OPTIMIZATION OF THE AMPLIFY-AND-FORWARD IN A WIRELESS SENSOR NETWORK USING COMPRESSED SENSING

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ABSTRACT

In this paper, we study a novel compressed sensing technique applied to wireless sensor networks with a star topology. In particular, we propose a amplify-and-forward transmission applied to wireless sensor networks with a star topology, a scheme to achieve a certain level of performance in terms of energy efficiency. In this approach, the key idea is twofold: the first is to take advantage of the time correlation properties present in most of the physical sensing scenarios and to perform a sparse version of the measured signal. The second one is to perform random projections using means of the channel matrix that models the path among transmitters, relays and receivers. To reconstruct the signal at the fusion center, we follow a l1-norm minimization approach. The simulation results show that our proposed distributed algorithm performs close to centralized compressed sensing techniques, presenting a reduction of the number of channel uses and significant energy savings. Furthermore, the trade-off between savings and the mean square error in the reconstruction is evaluated.

1. INTRODUCTION

Wireless Sensor Networks (WSNs) are nowadays one of the most promising and active fields of research in the communications area. Technically, WSNs are composed of many small and power limited devices. Usually, WSNs are designed to perform one specific task (such as the detection of some chemical agents; the measurement of temperature, humidity or light; location, estimation and positioning). Sensing nodes should operate under a set of power constraints. Since in most cases, their battery cannot be recharged, energy-efficient methods will definitely have an impact in the lifetime of the network.

Following with this motivation, we define a conventional star topology WSN (in this paper star-WSN) when all the nodes (i.e., sensing nodes) transmit their measurements to a central entity (i.e., fusion center). This approach provides the maximum accuracy in the signal reconstruction at the receiver. Conversely, it also results in the maximum expenditure in terms of energy (measured in number of transmissions) and in channel resources (measured in channel uses). This approach might not be the best in all situations and some trade-off may be more appropriate. Here, compressed sensing (CS) may be a good candidate to face this situation [1].

CS has not been yet widely extended in WSN maybe since its relative novelty. However, some solutions exist in the literature dealing with WSNs, e.g., [2–4]. Authors of [2] deal with compressing problems in tree-based networks.

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The information compression is only carried out by the data gathering nodes, but all sensors need to be active. In [3], projection methods for multihop networks are proposed. The message is distributed from the source to the sensors following a given route (projection), where each node adds its measurement. In such techniques all sensors have to be listening, and for the generic case, they need to transmit once per projection. It results in over-expensive star-WSNs in terms of energy consumption. To mitigate this effect, the authors modify the CS results with heuristics to provide better performance in terms of energy efficiency. Moreover, they do not present any distributed method to compress the measured signal. The approach in [4] does not require a routing structure. Instead, the sensors transmit all at the same time and phase-coherently, so it requires synchronization among nodes. This method reduces the in-network communication level compared to [3], but maintaining the same energy consumption level.

To overcome the problems above, we propose a CS method to select a subset of active sensing nodes to transmit, while the rest remains silent. We present a 3-step CS algorithm applied to star-WSNs, where a subset of sensors acts as relay nodes. It turns out to be energy-efficient and simplifies the Medium Access Control (MAC) by reducing the channel uses.

The main contributions of this paper are the following: i) we first propose a distributed method to deal with non-sparse but correlated signals (e.g., temperature measurements); ii) we perform the random projections using directly the channel of the system itself (i.e., the channel matrix modeling the communication path from the sensing nodes to the relay nodes and the fusion center); and iii) using the points i) and ii), we perform CS techniques in a distributed fashion for star-WSNs.

The rest is organized as follows: In Section 2, we formulate the problem. The CS background is presented in Section 3. Section 4 details the distributed approach of the proposed CS algorithm. Finally, simulation results are provided in Section 5, and conclusions are drawn in Section 6.

