ON WAVEFORM DESIGN IN INTERFERENCE-TOLERANT RANGE-DOPPLER ESTIMATION FOR WIDEBAND MULTISTATIC RADARS

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ABSTRACT

The problem of waveform selection for range-Doppler estimation in wide-band multistatic radars is addressed. The transmitted signal is assumed to be cyclostationary, and cyclostationarity properties are exploited for range-Doppler estimation in severe noise and interference environments. Simulation studies are carried out to investigate waveform parameter selection for better estimation performance.

1. INTRODUCTION

The detection and parameter estimation problem for a moving source by a two-sensor passive radar is addressed in the most general case in [5]. This problem is equivalent to the multistatic-radar problem with two receiver sensors when the transmitted signal is not available for processing. Several detection-estimation structures have been derived for a moving source/target scenario when the relative radial speeds between receiver sensors and source/target can be considered constant within the observation interval and the so-called narrow-band condition is satisfied. Such a condition holds when the product of transmitted signal bandwidth and observation interval is much smaller than the ratio of the medium propagation speed to the target radial speed. In such a case, the Doppler effect in the received signals can be modeled as a frequency shift and the time scale factor in the complex envelope can be assumed unity [14]. Both auto and cross-statistics of signals on the two sensors are involved in the locally optimum (i.e., for low signal-to-noise ratio (SNR)) detector-estimator in [5]. For each range-Doppler radar cell, under the narrow-band condition, a suboptimum detector-estimator is provided in [5]. The time-difference-of-arrival (TDOA) and frequency-difference-of-arrival (FDOA) of the signals on the two sensors are estimated by locating the maximum of the magnitude of the narrow-band cross-ambiguity function (NB-CAF) of the signals on the two sensors, provided that the peak exceeds a threshold which depends on the desired probability of false alarm and the test statistic distribution under the null hypothesis.

When large transmitted-signal bandwidths or large data-record lengths are adopted, then the narrow-band condition could not be satisfied and the range-Doppler estimation is performed by resorting to the wide-band cross-ambiguity function (WB-CAF) [8], [12]. In such a case, for each range-scale radar cell, a suboptimum detector-estimator consists in locating the peak of the magnitude of the WB-CAF of the signals on the two sensors and comparing it with an appropriate threshold.

Techniques based on the peak location of the NB-CAF or WB-CAF provide poor performance in severe noise and interference scenarios where interfering signals partially or completely overlap with the useful signal in both time and frequency domains.

Interference-tolerant algorithms for narrow-band signal detection and estimation of TDOA and FDOA that provide satisfactory performance have been proposed by exploiting the cyclostationarity properties of the involved signals. Almost-cyclostationary (ACS) signals are an appropriate model for almost all modulated signals adopted in communications, radar, sonar, and telemetry. They exhibit autocorrelation function which is a periodic or almost-periodic function of time [4]. An extensive analysis of cyclostationarity based techniques for TDOA estimation is presented in [2]. The problem of jointly estimating delay and Doppler is addressed in [6] in the narrow-band case by exploiting the joint cyclostationarity of transmitted and received signals. The detection-estimation problem addressed in [5] is applied in [3] to weak cyclostationary signals emitted by a stationary source (no Doppler). All the above mentioned cyclostationarity-based techniques assume that the narrow-band condition holds.

In the paper, in the wide-band scenario, an interference-tolerant cyclostationarity-based technique for estimating the time-scale ratio (TSR), FDOA, and TDOA of two signals impinging on two sensors and generated by a moving source or reflected by a moving target illuminated by a radar is proposed. In order to exploit cyclostationarity in radar, the transmitted signal is modeled as a finite-time segment of an ACS signal. Therefore, since ACS signals are finite-power, relatively long pulse trains should be considered. In addition, in order to get satisfactory performance in low SNR and signal-to-interference (SIR) conditions, sufficiently long observation intervals should be adopted for estimation of cyclic statistics. Consequently, the narrow-band condition could not be satisfied and a wide-band model for the received signal must be considered.

