DOA AND POLARIZATION ACCURACY STUDY FOR AN IMPERFECT DUAL-POLARIZED ANTENNA ARRAY

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ABSTRACT

In this paper, a realistic array model for dual-polarized elements that includes the cross polarization ratio is introduced. This parameter reveals element imperfections due to polarization impurity. For the introduced array model, the Cramér-Rao bound on the direction of arrival (DOA) and polarization accuracy is derived. This asymptotic estimation accuracy can be descriptively interpreted. In simulations, we investigated how the estimation accuracy is affected by the array and source parameters. It turns out that especially the polarization accuracy depends significantly on the polarization purity.

1. INTRODUCTION

Passive direction finding (DF) of multiple narrowband signals is of high interest in many applications like radar, sonar, radio astronomy, and wireless communication. For estimating the directions of arrival (DOAs), antenna arrays are employed, which are usually set up by identical uniformly polarized array elements arranged in a geometric systematic order. This antenna arrangement facilitates beam steering and calibration.

DF systems that employ these uniformly polarized antenna arrays may cause non-negligible bearing errors due to a mismatch between the signal polarization and the polarization of the array elements. To overcome this problem, diversely polarized antenna arrays or multiport antennas (e.g. crossed dipole pairs) can be used, that measure each polarization component separately. Such arrays are able to jointly estimate source DOAs and polarizations (Fig. 1), and are advantageous to resolve diversely polarized incoming signals.

Numerous DF algorithms have been extended to unknown source polarization, e.g. the maximum likelihood estimator (MLE) [1,2], the conventional multiple signal classification (MUSIC) method [3], the root-MUSIC algorithm [4], and the subspace fitting (SSF) approach [5]. An MLE for a source transmitting signals with time-varying polarization has been introduced in [6]. Furthermore, the DOA can be determined without explicitly computing the polarization parameters, which is computationally more efficient than a brute force search for all source parameters.

In [7, Sec. III], Weiss and Friedlander derived the Cramér-Rao bound (CRB) on the DOA and polarization accuracy of diversely polarized arrays. They presented numerical examples to interpret the CRB. Also in [5, Sec. IV.B], the asymptotic accuracy on the considered estimation problem has been derived. In all these cases, ideal array elements are assumed that purely measure the polarization components separately. Recently, the CRB for polarimetric arrays has been calculated in [8, Sec. III.C] taking the measured element patterns into account, but therein it is difficult to see how the bound is affected due to array imperfections.

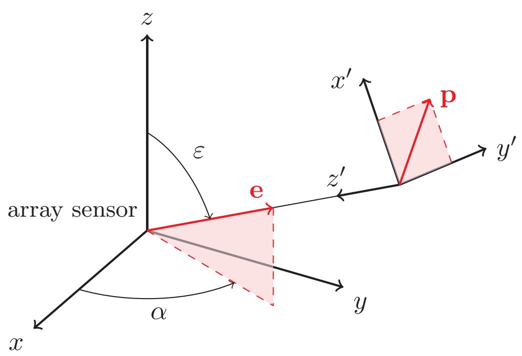

Figure 1: Bearing vector $\mathbf{e}$ and polarization vector $\mathbf{p}$ of an incident plane wave

In practice, especially for small array sensors with only few elements, array imperfections such as element coupling and high cross-polarization ratio (CPR) have a non-negligible effect. The CPR, i.e. the ratio of cross- to co-polarization amplitudes, is often used to the describe the polarization quality of an antenna. In this paper a realistic array model for linearly or circularly dual-polarized array elements is presented. Therefore, the CRB on the DOA and polarization accuracy is analytically derived including polarization impurity, to study the influence of the CPR on the estimation accuracy.

This paper is structured as follows: In Section 2, the data model for a single incoming arbitrarily polarized plane wave is described. Section 3 presents the derivation of the CRB of the underlying realistic array model and discusses the influence of the parameters on the estimation accuracy. The results of various Monte-Carlo simulations for the DOA and polarization estimation are presented in Section 4. Finally, Section 5 concludes this paper.

