. on Signal Processing*, 49(12):3123-3135, December 2001