2. PROBLEM STATEMENT

In our WSN scenario, a set of sensing nodes of cardinality $S$ is deployed over the measurement field to sense a given physical scalar magnitude. We assume $K$ sensing nodes transmitting while $Q = S - K$ remain silent. Furthermore $R$ sensing nodes act as active relay nodes. In CS nomenclature, $K$ also corresponds to the number of non-zero elements of the transmitted vector $x(n) \in \mathbb{R}^S$, and $R$ is the number of projections used in the reconstruction process.
We assume relays operating in Amplify-and-Forward (AF) mode. When the relays receive the data, they add their readings and retransmit to the fusion center.

The problem studied is how to distributively apply CS techniques to obtain the best trade-off in terms of reconstruction accuracy, energy consumption and channel uses.

In this framework, we will assume the following:

- Signals are strongly space-time correlated and are modeled as an $S$-dimensional stochastic process, namely,

$$
\mathbf{X} = \begin{bmatrix}
  x_1(1) & x_1(2) & \cdots & x_1(N) \\
  x_2(1) & x_2(2) & \cdots & x_2(N) \\
  \vdots & \vdots & \ddots & \vdots \\
  x_S(1) & x_S(2) & \cdots & x_S(N)
\end{bmatrix},
$$

where $N$ denotes the number of time samples in the observation window. We assume, without loss of generality, that these observations (i.e., column vectors) have zero mean $\mathbb{E}[\mathbf{x}(n)] = 0$, and $\mathbb{E}[\|\mathbf{x}(n)\|^2] = 1$, and present inner column (i.e., spatial) correlations with covariance matrix $\mathbf{R}_s$. Row (time) correlations are modeled by the covariance matrix $\mathbf{R}_r$. The latter is assumed to be known.

- The fusion center has full knowledge about the channels for all the communication links (i.e., from sensing nodes to relay nodes and to fusion center).

- We assume noiseless communication paths. Any real application measurement will be corrupted by at least a small amount of noise. However, the analysis of the robustness of CS techniques against the noise power is out of the scope of this paper (the interested reader can find further discussion on this topic in [1] and [5]).

- The relay nodes have been previously assigned and are assumed to be known. The communication between relay nodes and the fusion center is controlled by a certain MAC policy (e.g., CDMA), that spends $R$ channel uses.

3. COMPRESSIVE SENSING BACKGROUND

CS is a new signal processing technique, based on the fact that many natural signals (namely, $\mathbf{x} \in \mathbb{R}^S$) can be sparsely represented in a known linear basis $\Psi \in \mathbb{R}^{S \times S}$. The transformed vector is represented as $\mathbf{w} = \Psi^T \mathbf{x}$. In many cases, $\mathbf{w}$ is not purely sparse but with most of its coefficients close to zero. Hence, the smallest contributions can be discarded, conforming a new transformed vector $\mathbf{w}_K$ (means that only $K \ll S$ entries are different from zero).

CS measurements can be represented as $\mathbf{y} = \Phi \mathbf{w}_K$, where $\Phi \in \mathbb{R}^{R \times S}$ is the sampling matrix formed by $R$ random projections (where $K < R < S$). In the CS literature, random matrices with i.i.d. entries have been extensive used due to its simplicity and incoherence with any fixed basis $\Psi$ [1].

**Notation.** Boldface upper-case letters denote matrices, boldface lowercase letters denote column vectors, and italics denote scalars. $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ denote transpose, complex conjugate, and conjugate transpose (Hermitian) respectively. $[\mathbf{X}]_{i,j}$, $[\mathbf{x}]_i$ is the $(i, j)$th element of matrix $\mathbf{X}$, and $i$th position of vector $\mathbf{x}$, respectively. $[\mathbf{X}]_i$ denotes the $i$th column of $\mathbf{H}$. Let $a = \text{diag}([\mathbf{A}])$ correspond a column vector containing the diagonal elements of $\mathbf{A}$. Let $a = b, a_i$ indicate that vector $a$ is a sparse copy of $b$ where the $K$ larger entries remain equal and the rest are set to zero. $|\cdot|$ is the absolute value. Let $\hat{a}$ name the estimated value of variable $a$. $\mathbb{E}[\cdot]$ is the statistical expectation. $\mathbf{1}_N$ denotes the $N \times N$ identity matrix.