In the case of relative motion between transmitter and receiver, if the transmitted signal is ACS, under the wide-band condition transmitted and received signals are not jointly ACS but, rather, jointly spectrally correlated (SC) [10]. Analogously, in the case of multistatic radar with two receiver sensors and target with constant relative radial speeds with respect to the sensors, the received signals are jointly SC. Thus, cyclostationarity-based cross-correlation techniques such those in [6] cannot be adopted for delay and Doppler estimation. Jointly SC signals have Loève bifrequency cross-spectrum with support contained in lines whose slope is non-unit and depends on the motion parameters. Thus, the support is unknown and hence cross-statistics cannot be reliably estimated [9]. For this reason, the optimum detection-estimation procedure which requires the computation of cross-statistics between the signals received on the two sensors [5] cannot be implemented in practice. In contrast, by following the approaches in [1] and [11], the proposed suboptimum detection-estimation technique is only based on the computation of auto statistics of the signals on each sensor. These signals are ACS and, hence, under mild assumptions consistent estimators for their statistical functions exist [4].

The problem of waveform selection for the transmitted signal in a multistatic radar with two receiver sensors is addressed via numerical experiments and its performance is compared with techniques based on NB-CAF and WB-CAF.

The paper is organized as follows. In Section 2 the transmitted signal model is described and its cyclostationarity properties briefly reviewed. The received signals are modeled in Section 3. The proposed cyclostationarity-based detection and range-Doppler estimation technique is described in Section 4. Numerical results are presented in Section 5 for the transmitted waveform design. Conclusions are drawn in Section 6.

2. TRANSMITTED SIGNAL

The complex envelope $x_T(t)$ (with respect to the carrier frequency f_c) of the transmitted signal is the pulse train

$$x_T(t) = \sum_{k=1}^{N_b} a_k\, q(t-kT_p) \tag{1}$$

with pulse-repetition period T_p, $\{a_k\}$ binary equiprobable i.i.d. random variables, and linear frequency modulated (LFM) pulse

$$q(t) = \mathrm{rect}((t-T_d/2)/T_d)\, e^{j\pi\gamma_c t^2} \tag{2}$$

where T_d is the pulse width and γ_c is the chirp rate. The signal $x_T(t)$ can be modeled as a finite-time segment of an infinite-length pulse-amplitude-modulated (PAM) signal

$$x(t) = \sum_{k\in\mathbb{Z}} a_k\, q(t-kT_p)\,. \tag{3}$$

That is, $x_T(t) = x(t)\,\mathrm{rect}((t-T/2)/T)$, where $T = N_b T_p$ is the total duration of the transmitted pulse train $x_T(t)$. In the following, we will refer to the transmitted signal $x(t)$ and, accordingly, we will consider an infinite-length received signal. Therefore, the finite-length signals available in practice will be interpreted as segments of these infinite-length signals.

A finite-power complex-valued signal $x(t)$ is said to be second-order wide-sense ACS if its second-order moments are almost-periodic functions of time [4]:

$$\mathrm{E}\left\{x(t+\tau)\,x^{(*)}(t)\right\} = \sum_{\alpha\in\mathscr{A}} R^{\alpha}_{xx^{(*)}}(\tau)\, e^{j2\pi\alpha t} \tag{4}$$

where $(*)$ denotes an optional complex conjugation and the cycle frequencies α range in the countable set $\mathscr{A}$ (depending on $(*)$) containing possibly uncommensurate elements. When the elements of $\mathscr{A}$ are all multiples of the same fundamental frequency $1/T_p$, then the function in (4) is periodic with period T_p and the signal is said to be cyclostationary. In [13], it is shown that both autocorrelation function $\mathrm{E}\{x(t+\tau)\,x^*(t)\}$ and conjugate correlation function $\mathrm{E}\{x(t+\tau)\,x(t)\}$ are necessary for a complete second-order wide-sense characterization of the complex-valued signal $x(t)$. The coefficients

$$R^{\alpha}_{xx^{(*)}}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\mathrm{E}\left\{x(t+\tau)\,x^{(*)}(t)\right\} e^{-j2\pi\alpha t}\,\mathrm{d}t \tag{5}$$

of the (generalized) Fourier series expansion in (4) are referred to as (conjugate) cyclic autocorrelation functions. By double Fourier transforming with respect to $t_1 = t+\tau$ and $t_2 = t$ both sides of (4) one obtains the Loève bifrequency spectrum [7]