The following notations are used throughout this paper: $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively; $\mathbf{I}_n$ and $\mathbf{0}_n$ denote the $n \times n$-dimensional identity and zero matrix, respectively; and $\mathrm{E}\{\cdot\}$ denotes the expectation operation.

2. DOA AND POLARIZATION ESTIMATION PROBLEM

Let us consider a transverse electromagnetic wave impinging on an antenna array from direction (α, ε), where $\alpha \in (-\pi, \pi]$ denotes the azimuth angle and $\varepsilon \in [0, \frac{\pi}{2}]$ is the elevation angle (Fig. 1). The signal polarization is defined by the phase difference $\beta \in (-\pi, \pi]$ between the electric field components and the auxiliary polarization angle $\gamma \in [0, \frac{\pi}{2}]$. Fig. 2 displays the shape of the polarization ellipse depending on polarization parameters. If $\gamma = 0$, $\gamma = \frac{\pi}{2}$, $\beta = 0$, or $\beta = \pi$, the signal is linearly polarized. If $\gamma = \frac{\pi}{4}$ and $\beta = \pm\frac{\pi}{2}$, the signal has a left or right hand circular polarization (LHCP or RHCP), respectively. In the remaining cases the signal is elliptically polarized. The signal DOA and polarization is completely defined by the following unit vectors, respectively:

$$\mathbf{e}(\alpha, \varepsilon) = \begin{pmatrix} \cos\alpha \sin\varepsilon \\ \sin\alpha \sin\varepsilon \\ \cos\varepsilon \end{pmatrix}, \tag{1}$$

$$\mathbf{p}(\beta, \gamma) = \begin{pmatrix} \cos\gamma \\ \sin\gamma \, \mathrm{e}^{j\beta} \end{pmatrix}. \tag{2}$$

Sometimes the polarization vector $\mathbf{p}$ is referred to as the Jones vector.

2.1 Data Model

To formulate the estimation problem, it is important to establish a simple mathematical relationship between the received array data and the source parameters $\boldsymbol{\vartheta} = (\alpha, \varepsilon, \beta, \gamma)^T$. The common array data model assumes that a set of narrowband signals with wavelength λ impinges as plane waves on an array sensor with M dual-polarized elements. For the case of a single source and using complex envelope notation, the k-th sample of the random (measurement) vector $\mathbf{z}_k \in \mathbb{C}^{2M}$ can be expressed by the signal and the additional random noise vector $\mathbf{n}_k \in \mathbb{C}^{2M}$ as

$$\mathbf{z}_k = \mathbf{a}(\boldsymbol{\vartheta})\, s_k + \mathbf{n}_k\,, \tag{3}$$

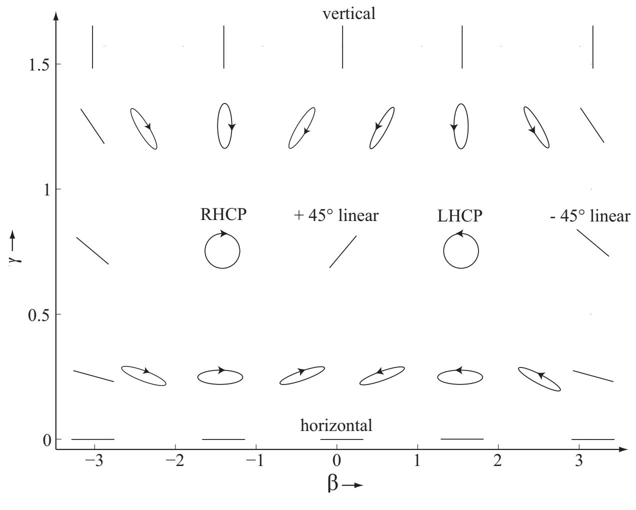

Figure 2: Polarization figures in dependence of β and γ

where s_k, $k = 1, ..., K$, are the unknown source signal snapshots, and $\mathbf{a}(\boldsymbol{\vartheta})$ is referred to as the array transfer vector. The array transfer vector expresses the complex array response to a unit wavefront arriving from the DOA with the polarization vector $\mathbf{p}(\beta, \gamma)$. For the considered far-field source, the array response of the m-th antenna element is given by

$$\mathbf{a}_m(\boldsymbol{\vartheta}) = \mathbf{G}_m(\alpha, \varepsilon)\, \mathbf{p}(\beta, \gamma)\, \mathrm{e}^{j\frac{2\pi}{\lambda}\mathbf{e}^T(\alpha,\varepsilon)\,\mathbf{r}_m}\,. \tag{4}$$