To obtain an approximation of $\mathbf{w}_K$ we solve a $l^1$-norm minimization program, following a convex relaxation of the original NP-hard problem (with the $l^0$-norm) as in [6], [7]:

$$
\begin{align*}
\text{minimize} & \quad \|\hat{\mathbf{w}}\|_1 \\
\text{subject to} & \quad \mathbf{y} = \Phi \hat{\mathbf{w}}.
\end{align*}
$$

Moreover, CS techniques claim that when the signal $\mathbf{w}_K$ is sufficiently sparse, the recovery via $l^1$-norm minimization is exact [1]. Finally, to reconstruct the signal, one must compute $\hat{\mathbf{x}} = \Psi \hat{\mathbf{w}}$.

4. COMPRESSIVE SENSING IN AMPLIFY AND FORWARD

We propose a CS solution for the AF case in a star-WSN. The algorithm will be divided in three phases, i) the sensing phase, ii) the projection phase, and iii) the signal reconstruction phase, carried out by the fusion center (see Figure 1).

4.1 Sensing phase

As we mentioned before, CS is a powerful technique to deal with sparse signals. A priori, physical measurements are not necessarily sparse, but expected to be space-time correlated. Thus, correlated signals may be sparse or compressible when expressed in a proper basis. Wavelets are in general considered a good candidate to construct $\Psi$ [8]. The main difficulty in projecting a signal into a given wavelet basis is that this is a centralized problem. Hence in WSNs, some central entity is needed to gather all the measurements (i.e., collect $\mathbf{x}(n)$) and compute all the wavelet coefficients (or at least the $K$ most important). In star-WSNs, it implies that all $S$ sensing nodes transmit their readings to this central entity. One can see that this mechanism is signaling intensive and highly energy consuming.

However, knowing a priori that the signals are statistically time-correlated (which happens in most of measurable
tributions, i.e.,

sion. Hence, the
of the projection at the same time, using coherent transmis-

all the contributions forming one projection.

from the source through the network and each node adds its

them in the gathering nodes [2]. Others circulate the messag

projections by assuming a tree-based WSN and computing

the sensor remains silent. Algorithm 1 reviews the action

cast their readings. The rest

information

physical phenomena), one can differentially compress the

readings in a distributed manner with respect to some prior

information.

The prior information vector \( \mathbf{x} \in \mathbb{R}^S \) can be computed as

a linear combination of \( N \) previous decoded readings. As-

suming the temporal covariance matrix \( \mathbf{R}_s \in \mathbb{R}^{N \times N} \) known

in each sensor, one may use the linear Wiener filter LWF so-

\[
[\mathbf{x}]_s = \mathbf{w}^H \mathbf{x}_{N,s},
\]

(3)

where \( \mathbf{x}_{N,s} \in \mathbb{R}^N \) collects the \( N \) past readings of sensor \( s \).

The \( N \)-dimensional weighting vector is given by \( \mathbf{w} = \mathbf{R}_s^{-1} \mathbf{x} \),

where \( \mathbf{x} \) denotes the \( N \times 1 \) cross-correlation vector between

the past stored samples and the desired measurement \( [\mathbf{x}]_s \). Hence,

one can compose \( \mathbf{w} = \mathbf{x} - \mathbf{x} \), where most of the en-
tries will be close to zero. Therefore, let \( \mathbf{w}_K \) represent the

\( K \)-sparse version of \( \mathbf{w} \) (i.e., only loaded with the \( K \) entries

with higher level of ‘uncertainty’). Following this approach,

only a set of sensors \( \mathcal{S} \subset \mathcal{N} \) of cardinality \( K \) will broad-
cast their readings. The rest \( \{ \mathcal{N} - \mathcal{S} \} \) will remain silent.

From a practical point of view, the \( s \)-th sensor will transmit

if \( [\mathbf{w}_K(n)]_s \geq \Delta x \) and will remain silent otherwise. The value

of \( \Delta x \) can be chosen to ensure \( K \) active transmitters in mean.

