SEQUENTIAL AND DIRECT ACCESS OF HEAD-RELATED TRANSFER FUNCTIONS (HRTFS) FOR QUASI-CONTINUOUS ANGULAR POSITIONS

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ABSTRACT

Binaural rendering is based on head-related transfer functions (HRTFs) to reproduce the soundfield around the listener using stereo-headphones. Especially, for the perception of moving sound sources, a high angular resolution of the measured HRTFs is needed.

A continuous HRTF representation with quasi-infinite azimuthal resolution can be obtained with a system identification approach which relies on a sequentially recursive adaptation process. As a consequence, it enables by nature the sequential access of HRTFs, e.g., for the virtualization of rotating sound sources. However, so far the direct access of a HRTF of an arbitrary azimuthal position, for a sudden change of the sound source position, is not possible without knowledge of neighboring HRTFs.

To overcome this limitation, we present a new implementation strategy enabling a flexible, i.e., a sequential and direct, access of 'Perfect Sequence LMS based HRTFs'. We will prove that we obtain numerically the same high-quality HRTFs as before, without increase of storage requirements, and a significantly reduced computational complextity.

1. INTRODUCTION

Head-related transfer functions (HRTFs) or in the time-domain head-related impulse responses (HRIRs), respectively, are usually obtained from measurements on human subjects or dummy heads taken from systematic measurements over a discrete set of angles, at discrete frequencies and time instants. To produce a realistic illusion of moving sound sources and changes in listener position with real-time virtual auditory spaces, a very dense grid of HRIRs is required. Typically, practical systems interpolate the necessary functions between measured HRIRs [1], [2]. However, the validity and feasibility of such interpolation methods can still be considered as a critical point due to complexity and spatial perception.

In [3]-[5] an alternative concept has been proposed which is based on a rotating, dynamical measurement setup. The quasi-continuous HRIR acquisition at any azimuth relys on a *least mean square-* (LMS)-type adaptive filtering method. In [6] the same principle has been used, except the stimulus signal, which was replaced by a so called *perfect sequence* (PSEQ) [7]. The excitation with a PSEQ guarantees a rapid convergence speed as it represents — due to its specific correlation properties — the *optimal* excitation signal of the LMS-algorithm [8]. With an appropriate choice of the system components, such as rotational speed, sampling rate and filter length, algorithms of this kind are capable to continuously track the dynamic changes and to provide in each time instant with the so-called PSEQ-LMS based HRIRs a relevant reconstruction of the actual HRIRs. As a result, HRIRs are obtained at quasi all angular positions along the horizontal plane (with respect to the sampling rate) avoiding the need of interpolation. The low effort for the actual measurement procedure in combination with the resulting high resolution and quality of the PSEQ-LMS based HRIRs makes this approach especially attractive for binaural rendering systems.

For a realistic percept of virtual sound source positioning different kinds of changes have to be considered, such as continuous head movements as well as sudden changes of a source position. Consequently, a realistic rendering system necessitates a *sequential* and a *direct* access of PSEQ-LMS based HRIRs. The recursive nature of the LMS-type adaptive filtering process enables easily the *sequential* access, however, it is understood that the *direct* access of PSEQ-LMS based HRIRs at any azimuthal angle (referring to a sudden shift in position) is a problem so far.

In this paper we will show how to overcome this limitation. We will develope an algorithm which

- provides us the flexibility of both: a *sequential* and *direct* access
- provides the same PSEQ-LMS based HRIRs as before (in terms of numerical accuracy)
- shows no increase of memory capacity and
- a reduced computational complexity by a factor of 2.

Section 2 describes briefly the conventional approach for the acquisition of PSEQ-LMS based HRIRs, i.e., the measurement setup and the algorithmic aspects. Section 3 outlines basic measures to reduce the computational complexity of the approach. In Sec. 4 and 5 we present the modifications needed for the *direct* and *sequential* access of the HRIRs. To prove the accuracy of the proposed algorithm, we perform a numerical comparison with the conventional HRIRs in Sec. 6 and present finally in Sec. 7 a design example to show the benefit of the new implementation strategy.