As can be seen from (4), the array response depends on the desired source parameters $\boldsymbol{\vartheta}$, the array element locations $\mathbf{r}_m \in \mathbb{R}^3$, $m = 1, ..., M$, relative to a reference point commonly in the center of the array, and the element pattern of the m-th array element

$$\mathbf{G}_m(\alpha, \varepsilon) = \begin{bmatrix} g_{\mathrm{h}1,m}(\alpha, \varepsilon) & g_{\mathrm{v}1,m}(\alpha, \varepsilon) \\ g_{\mathrm{h}2,m}(\alpha, \varepsilon) & g_{\mathrm{v}2,m}(\alpha, \varepsilon) \end{bmatrix}. \tag{5}$$

Herein, $g_{\mathrm{h}p,m}$ and $g_{\mathrm{v}p,m}$ are the linearly polarized characteristic components of the p-th port, $p \in \{1, 2\}$, of the m-th element, $m = 1, ..., M$. Commonly, the element patterns are described by their linearly polarized components, because the element characteristics can be measured adequately with a linearly polarized antenna (e.g. horn antenna). Any other orthonormal polarization basis can be calculated by using a suitable transformation matrix $\mathbf{T}$.

2.2 Realistic Array Model

In this section, an array data model is introduced where the element characteristics and the polarization vector are described by their co- and cross-polarized components:

$$\bar{\mathbf{G}}_m(\alpha, \varepsilon) = \mathbf{G}_m(\alpha, \varepsilon)\, \mathbf{T}\,, \tag{6}$$

$$\bar{\mathbf{p}}(\beta, \gamma) = \mathbf{T}^{-1}\, \mathbf{p}(\beta, \gamma)\,. \tag{7}$$

The terms mentioned in (6) and (7) are replaced by the corresponding terms in the array transfer vector (4). For example, the transformation matrix for linearly dual-polarized elements is the identity matrix, and for circularly dual-polarized elements, the transformation is given by

$$\mathbf{T} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix}. \tag{8}$$

Especially a mismatch between the real and assumed (measured) phase characteristics of the antenna leads to biased bearing results. However, for the accuracy analysis carried out, the phase patterns are neglected, i.e. only the amplitude characteristics are considered. Typically, array elements may be not omni-directional due to element coupling effects, reflections of the sensor platform, etc. Nevertheless, the array elements can be assumed as constant in the vicinity of the DOA or can be approximated by an average amplitude value resulting from the amplitude characteristic in the vicinity of the DOA. Furthermore, we assume that both ports of the dual-polarized elements have the same characteristic, and that all elements are identical. These assumptions lead to a simple but realistic array model for dual-polarized array elements stated as follows:

$$\bar{\mathbf{G}}_m \approx \begin{bmatrix} 1 & \kappa \\ \kappa & 1 \end{bmatrix} , \qquad (9)$$

$m = 1, ..., M$, where $\kappa \in \mathbb{R}$ denotes the CPR of the array elements. This parameter can be interpreted as the polarization purity of the array elements. For example, for the ideal case that $\kappa = 0$, both ports of the dual-polarized element show purely co-polarized components, and for the unfavorable case that $\kappa = 1$, both co- and cross-polarization components have the same strength.

2.3 Problem Statement

The estimation problem is stated as follows: Estimate all source parameters $\boldsymbol{\vartheta}$ from the received data set $\mathbf{z}_k$, $k = 1, ..., K$, while the unknown source signals s_k, $k = 1, ..., K$, are nuisance parameters, i.e. these parameters are also unknown but not of interest.