For the next iteration, the sensors store either the reading

in the case of transmission or the prior information when

the sensor remains silent. Algorithm 1 reviews the action
executed by the sensing nodes.

4.2 Projection phase

Some existing CS examples solve the problem of making

projections by assuming a tree-based WSN and computing

them in the gathering nodes [2]. Others circulate the message

from the source through the network and each node adds its

contribution [3]. Then, the message returns to the sink with

all the contributions forming one projection.

On the contrary, in [4] each sensor sends its contribution

of the projection at the same time, using coherent transmis-

sion. Hence, the fusion center receives the sum of all con-

tributions, i.e., \( y_i = [\mathbf{P}]_i [\mathbf{w}_K(n)] \). This algorithm is iteratively

repeated \( R \) times for each projection vector \( [\mathbf{P}]_i \).

Obviously, the approaches of [3] and [4] may become

very energy expensive, since in the worst case, it results in

Algorithm 1 Sensing nodes

for \( n = 1 \) to end do

during the sensing phase

for each \( s \in \mathcal{S} \) do

get the \( s \)-th measurement \([\mathbf{x}(n)]_s\)

compute prior information \([\mathbf{x}]_s\)

compute \([\mathbf{w}_K(n)]_s = [\mathbf{x}(n)]_s - [\mathbf{x}]_s\)

if \([\mathbf{w}_K(n)]_s > \Delta x\) then

broadcast \([\mathbf{w}_K(n)]_s\)

store \([\mathbf{x}]_s\)

else

stay silent.

store \([\mathbf{x}]_s\)

end if

end for

end for

\[ \text{Algorithm 2 Relay nodes} \]

for \( n = 1 \) to end do

for each \( r \in \mathcal{R} \) do

while during the sensing phase do

get the \( r \)-th measurement \([\mathbf{x}(n)]_r\)

compute \([\mathbf{w}_K(n)]_r = [\mathbf{x}(n)]_r - [\mathbf{x}]_r\)

collect readings from \( K \) transmitting sensors.

end while

compute projection \( y_r(n) = [\mathbf{P}_r]_r [\mathbf{w}_K] \)

transmit \( y_r(n) \) to the fusion center.

end for

end for

R \cdot S \) transmissions to the fusion center for each field measure-

ment. To cope with this situation, the authors of [3] search

(heapistically) \( K \)-sparse projection vectors to compensate

the high energy costs (at the sacrifice of reconstruction accu-

racy). Differently, our proposed algorithm works with non-
sparse projection vectors with a cost of \( (K + R) \ll (R \cdot K) \)

transmissions.

First, let \( q_r \) define the signal at the \( r \)-th relay node during

the sensing phase as:

\[
q_r = \sum_{i \in \mathcal{S}} [\mathbf{H}]_{ri} [\mathbf{w}_K(n)],
\]

(4)

where \( [\mathbf{H}]_{ri} \) is the channel contribution from the active

sensor \( i \) towards the relay \( r \). Then, the relay will add its own

reading to \( q_r \) and retransmits the projection \( y_r(n) = q_r +

[\mathbf{w}_K(n)]_r \) (this process is summarized in Algorithm 2).

Mathematically, what we propose is to make use of the

channel matrix \( \mathbf{H} \in \mathbb{R}^{R \times (S-R)} \) to construct the sensing ma-

trix. It is modeled by \( \mathbf{P} \in \mathbb{R}^{R \times S} \), namely,

\[
\mathbf{P} = \begin{bmatrix} \mathbf{H} & \mathbf{I}_R \end{bmatrix}
\]

(5)

The matrix \( \mathbf{P} \) is formed by two blocks. The channel

matrix \( \mathbf{H} \) models the implicit communication path from the

\( S-R \) transmitters (all sensors but the relays, even though

only \( K \) will be active) to the relay nodes (i.e., \( S-R \) transmis-

sers per \( R \) receivers). From the CS point of view, this block

models the random projections. On the other hand, the read-
ings added by the relays are denoted by an identity matrix

\( \mathbf{I}_R \). The latter part is deterministic and may degrade the

non-coherence properties of the random matrices. However,

it allows us to insert more sensor data without any energy cost

given the fact that the relay nodes are already active.