$$\mathrm{E}\left\{X(f_1)\,X^{(*)}(f_2)\right\} = \sum_{\alpha\in\mathscr{A}} S^{\alpha}_{xx^{(*)}}(f_1)\,\delta(f_2 - _{(-)}(\alpha - f_1)) \tag{6}$$

where the Fourier transform $X(f)$ of $x(t)$ is assumed to exist with probability 1 (at least) in the sense of distributions. In (6), $\delta(\cdot)$ denotes Dirac delta, $_{(-)}$ is an optional minus sign linked to $(*)$, and the (conjugate) cyclic spectra $S^{\alpha}_{xx^{(*)}}(f)$ are the Fourier transforms of the (conjugate) cyclic autocorrelation functions $R^{\alpha}_{xx^{(*)}}(\tau)$. From (6) it follows that for ACS signals the support of the Loève bifrequency spectrum is contained in lines with slopes ± 1 and correlation exists between spectral components that are separated by quantities equal to the cycle frequencies (that belong to the countable set $\mathscr{A}$). In contrast, for wide-sense stationary signals, the set $\mathscr{A}$ contains the only element $\alpha = 0$, that is, second-order moments in (4) do not depend on t and distinct spectral components are uncorrelated.

The PAM signal defined in (3) is second-order cyclostationary. Expressions of its (conjugate) cyclic autocorrelation functions and (conjugate) cyclic spectra can be found in [4].

3. RECEIVED SIGNALS

In the case of relative motion between transmitter and receiver, if the relative radial speed can be assumed constant within the observation interval, then the transmitted signal $x(t)$ experiences a linearly time-varying delay. Thus, the complex envelope $y(t)$ of the received signal can be written as [14, pp. 240-242]

$$y(t) = b\,x(st-d)\,e^{j2\pi\nu t} \tag{7}$$

where b is the complex gain, d the time delay, s the time-scale factor, and ν the frequency shift. In (7), for stationary radar, in the case of moving target (modeled as a transmitting source with carrier frequency f_c') it results $d = R_0/(c+\mathrm{v})$, $s = c/(c+\mathrm{v})$, and $\nu = -(\mathrm{v}/(c+\mathrm{v}))f_c' = (s-1)f_c'$, where R_0 is the target range at $t=0$, v the relative radial speed between receiver and target, and c is the medium propagation speed.

In [14, pp. 240-242] it is shown that the time-scale factor s can be considered unity in the argument of the complex envelope $x(\cdot)$ provided that the "narrowband condition"

$$BT \ll c/|\mathrm{v}| = |s/(1-s)| \tag{8}$$

is satisfied, where B is the bandwidth of $x(t)$ and T is the length of the observation interval.

Let $x_0(t)$ be the complex envelope with respect to the carrier frequency f_c' of the signal transmitted by a source in relative motion with respect to two sensors and let $r_1(t)$ and $r_2(t)$ denote the received signals. If the relative radial speeds of the source with respect to the sensors can be assumed to be constant within the observation interval, then, in accordance with (7), by some straightforward calculation the received signals on the two sensors can be written as

$$r_1(t) = x(t) + n_1(t) \tag{9a}$$
$$r_2(t) = b\,x(s(t-\bar{\tau}))\,e^{j2\pi\nu t} + n_2(t)\,. \tag{9b}$$

In (9a) and (9b), denoting by b_i, s_i, ν_i, and $d_i = s_i\tau_i$, the complex gain, time-scale factor, frequency shift, and delay on the ith sensor $(i = 1,2)$,

$$s \triangleq s_2/s_1, \qquad \nu \triangleq \nu_2 - (s_2/s_1)\nu_1,$$
$$d = s\bar{\tau} \triangleq (s_2/s_1)\tau_2 - \tau_1, \quad b \triangleq (b_2/b_1)\,e^{j2\pi\nu_1((s_2/s_1)\tau_2-\tau_1)} \tag{10}$$

are TSR, FDOA, TDOA, and complex-gain ratio (CGR), respectively. In addition,

$$x(t) \triangleq b_1\,x_0(s_1(t-\tau_1))\,e^{j2\pi\nu_1 t} \tag{11}$$
$$s_2\bar{\tau} = s_2\tau_2 - s_1\tau_1 = d_2 - d_1\,. \tag{12}$$

4. CYCLOSTATIONARITY-BASED DETECTION AND RANGE-DOPPLER ESTIMATION

In this section, the proposed range-Doppler estimation method is presented. Let the transmitted signal $x_0(t)$ exhibit cyclostationarity with cycle frequency α_0 and conjugate cyclostationarity with conjugate cycle frequency β_0.