2. SYSTEM DESCRIPTION

The acquisition of PSEQ-LMS based HRIRs is performed in two steps:

(I) acoustic measurement procedure
→ recording of the system reaction

(II) system identification via LMS
→ computation of the HRIRs

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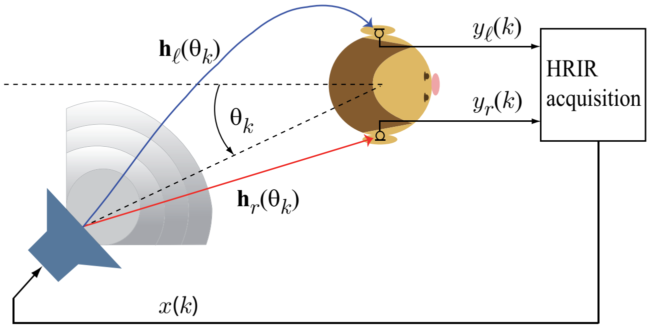

Figure 1: Measurement setup for the acquisition of HRIRs.

The measurement setup (I) is illustrated in Fig. 1. For the parameters for our experiment we choose

- a sampling rate of $f_s = 1/T_s = 44.1\,\mathrm{kHz}$
- a revolution time of $T_{360} = 20\,\mathrm{s}$ and
- a filter length of $N = 308$, which is also the period of the PSEQ.

The excitation signal $x(k)$ in our case a periodically applied PSEQ, is emitted via loudspeaker, where k denotes the discrete time-instant. Signal $x(k)$ provokes the reaction of the real HRIRs $\mathbf{h}_\ell(\theta_k)$ and $\mathbf{h}_r(\theta_k)$ in terms of the signals $y_\ell(k)$ and $y_r(k)$ observed in the left and right ear canal of the subject, respectively, where $\theta_k = 2\pi k T_s/T_{360}$ denotes the azimuth of the sequentially rotating head, see Fig. 1.

The system identification approach relies on the LMS-algorithm which is a linear adaptive filtering algorithm that consists of an *adaptive process* performing the adjustment of the filter taps. The computation of the binaural PSEQ-LMS based HRIRs reduces to two main equations: the actual *system identification* of the HRIRs in terms of

$$\widehat{\mathbf{h}}_{\ell,r}(k+1) = \widehat{\mathbf{h}}_{\ell,r}(k) + \mu_0 \frac{e_{\ell,r}(k)\,\mathbf{x}(k)}{||\mathbf{x}(k)||^2} \tag{1}$$

and the generation of the required *estimation errors*

$$e_{\ell,r}(k) = y_{\ell,r}(k) - \widehat{\mathbf{h}}^T_{\ell,r}(k)\,\mathbf{x}(k) \tag{2}$$

for the left and right channel, respectively. Vector

$$\mathbf{x}(k) = \big(x(k), x(k-1), \ldots, x(k-N+1)\big)^T$$

represents N samples of the excitation signal and

$$\widehat{\mathbf{h}}_{\ell,r}(k) = \big(\widehat{h}_{\ell,r}(0,k),\, (\widehat{h}_{\ell,r}(1,k), \ldots, \widehat{h}_{\ell,r}(N-1,k)\big)^T$$

is the vector representation of the computed HRIRs of length N at time-instant k (or angle $\theta_k = 2\pi k T_s/T_{360}$). Parameter μ_0 denotes the stepsize factor and $||\mathbf{x}(k)||$ the norm of the excitation vector.

In the next sections we will inspect (1) and (2) to reduce the complexity and to enable the computation of $\widehat{\mathbf{h}}_{\ell,r}(k)$ without knowledge of $\widehat{\mathbf{h}}_{\ell,r}(k-1)$, i.e., the *direct* access of any HRIRs. For simplicity, we will drop from now on the index ℓ, r marking the difference between the left and right audio channel, as the subsequent theoretical analysis holds for both channels exactly in the same way.