From a MAC point of view, there is no need to make or-

thogonal transmissions, the sensing phase has a cost of one

channel use as depicted in (4). Differently, during the pro-

jection phase, it is assumed that the relay nodes use a certain

MAC technique to send a coded version of the projected values.

This phase has a total cost of \( R \) channel uses.

4.3 Signal reconstruction phase

The fusion center gathers all the received projections,

namely, \( y = [y_1 y_2 \cdots y_R]^T \). Hence, to reconstruct an approxi-

mation of \( \mathbf{w}_K \) the fusion center needs to solve (2) and then

compute \( \hat{\mathbf{x}} = \hat{\mathbf{w}} + \mathbf{x} \) (see Algorithm 3). Notice that the fusion

center needs to make use of the knowledge of \( \mathbf{H} \) in (2).

In this scenario, the reconstruction errors come from two

main sources; i) from the \( l^1 \)-norm minimization problem,
Algorithm 3 fusion center

for \( n = 1 \) to \( \text{end} \) do
  while during the sensing phase do
    compute prior information \( x(n) \).
  end while
  while during the projection phase do
    collect \( y_1(n) \ldots y_K(n) \) projection from relays.
  end while
  solve \( l^1 \)-norm minimization (2) to recover \( \hat{\omega}(n) \).
  reconstruct \( x(n) = \hat{\omega}(n) + x(n) \).
end for.

and ii) from the fact that the fusion center version of the prior information vector might differ from the one of the sensors. To mitigate the cumulative error that ii) may introduce, one possible solution is to transmit periodically \( N \) uncompressed readings in order to restore the accumulated error.

Furthermore, we assume in both sides a perfect knowledge of the time correlation parameters \( R_t \) and \( r_s \). In practice, one can use different methods to reduce the number of snapshots needed to obtain good correlation estimators [9].

5. NUMERICAL RESULTS

In this section, we provide simulation results to show the performance of the proposed methodology.

The parameters that configure the basic setup of the simulation environment are as follows:

- Number of sensing nodes: \( S = 200 \).
- Number of past samples: \( N = 10 \).
- Correlation model: The time correlation coefficient between measurements \( x(n) \) and \( x(n-k) \) is \( \rho^{|k|} \) where \( \rho = 0.95 \).
- We also define the following figures of merit.
- Relative energy consumption (relative to conventional star-WSNs measured in number of transmissions): \( \frac{E_{\text{K}}}{E_{\text{K}}} \).
- Channel uses: \( R + 1 \).
- Mean Square Error (MSE): \( \| x - \hat{x} \|_2^2 \).

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Furthermore, we assume in both sides a perfect knowledge of the time correlation parameters \( R_t \) and \( r_s \). In practice, one can use different methods to reduce the number of snapshots needed to obtain good correlation estimators [9].

\begin{algorithm}
\begin{algorithmic}
  \For {\( n = 1 \) to \( \text{end} \)}
    \While {during the sensing phase}
      compute prior information \( x(n) \).
    \EndWhile
    \While {during the projection phase}
      collect \( y_1(n) \ldots y_K(n) \) projection from relays.
    \EndWhile
    solve \( l^1 \)-norm minimization (2) to recover \( \hat{\omega}(n) \).
    reconstruct \( x(n) = \hat{\omega}(n) + x(n) \).
  \EndFor.
\end{algorithmic}
\end{algorithm}

5.1 MSE performance

In this subsection, we evaluate the performance of the proposed algorithm in terms of MSE. In Fig 2, the MSE is represented as a function of the number of active nodes, \( K \), and for three relay configurations (20%, 25%, and 30% of sensors acting as relays). The analysis is divided in two parts.