Let us make a few comments on the signal model and the noise model. The deterministic signal model - sometimes also named as conditional model - is used, where each sample s_k, $k = 1, ..., K$, is regarded to be fixed and unknown, i.e. each sample is an unknown deterministic parameter that needs to be estimated. This model does not exclude the possibility that the signals are sampled from a random process. Furthermore, the random noise vectors $\mathbf{n}_k$, $k = 1, ..., K$, are assumed to be zero-mean complex Gaussian, and are temporally and spatially uncorrelated, i.e. the noise vectors are given by

$$\mathrm{E}\left\{\mathbf{n}_k\, \mathbf{n}_{k'}^H\right\} = \sigma_\mathrm{n}^2\, \delta_{k,k'}\, \mathbf{I}_{2M} ,$$
$$\mathrm{E}\left\{\mathbf{n}_k\, \mathbf{n}_{k'}^T\right\} = \mathbf{0}_{2M} ,$$

where σ_n^2 denotes the receiver noise variance and $\delta_{k,k'}$ the Kronecker delta. For simplicity, it is assumed that the emitted source signal has a constant amplitude, i.e. $|s_k| = s$. With this the signal-to-noise ratio w.r.t. a single element can be defined as $\mathrm{SNR} = s^2/\sigma_\mathrm{n}^2$.

2.4 DOA and Polarization Estimation

As already mentioned in Section 1, several DF methods have been proposed based on standard DF algorithms [1–5]. A basic algorithm to estimate the unknown four-dimensional parameter vector $\boldsymbol{\vartheta}$ is the considered conventional beamformer:

$$\hat{\boldsymbol{\vartheta}} = \arg\max_{\boldsymbol{\vartheta}} \frac{\mathbf{a}^H(\boldsymbol{\vartheta})\, \hat{\mathbf{R}}\, \mathbf{a}(\boldsymbol{\vartheta})}{\mathbf{a}^H(\boldsymbol{\vartheta})\, \mathbf{a}(\boldsymbol{\vartheta})} , \qquad (10)$$

where $\hat{\mathbf{R}} = \frac{1}{K} \sum_k \mathbf{z}_k\, \mathbf{z}_k^H$ denotes the estimated data covariance matrix.

It is mentioned that for DOA and polarization parameter estimation, a solution can be found that is computationally more efficient than a brute force search for all parameters simultaneously. By transforming the classical beamformer into the Rayleigh quotient, according to [6], the four-dimensional estimation problem can be reduced to a two-dimensional DF problem. Then, the DOA estimates can be used to compute the polarization coefficients.

3. PERFORMANCE ANALYSIS

For judging an estimation problem, it is important to know the maximum estimation accuracy that can be attained with the given measurements. The CRB provides a lower bound on the estimation accuracy for any *unbiased* estimator. Note that nonlinear estimation problems result in biased estimators, which are only asymptotically unbiased. Therefore, e.g., for weak sources the CRB is not valid. Moreover, a misspecified array model leads to biased bearing estimates. These errors are not covered by the CRB, but its parameter dependencies reveal characteristic features of the estimation problem.

3.1 Preliminaries to the CRB

Let $\hat{\mathbf{x}}(\mathbf{z})$ denote some unbiased estimate of the unknown target parameters $\mathbf{x}$ based on the measurements $\mathbf{z}$. The covariance matrix $\mathbf{C}$ of the estimation error $\triangle\mathbf{x} = \mathbf{x} - \hat{\mathbf{x}}(\mathbf{z})$ satisfies the multi-dimensional Cramér-Rao inequality

$$\mathbf{C} = \mathrm{E}\left\{\triangle\mathbf{x}\, \triangle\mathbf{x}^T\right\} \geq \mathbf{J}^{-1}(\mathbf{x}) , \qquad (11)$$

where the inequality is interpreted as stating that the matrix difference is positive semidefinite. If equality holds, the estimator is called efficient. The CRB is given by the inverse Fisher information matrix (FIM):

$$\mathbf{J}(\mathbf{x}) = \mathrm{E}\left\{ \left(\frac{\partial \ln L(\mathbf{x};\mathbf{z})}{\partial \mathbf{x}}\right)^T \left(\frac{\partial \ln L(\mathbf{x};\mathbf{z})}{\partial \mathbf{x}}\right) \right\} , \qquad (12)$$