Accounting for the definition of $x(t)$ in (11), and taking the Fourier transforms of the (conjugate) cyclic autocorrelation functions $R^{s_i\alpha_0}_{r_ir_i^*}(\tau)$ and $R^{s_i\beta_0+2\nu_i}_{r_ir_i}(\tau)$, the following expressions for the (conjugate) cyclic spectra on the two sensors are obtained:

$$\begin{cases} S^{s_1\alpha_0}_{r_1r_1^*}(f) = S^{s_1\alpha_0}_{xx^*}(f) + S^{s_1\alpha_0}_{n_1n_1^*}(f) \\ S^{s_2\alpha_0}_{r_2r_2^*}(f) = \dfrac{|b|^2}{|s|}\,e^{-j2\pi\alpha_1 s\bar{\tau}}S^{\alpha_1}_{xx^*}\left(\dfrac{f-\nu}{s}\right) + S^{s_2\alpha_0}_{n_2n_2^*}(f) \end{cases} \tag{13}$$

$$\begin{cases} S^{s_1\beta_0+2\nu_1}_{r_1r_1}(f) = S^{s_1\beta_0+2\nu_1}_{xx}(f) + S^{s_1\beta_0+2\nu_1}_{n_1n_1}(f) \\ S^{s_2\beta_0+2\nu_2}_{r_2r_2}(f) = \dfrac{b^2}{|s|}\,e^{-j2\pi\beta_1 s\bar{\tau}}S^{\beta_1}_{xx}\left(\dfrac{f-\nu}{s}\right) + S^{s_2\beta_0+2\nu_2}_{n_2n_2}(f) \end{cases} \tag{14}$$

where $\alpha_1 = s_1\alpha_0$ and $\beta_1 = s_1\beta_0 + 2\nu_1$ are cycle frequency and conjugate cycle frequency, respectively, of $x(t)$, $S^{\alpha}_{x_0x_0^*}(f)$ and $S^{\beta}_{x_0x_0}(f)$ are the cyclic spectrum and the conjugate cyclic spectrum, respectively, of $x_0(t)$, and $S^{\alpha}_{n_in_i^*}(f)$ and $S^{\beta}_{n_in_i}(f)$ those of $n_i(t)$, $(i = 1,2)$.

The joint statistical characterization of $r_1(t)$ and $r_2(t)$ can be made by the Loève bifrequency cross-spectrum [7]. It can be shown that $r_1(t)$ and $r_2(t)$ exhibit a jointly spectrally correlated component with spectral supports contained in lines with slopes $_{(-)}s_2/s_1$ in the bifrequency plane.

In the addressed problem, parameters s_1 and s_2 are linked to the state of motion of the source, are unknown, and need to be estimated. Consequently, even in the noise-free case, the spectral cross-correlation density cannot be reliably estimated due to the lack of knowledge of the support lines [9]. For this reason, in the following the detection and estimation problem will be addressed by suboptimal techniques avoiding to use cross statistics but only using auto statistics. It is worthwhile to underline that the difficulty in estimation arises from the fact that the time-scale factors cannot be assumed unity due to the wideband model. In contrast, when s_1 and s_2 can be both assumed equal to 1, then $r_1(t)$ and $r_2(t)$ are jointly ACS and the techniques proposed in [6] in the case of Doppler shift and those in [2], [3] when the Doppler shift is absent can be utilized.