5.1.1 Impact of the number of active sensing nodes, \( K \)

When \( R \) is fixed, the evolution of the MSE as a function of \( K \) follows a quasiconvex shape. The intuition behind this is that for lower values of \( K \) the signal is sparse enough to be reconstructed (i.e., the condition \( K \ll R \) holds and hence \( \omega_K = \omega_K \) [11]). However, the MSE is not equal to zero since the signal is not perfectly sparse, i.e., \( \| \hat{\omega}_K - \omega \|_2 > 0 \). One can decrease \( \| \hat{\omega}_K - \omega \|_2 \) by including more non-zero values in \( \omega_K \).

Doing so, there is a point where the condition \( K \ll R \) does not hold anymore and it becomes \( K \approx R \). In this situation, the signal \( \omega_K \) cannot be reconstructed with zero error, letting the MSE increase.

5.1.2 Impact of the number of relay nodes, \( R \)

For higher values of \( R \), the number of projections also increases, and hence the condition \( K \ll R \) can be fulfilled for larger values of \( K \).

In this situation, less sparse signals can also be reconstructed, providing a better accuracy.

On the other hand, when \( R \) increases, the energy spent and the number of channel uses grow linearly.

5.2 CS comparison techniques: distributed versus centralized.

In order to graphically measure the performance obtained by our distributed solution, we compare it with a centralized approach. Authors of [3] show that a spatial correlated signals (e.g., temperature) are compressible in the Discrete Cosine Transform (DCT) basis. Hence, we need to assume a genius entity that gathers all the \( S \) readings at zero-cost and then, for a fair comparison, we set the spatial and temporal correlation parameters to the same order of magnitude.

Figure 2: MSE performance curve for different relay configurations. This figure has been averaged over 50 realizations.

Figure 3: Visual reconstruction of one realization. For \( K = 15 \), \( R = 60 \).
performs the DCT. Only the $K$ highest coefficients will be transmitted. Obviously, this technique is hardly applicable in practice, however, it will be used to explore the performance of our distributed CS approach in comparison with centralized methods.

5.2.1 Original signal versus reconstruction

Fig. 3 shows a graphical reconstruction of one realization. The scenario is composed by 15 active nodes plus 60 relays. It leaves a total of 125 nodes in silent mode. Visually, one can see that our proposed method performs better in such conditions than DCT. This is because the distributed algorithm takes advantage of the prior information. To have this, the price to pay is that each node needs to compute its temporal correlation statistics.

In this configuration, the resulting energy consumption is 0.375 and the channel uses are reduced by 70% compared to a conventional star-WSN (61 channel uses in front of 200).

5.2.2 Performance comparison

In case the correlation parameters are not available, we also propose a simplified version to compute the prior information by taking into account only the last reading. It is not optimal in terms of MSE, but approximates it for high correlation situations. Fig. 4 shows a comparison of the MSE obtained with the three evaluated techniques in terms of energy consumption. We assume the condition $K \ll R$ holds and follows the practical rule of four-to-one [1].

For low relative energy consumption, the proposed approach provides better MSE. It is due to the already-mentioned effect of the prior information. When $K < R$ increases, the contribution of the prior information is reduced (because $\omega_K$ is less sparse so less readings should be estimated), and hence, all the techniques converge to a similar MSE performance.

6. CONCLUSIONS

This paper has proposed a 3-step distributed CS algorithm for star-WSNs. Assuming time-correlated signals, the algorithm selects a reduced set of sensors to transmit distributedly, i.e., the ones with ‘less predictable’ measurements, where some of them act as relays. The rest of measurements are estimated from the previous ones. The presence of relay sensors is used to distributely perform the random projections employing the channel itself, and hence, no central entity is needed. To recover the signal from the received projections, the fusion node solves a $1^{st}$-norm minimization program. Simulation results study the trade-off between the energy consumption, the channel uses and the MSE performance. It is shown that even for high energy savings, a low level of MSE in the reconstruction can be guaranteed. The optimal selection of the subset of active and relay nodes remains as an open issue for future work.

References