where $L(\mathbf{x};\mathbf{z})$ denotes the likelihood function that describes how likely the source parameters given the measurements are. The drivation of the CRB for the multi-source case can be found e.g. in [5,7]. For the considered single source case, the log-likelihood function reads

$$\ln L(\boldsymbol{\vartheta}, s_1, \ldots, s_K; \mathbf{z}_1, \ldots, \mathbf{z}_K)$$
$$= -2KM \ln(\pi\sigma_\mathrm{n}^2) - \frac{1}{\sigma_\mathrm{n}^2} \sum_{k=1}^{K} |\mathbf{z}_k - \mathbf{a}(\boldsymbol{\vartheta})\, s_k|^2 . \qquad (13)$$

3.2 Derivation of the CRB

Using the assumptions mentioned in Section 2.3, the CRB for the estimation problem can be calculated by inserting (13) in (12) and taking the expectation [9]. Then, the CRB on the DOA and polarization estimation accuracy is given as follows

$$\mathbf{J}^{-1}(\boldsymbol{\vartheta}) = \frac{1}{2K\mathrm{SNR}} \left[\mathbf{J}_1(\boldsymbol{\vartheta}) - \mathbf{J}_2(\boldsymbol{\vartheta})\right]^{-1} \qquad (14)$$

with

$$\mathbf{J}_1(\boldsymbol{\vartheta}) = \mathrm{Re}\left\{\mathbf{D}^H(\boldsymbol{\vartheta})\, \mathbf{D}(\boldsymbol{\vartheta})\right\} ,$$
$$\mathbf{J}_2(\boldsymbol{\vartheta}) = \mathrm{Re}\left\{\mathbf{D}^H(\boldsymbol{\vartheta})\, \mathbf{a}(\boldsymbol{\vartheta})\, \mathbf{a}^+(\boldsymbol{\vartheta})\, \mathbf{D}(\boldsymbol{\vartheta})\right\} ,$$
$$\mathbf{D}(\boldsymbol{\vartheta}) = \left[\frac{\partial}{\partial\alpha}, \frac{\partial}{\partial\varepsilon}, \frac{\partial}{\partial\beta}, \frac{\partial}{\partial\gamma}\right] \mathbf{a}(\boldsymbol{\vartheta}) \in \mathbb{C}^{2M\times 4} ,$$

where $(\cdot)^+$ denotes the Moore-Penrose pseudoinverse. The term $\mathbf{J}_2(\boldsymbol{\vartheta})$ can be interpreted as a measure how much the estimation accuracy degrades by the circumstance that the signal nuisance parameters are unknown. If the signal samples are known, then $\mathbf{J}_2(\boldsymbol{\vartheta}) = \mathbf{0}_4$. As expected, (14) reveals that the estimation accuracy increases with a growing number of samples K or SNR, but it is difficult to see how the bound is affected by the remaining array and source parameters. To get a deeper insightful interpretation of the CRB, the realistic array model in (9) is used and a planar array is assumed: $\mathbf{r}_m = (x_m, y_m, 0)^T$, $m = 1, ..., M$. Then, it follows

$$\mathbf{J}_1(\boldsymbol{\vartheta}) = \begin{bmatrix} \mathbf{J}_1(\alpha,\varepsilon) & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{J}_1(\beta,\gamma) \end{bmatrix}, \tag{15}$$

$$\mathbf{J}_2(\boldsymbol{\vartheta}) = \begin{bmatrix} \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{J}_2(\beta,\gamma) \end{bmatrix}. \tag{16}$$

Since the reference point is placed in the center of the array, i.e. $\sum_m x_m = \sum_m y_m = 0$, the DOA estimation accuracy is not influenced by the unknown signal samples, while the polarization accuracy degrades. Due to the block-diagonal form of (15) and (16), the DOA and polarization parameters are uncorrelated. Interpreting the polarization parameters as nuisance parameters and using the computational efficient approach mentioned in Section 2.4, the matrices reveal that this leads to no performance benefit.