3. REDUCTION OF COMPUTATIONAL COMPLEXITY

The computation of the PSEQ-LMS based HRIRs for one channel reduces to the simplified notation

$$\widehat{\mathbf{h}}(k+1) = \widehat{\mathbf{h}}(k) + \mu_0 \frac{e(k)\,\mathbf{x}(k)}{||\mathbf{x}(k)||^2} \tag{3}$$

$$e(k) = y(k) - \widehat{\mathbf{h}}^T(k)\,\mathbf{x}(k)\,. \tag{4}$$

The computational load per iteration, i.e., per HRIR, ranges in an order of $2N$ multiply-add operations.

Observing the identification process with one set of measurement signals for the left and right channel (here only represented by one $y(k)$), obviously always the same set of HRIRs is produced. It is understood that one possibility to reduce the complexity could be to calculate all HRIRs over the complete 360° horizontal plane and store them on a data medium. As we obtain for each observed signal $y(k)$ a HRIR of length N, the required amount of storage would increase by factor N. Keeping the typical dimensions of f_s, T_{360}, and N of our experiments in mind, for a real-time application we preferably choose a trade-off between storage and computational load, cf. Sec. 7.

As stated before one measurement will always lead to the same set of HRIRs but coincidently also to the *same* error signal $e(k)$. In order to reduce the computational complexity we run once the identification process conventionally. Besides the HRIRs, we compute all error samples $e(k)$ and store them. Now the observed signal $y(k)$ is dropped and replaced by the new set of error samples $e(k)$. Consequently, the storage requirements remain the same.

From now on we can run the identification process in our real-time application by solving only equation (3). As the error signal is available, the computation of (4) can be omitted, which halves the computational load to an order of N multiply-add operations.

Further effort can be saved if in addition a precalculation of the product

$$\tilde{e}(k) = \mu_0 \frac{e(k)}{||\mathbf{x}(k)||^2}$$

is performed in advance, as the stepsize μ_0 and the norm of the *periodic* excitation signal $||\mathbf{x}(k)||^2$ are constant with time. The storage of the modified error signal $\tilde{e}(k)$ (instead of $e(k)$) simplifies the system identification problem (3) to

$$\widehat{\mathbf{h}}(k+1) = \widehat{\mathbf{h}}(k) + \tilde{e}(k)\,\mathbf{x}(k)\,, \tag{5}$$

where only $\tilde{e}(k)$ and one period of $x(k)$ are stored.

4. DIRECT ACCESS OF PSEQ-LMS BASED HRIRS

For a realistic synthesis of vitual auditory scenes all kinds of changes of sound source positions, e.g., by head movements, have to be considered. This includes movements as well as sudden changes corresponding to the possibility of a *sequential* and *direct* access of HRIRs, respectively.

The recursive nature of the system identification process (see (5)) enables easily a *sequential* access (clockwise or counter-clockwise and with different rotational speeds), however, the *direct* access of PSEQ-LMS based HRIRs at any azimuthal angle is a problem.

In this section we will provide a solution for the direct access starting with a reformulation of the LMS-algorithm. The key of the solution is to use the *periodicity* of the excitation signal $x(k)$ and to store the recursive information in a *new error signal* without increase of storage.