The derivation of CRB for linearly and circularly dual-polarized elements leads to slightly different results that can be generalized by introducing the term

$$c(\kappa,\beta,\gamma) = \begin{cases} 4\kappa\cos\gamma\sin\gamma\cos\beta + \kappa^2 + 1 & \text{, if linear} \\ 4\kappa\cos^2\gamma + (\kappa-1)^2 & \text{, if circular} \end{cases}.$$

In the ideal case $\kappa = 0$, we have $c = 1$. The derivation of the CRB for another polarization base can be determined straightforward analogous to the presented calculations.

For a symmetric antenna array with $\sum_m x_m y_m = 0$, the azimuth and elevation angles are uncorrelated. From this it follows that

$$\mathbf{J}_1(\alpha,\varepsilon) = \frac{4\pi^2 c}{\lambda^2} \begin{bmatrix} \sin^2\varepsilon \sum_m x_m^2 & 0 \\ 0 & \cos^2\varepsilon \sum_m y_m^2 \end{bmatrix}. \tag{17}$$

For the ideal case $\kappa = 0$, (17) corresponds to a conventional single-port array with a matching signal polarization, i.e. in this case the dual-polarized array elements offer no DF performance benefit. We briefly mention that the DF accuracy depends on the element number M, the array geometry, and the elevation ε itself. Moreover w.r.t. the uniqueness condition, a decreasing signal wavelength λ leads to a decreasing antenna beamwidth and therewith an increasing DOA accuracy.

Analogous to (17) follows

$$\mathbf{J}_1(\beta,\gamma) - \mathbf{J}_2(\beta,\gamma) = \frac{M(1-\kappa^2)^2}{c} \begin{bmatrix} \sin^2\gamma\cos^2\gamma & 0 \\ 0 & 1 \end{bmatrix}. \tag{18}$$

The polarization estimation accuracy depends on the number of antenna elements M, but is independent from DOA, signal wavelength and array geometry. Furthermore, the signal polarization can be determined with a single dual-polarized array element. The first entry of (18) reveals, that the phase difference β cannot be identified, if the signal is linearly polarized, i.e. $\gamma = 0$ or $\gamma = \frac{\pi}{2}$. Fig. 2 illustrates that for a linearly polarized signal, the polarization ellipse is uniquely described by the auxiliary polarization angle γ. Furthermore as expected, the polarization accuracy depends significantly on the CPR. If $\kappa = 1$, then the polarization is not observable. Indeed in this case, (17) reveals that the DOA is observable and even the DOA accuracy slightly increases, because the cross-polarization components cause a higher SNR. Note, especially for small array antennas a polarization mismatch leads to unpredictable DF results.

4. SIMULATION RESULTS

In this section, Monte Carlo simulations with 1000 trials have been carried out. The conventional beamformer described in Section 2.4 has been used to solve the estimation problem. A triangular array with $M = 3$ circularly dual-polarized elements is considered, where the first port is RHCP and the second port is LHCP. The array elements were located at $\mathbf{r}_m = \frac{\lambda}{3}(\sin(\frac{2\pi m}{3}), \cos(\frac{2\pi m}{3}), 0)^T$, $m = 1, ..., 3$. The sensor collects $K = 100$ samples per DOA estimate. A linearly polarized signal is considered with $\beta = 0$ and $\gamma = \frac{\pi}{4}$.

For an SNR = 0 dB and $\kappa = 0.4$, Fig. 3 depicts the simulated DF covariance and CRB ellipses for a set of DOAs placed in the quarter of the sensor field of view. The comparison of the covariance and bound ellipses reveals that the covariance ellipses lie close to the bound ellipses, i.e. the conventional beamformer is approximately efficient. Due to the nature of the antenna coordinates, the covariance ellipses become circles for the regarded array geometry. Thus, at the horizon these ellipses offer a high azimuth variance and a low elevation variance, whereas the covariance ellipse at $\varepsilon = 0°$ behaves in the opposite sense.