4.1 Reformulation of the LMS-Algorithm

For the direct access of the HRIRs, i.e., without extentive iterations starting from the beginning, we introduce an alternative formulation of the LMS-algorithm. Based on our error signal $\tilde{e}(k)$ and an initialization with $\widehat{\mathbf{h}}(0) = \mathbf{0}$ we obtain for the first N HRIRs:

$$\begin{aligned}
\widehat{\mathbf{h}}(1) &= \widehat{\mathbf{h}}(0) + \tilde{e}(0)\cdot\mathbf{x}(0) = \tilde{e}(0)\cdot\mathbf{x}(0) \\
\widehat{\mathbf{h}}(2) &= \widehat{\mathbf{h}}(1) + \tilde{e}(1)\cdot\mathbf{x}(1) \\
&= \tilde{e}(0)\cdot\mathbf{x}(0) + \tilde{e}(1)\cdot\mathbf{x}(1) \\
&\;\vdots \\
\widehat{\mathbf{h}}(N) &= \tilde{e}(0)\cdot\mathbf{x}(0) + \\
&\qquad \tilde{e}(1)\cdot\mathbf{x}(1) + \cdots + \\
&\qquad\qquad \tilde{e}(N-1)\cdot\mathbf{x}(N-1)\,.
\end{aligned}$$

Now we use the fact that the excitation signal is *periodic* in N. We reformulate the following recursions according to

$$\begin{aligned}
\widehat{\mathbf{h}}(N+1) &= [\tilde{e}(0)+\tilde{e}(N)]\cdot\mathbf{x}(0) + \\
&\qquad \tilde{e}(1)\cdot\mathbf{x}(1) + \cdots + \\
&\qquad\qquad \tilde{e}(N-1)\cdot\mathbf{x}(N-1) \\
&\;\vdots \\
\widehat{\mathbf{h}}(2N) &= [\tilde{e}(0)+\tilde{e}(N)]\cdot\mathbf{x}(0) + \\
&\qquad [\tilde{e}(1)+\tilde{e}(N+1)]\cdot\mathbf{x}(1) + \cdots + \\
&\qquad\qquad [\tilde{e}(N-1)+\tilde{e}(2N-1)]\cdot\mathbf{x}(N-1) \\
&\;\vdots \\
\widehat{\mathbf{h}}(\gamma N) &= [\tilde{e}(0)+\cdots+\tilde{e}((\gamma-1)N)]\cdot\mathbf{x}(0) + \\
&\qquad [\tilde{e}(1)+\cdots+\tilde{e}((\gamma-1)N+1)]\cdot\mathbf{x}(1) + \cdots + \\
&\qquad\qquad [\tilde{e}(N-1)+\cdots+\tilde{e}(\gamma N-1)]\cdot\mathbf{x}(N-1)
\end{aligned}$$

with $\gamma \geq 1$ and $N,\gamma \in \mathbb{N}$.

4.2 Direct Access

In order to develope the idea of the direct access we first focus on HRIRs at time instant $k = \gamma N$, $\gamma \geq 1$. Later, this restriction will be omitted.

The HRIRs of these time instants can be expressed in compact form

$$\widehat{\mathbf{h}}(\gamma N) = \sum_{i=0}^{N-1}\left[\sum_{\eta=0}^{\gamma-1}\tilde{e}(i+\eta N)\right]\cdot\mathbf{x}(i)\,.$$

With a new abbreviation for the sum in squared brackets according to

$$\varepsilon_i(\gamma N) = \sum_{\eta=0}^{\gamma-1}\tilde{e}(i+\eta N) \qquad (6)$$

we get

$$\widehat{\mathbf{h}}(\gamma N) = \sum_{i=0}^{N-1}\varepsilon_i(\gamma N)\cdot\mathbf{x}(i)\,. \qquad (7)$$

With $\varepsilon_i(\gamma N)$ for all $\gamma \geq 1$ and $i \in \{0,1,\ldots N-1\}$ we obtain a new set of data which comprises the information of the error signal $\tilde{e}(k)$ in a new *recursive* structure. This inherit recursive information (as part of the LMS) enables the direct access of HRIRs at any time instant $k = \gamma N$.

For the set-up of our algorithm we run through the following steps for the left and right channel:

- measure microphone signal $y(k)$
- run the LMS-algorithm and determine $e(k)$ and $\tilde{e}(k)$
- calculate the *recursive* error signal $\varepsilon_i(\gamma N)$ with (6)
- store $\varepsilon_i(\gamma N)$ instead of $\tilde{e}(k),\ e(k)$, and $y(k)$.

Note, that all these steps can be performed off-line at one-time and that the total amount of storage for a practical application does not increase.

As the *recursive* error signal $\varepsilon_i(\gamma N)$ is now available, the HRIR at time instant $k = \gamma N$ can directly be accessed by solving (7).

The basic load for the system identification process according to (5) ranges in an order of N, see Sec. 3. The complexity of (7), however, amounts to $N \times N$ multiply-add operations. Such a peak load is mostly undesirable for a real-time implementation. To keep the computational complexity in an order of N, we propose to spread the work-load over N iterations, i.e., in each time-instant a single scalar product in terms of $\varepsilon_i(\gamma N)\cdot\mathbf{x}(i)$ is computed and successively added up during N iterations. The resulting small delay of N/f_s ranges typically in an order of 6-7 ms, which is acceptable in many real applications.

So far, we considered only the direct access of the filter coefficients at time instants $k = \gamma N$ with $\gamma \geq 1$. In practice, a resolution of N, which correponds to an angle of $(360^\circ \cdot N)/(T_{360}\cdot f_s) = 0.13^\circ$, is absolutely sufficient, as the maximum spatial resolution of the auditory system is about 1° for changes in the azimuth [9]. However, for the sake of completeness, we can formulate the solution also for all HRIRs in-between with

$$\widehat{\mathbf{h}}(\gamma N+\lambda) = \left[\sum_{i=0}^{\lambda-1}\varepsilon_i((\gamma+1)N) + \sum_{i=\lambda}^{N-1}\varepsilon_i(\gamma N)\right]\cdot\mathbf{x}(i) \qquad (8)$$

and $1 \leq \lambda \leq N-1$. The combination of (7) and (8) provides the general solution. Thus, the storage of the *recursive* error signal $\varepsilon_i(\gamma N)$ enables us to directly access any HRIR in the horizonztal auditory scene.

5. MODIFIED SEQUENTIAL-AZIMUTH ACQUISITION

In Sec. 4.2 we replaced the conventional error signal $e(k)$ (or $\tilde{e}(k)$), which is normally required for the *sequential* access of PSEQ-LMS based HRIRs, by the *recursive* error signal $\varepsilon_i(\gamma N)$. Now the question is answered of how to perform the *sequential* access on the basis of the *recursive* error signal.

We distinguish between the two possibilities of clockwise and counter-clockwise rotation and assume without loss of generality that $\widehat{\mathbf{h}}(\gamma N)$ is known, e.g., via direct access.

5.1 Counter-Clockwise Recursion

Beginning with the LMS-algorithm according to (5) for the computation of $\widehat{\mathbf{h}}(\gamma N+1)$ we obtain

$$\begin{aligned}\widehat{\mathbf{h}}(\gamma N+1) &= \widehat{\mathbf{h}}(\gamma N)+\tilde{e}(\gamma N)\cdot \mathbf{x}(\gamma N)\\ &= \widehat{\mathbf{h}}(\gamma N)+\tilde{e}(\gamma N)\cdot \mathbf{x}(0)\,.\end{aligned}$$

As $\tilde{e}(\gamma N)$ itself is not available, we express the error signal in terms of the *recursive* error signal defined in (6)

$$\begin{aligned}\varepsilon_0\big((\gamma+1)N\big)-\varepsilon_0\big(\gamma N\big) &= \sum_{\eta=0}^{\gamma}\tilde{e}(\eta N)-\sum_{\eta=0}^{\gamma-1}\tilde{e}(\eta N)\\ &= \tilde{e}(\gamma N)\,.\end{aligned}$$

Thus, the modified LMS-recursion is given by

$$\widehat{\mathbf{h}}(\gamma N+1)=\widehat{\mathbf{h}}(\gamma N)+\left[\varepsilon_0\big((\gamma+1)N\big)-\varepsilon_0(\gamma N)\right]\mathbf{x}(0)\,.$$