In Fig. 4 – Fig. 6 the signal impinges from the DOA $\alpha = 60°$ and $\varepsilon = 30°$. We have calculated the root mean square error (RMSE) and the square root of the corresponding CRB elements for both the DOA estimation accuracy (Fig. 4) and the polarization estimation

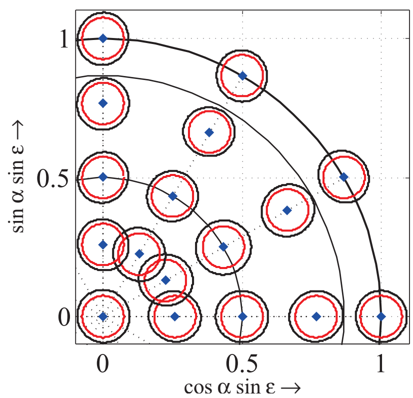

Figure 3: Simulated (black lines) and theoretical (red lines) uncertainty ellipses with true DOA (blue dots)

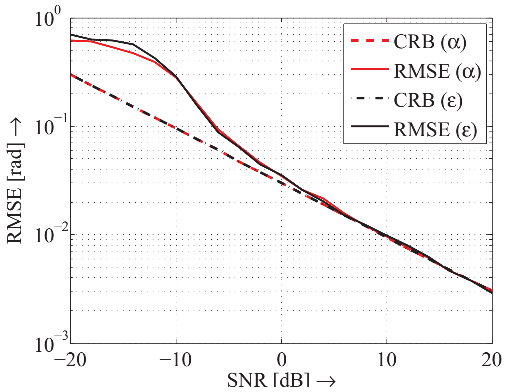

Figure 4: DOA estimation accuracy versus SNR

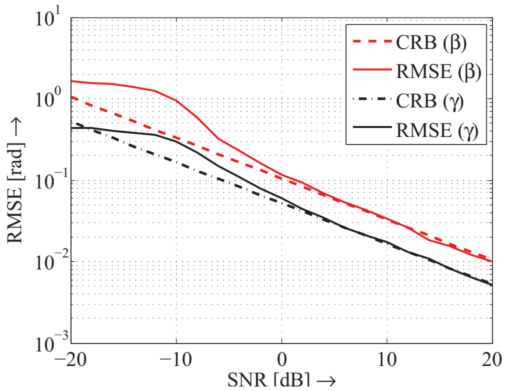

Figure 5: Polarization estimation accuracy versus SNR

accuracy (Fig. 5). For an SNR > 0 dB the conventional beamformer attains the CRB with the expected asymptotic performance, and for lower SNR, the RMSE deviates increasingly from the CRB and finally approaches a limit of $\pi^2/6$ or $\pi^2/24$ for the azimuth and elevation variance, respectively. This occurs when the DOA estimates are uniformly distributed in the visible angular region. Similar limits can be found for the polarization variances.

For an SNR = 0 dB, Fig. 6 shows the simulation result for a varying CPR. The polarization estimation variances lie close to the CRB across the entire range of κ. As expected from (18), the CRB increases with growing CPR, but a value of $\kappa \leq 0.3$ is still acceptable. Typically, linearly polarized antennas have a smaller CPR than circularly polarized antennas. For example, a dual-polarized array antenna with $\kappa > 0.3$ leads to significant DOA and polarization errors.

5. CONCLUSION

In this paper, a realistic array model for dual-polarized array elements has been introduced considering the CPR of the elements. The CRB has been derived analytically for the single source case, and has been interpreted. The presented simulation results exhibit that especially the polarization accuracy depends on the CPR. In further studies, we will investigate the DOA and polarization estimation bias that results from differences between the

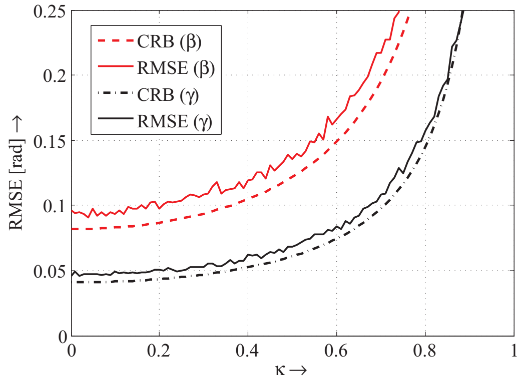

Figure 6: Polarization estimation accuracy versus CPR

real array model (6) and the assumed array model (9).

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