Above equation shows that each sample $\tilde{e}(k)$ can be represented by the difference of two appropriate *recursive* error samples. Thus, also the recursive LMS-algorithm can be expressed by the available *recursive* error samples. The general equation is given by

$$\widehat{\mathbf{h}}(\gamma N+i+1)=\widehat{\mathbf{h}}(\gamma N+i)+\left[\varepsilon_i\big((\gamma+1)N\big)-\varepsilon_i(\gamma N)\right]\mathbf{x}(i) \tag{9}$$

with $0\leq i<N$ and $0\leq\gamma<T_{360}\cdot f_s/N$.

5.2 Clockwise Recursion

The computation of the clockwise recursion follows the same principal procedure. Given the HRIR $\widehat{\mathbf{h}}(\gamma N)$ the LMS-recursion provides the preceeding sample with

$$\begin{aligned}\widehat{\mathbf{h}}(\gamma N-1) &= \widehat{\mathbf{h}}(\gamma N)-\tilde{e}(\gamma N-1)\cdot\mathbf{x}(\gamma N-1)\\ &= \widehat{\mathbf{h}}(\gamma N)-\left[\varepsilon_{N-1}((\gamma-1)N)-\varepsilon_{N-1}\big(\gamma N\big)\right]\cdot\mathbf{x}(N-1)\end{aligned}$$

or in general form

$$\begin{aligned}\widehat{\mathbf{h}}(\gamma N-i) &= \widehat{\mathbf{h}}(\gamma N-i+1)-\\ &\quad\left[\varepsilon_{N-i}((\gamma-1)N)-\varepsilon_{N-i}\big(\gamma N\big)\right]\cdot\mathbf{x}(N-i)\end{aligned} \tag{10}$$

with $0\leq i<N$ and $0\leq\gamma<T_{360}\cdot f_s/N$ and again the required error signal is replaced by an appropriate difference of *recursive* error samples.

As the additional difference in (9) and (10), respectively, has to be computed only once per time instant, the ongoing identification process takes again mainly N multiply-add operations, i.e., it is comparable to the work load of recursion (5).

We can conclude, that the storage of the *recursive* error signal with its inherit recursive information allows us the *sequential* as well as the *direct* access of PSEQ-LMS based HRIRs for the simulation of different kinds of sound source position changes. In comparison to (5) this can be achieved at comparable computational and memory costs. Compared to the classic approach by executing (3) and (4) the computational complexity could be halved. In contrast to a complete storage of all HRIRs, we need some computational load but only require a reasonable and manageable amount of memory space.

6. NUMERICAL ACCURACY

In our application we deal with a recursion length of up to $T_{360}\times f_s=882\,000$ for a complete rotation. In order to exclude any kind of error propagation we will examine the numerical accuracy of the new approach for the *sequential* and *direct* access. The conventional PSEQ-LMS based HRIRs compute with (3) and (4) serve as reference $\mathbf{h}_{\text{ref}}(k)$.

The numerical comparison of the accuracy is performed in terms of the normalized *system distance*

$$D(k)=||\mathbf{h}_{\text{ref}}(k)-\widehat{\mathbf{h}}(k)||^2/||\mathbf{h}_{\text{ref}}(k)||^2\,. \tag{11}$$

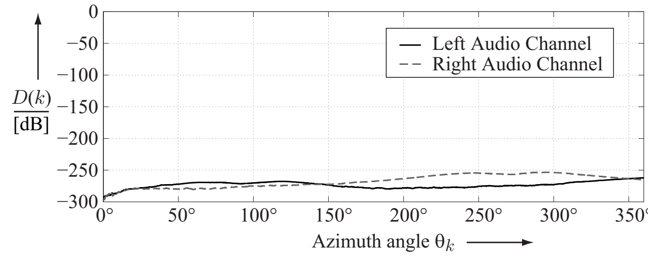

Figure 2: Comparison of $T_{360}\times f_s$ HRIRs computed via conventional LMS and *sequential* access.

Figure 2 shows for the left and right audio channel the corresponding system distance between the HRIRs determined with (3) and (4) and the new counter-clockwise recursion (9) for all k in the 360° horizontal plane. Obviously, both approaches lead to the same HRIRs with respect to the provided computational precision.

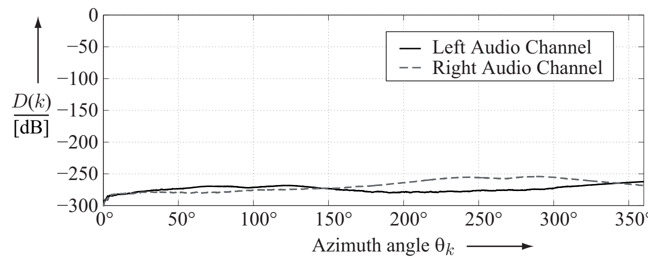

Figure 3: Comparison of $T_{360}\times f_s/N$ HRIRs computed via conventional LMS and *direct* access.

Figure 3 confirms these results also for the *direct* access. In this case the system distance is built only for time instants $k=\gamma N$ between the HRIRs $h_{\text{ref}}(\gamma N)$ picked out of the sequential conventional process by executing (3) and (4) and the HRIRs $\widehat{\mathbf{h}}(\gamma N)$ according to the *direct* access (7).

Thus, both types of the new HRIR access, *sequentially* or *direct*, provide the same exact results as the conventional LMS-type algorithm. No numerical divergence due to the long recursion length does occur. Besides, no audible differences were observed by informal listening tests.

7. DESIGN EXAMPLE

Aiming at low complexity and a flexible access of PSEQ-LMS based HRIRs it is understood that a precalculation and storage of all HRIRs would be one possible solution. The complete storage of all HRIRs in 16 bit resolution and for two channels would result in a memory space of

$$f_s \cdot T_{360} \cdot N \cdot 2 \cdot 2\,\text{Byte} = 1.1\,\text{GByte}\,.$$

For many applications, this huge amount of memory requirements is not practical.

The new implementation strategy, however, enables a trade-off between memory space and computational complexity. In this solution we store only the *recursive* error signals for the left and right channel (and one period of the excitation signal) leading to

$$f_s \cdot T_{360} \cdot 2 \cdot 2\,\text{Byte} = 3.5\,\text{MByte}\,,$$

thus, reducing the required memory space by factor $N = 308$. At the expense of N multiply-add operations per sample the desired HRIRs can be computed via *sequential* or *direct* access depending on the movements of the sound sources. For many implementations in real-time this compromise between storage and complexity is more realistic with respect to the given resources.

8. CONCLUSIONS

Based on a new implementation strategy a solution has been found for the problem of a *direct* access of PSEQ-LMS based HRIRs. Using the periodicity of the PSEQ as special characteristic of the excitation signal we reformulated the conventional LMS-algorithm. The recursive information as part of the LMS iterations is explicitely stored in a new *recursive* error signal and replaces the conventional error signal. As the recursive information is now existent within a stored signal, we can compute any desired HRIR without knowledge of neighboring HRIRs.

In this paper we also have shown that the error signal and thus the *sequential* access can also be traced back to the difference between appropriate *recursive* error samples.

As a result, the proposed flexible solution provides both possibilities, the *direct* and *sequential* access of HRIRs with quasi-infinite resolution, enabling the synthesis of sudden azimuthal shifts or continuous rotations of one or multiple virtual sound sources. Due to the elegant solution this flexibility is obtained without increase of memory requirements and the computational complexity could even be reduced by a factor of 2. The HRIRs produced according to the new implementation strategy are within the same numerical precision as the classic approach. At the same time it benefits from the compromise between storage and complexity requirements and fits well in todays given hardware resouces, e.g., for real-time binaural rendering applications.

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