The parameters s_i, ν_i can be estimated starting from the technique proposed in [11]. Let

$$\lambda_{r_ir_i^{(*)}}(\alpha) \triangleq \int_{\mathbb{R}} \left|\widehat{S}^{\alpha}_{r_ir_i^{(*)}}(f)\right|^2 \mathrm{d}f \tag{15}$$

where $\widehat{S}^{\alpha}_{r_ir_i^{(*)}}(f)$, $(i = 1,2)$, denote the (conjugate) frequency-smoothed cyclic periodograms which are consistent estimators of the (conjugate) cyclic spectra obtained observing signals in $[0,T]$ [4].

Let us assume that the values of s_i and ν_i are such that for some $\Delta\alpha$ and $\Delta\beta$ the useful signal in $r_i(t)$, say $y_i(t)$, has only one cycle frequency in the set $J(\alpha_0,\Delta\alpha) \triangleq [\alpha_0 - \Delta\alpha/2, \alpha_0 + \Delta\alpha/2]$ and only one conjugate cycle frequency in the set $J(\beta_0,\Delta\beta)$, and, moreover $S^{\alpha}_{n_in_i^*}(f) = 0$ for $\alpha \in J(\alpha_0,\Delta\alpha)$ and $S^{\beta}_{n_in_i}(f) = 0$ for $\beta \in J(\beta_0,\Delta\beta)$. In the ideal case of perfect measurements (that is, for infinite data-record length and infinitely small spectral resolution so that $\widehat{S}^{\alpha}_{r_ir_i^*}(f) = S^{\alpha}_{r_ir_i^*}(f)$ and $\widehat{S}^{\beta}_{r_ir_i}(f) = S^{\beta}_{r_ir_i}(f)$ with probability 1 (w.p.1)), when $\alpha \in J(\alpha_0,\Delta\alpha)$ and $\beta \in J(\beta_0,\Delta\beta)$, accounting for (13) and (14) the statistics $\lambda_{r_ir_i^*}(\alpha)$ and $\lambda_{r_ir_i}(\beta)$ are different from zero w.p.1 only for $\alpha = \alpha_i \triangleq s_i\alpha_0$ and $\beta = \beta_i \triangleq s_i\beta_0 + 2\nu_i$, respectively. Thus, in the case of finite data-record length and spectral resolution, the following estimates for α_i and β_i can be considered:

$$\widehat{\alpha}_i = \arg\max_{\alpha\in J(\alpha_0,\Delta\alpha)} \lambda_{r_ir_i^*}(\alpha) \tag{16}$$
$$\widehat{\beta}_i = \arg\max_{\beta\in J(\beta_0,\Delta\beta)} \lambda_{r_ir_i}(\beta)\,. \tag{17}$$

In accordance with [3], the detection problem, *separately on each sensor*, can be formulated by one of the two alternative hypothesis tests (H_1 = $y_i(t)$ present, H_0 = $y_i(t)$ absent)

$$\lambda_{r_ir_i^*}(\widehat{\alpha}_i) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_i^{(1)} \quad i = 1,2 \tag{18}$$
$$\lambda_{r_ir_i}(\widehat{\beta}_i) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_i^{(2)} \quad i = 1,2 \tag{19}$$

where the thresholds $\lambda_i^{(1)}$ and $\lambda_i^{(2)}$ depend on the false-alarm probability and the noise and interference statistics. Note that, in general, the two detection tests (18) and (19) have different performance and these detection strategies are not optimum in the case of two-sensor receiver [3], [5]. In fact, the optimum detection test involves cross statistics between signals on the two sensors. However, these statistics cannot be consistently estimated when the time-scale parameters s_1 and s_2 are unknown [9]. Then, under H_1, accounting for (13) and (14), estimates of s_i and ν_i can be obtained by $\widehat{s}_i = \widehat{\alpha}_i/\alpha_0$

and $\widehat{\nu}_i = (\widehat{\beta}_i - \widehat{s}_i\beta_0)/2$. Consequently, estimates of the TSR $s = s_2/s_1$ and FDOA $\nu = \nu_2 - s\nu_1$ are given by $\widehat{s} = \widehat{\alpha}_2/\widehat{\alpha}_1$ and $\widehat{\nu} = (\widehat{\beta}_2 - \widehat{s}\widehat{\beta}_1)/2$. It is worthwhile to underline that the estimates of TSR and FDOA do not require the knowledge of α_0 and β_0.

From equations (13) it follows that the estimate of $\bar{\tau}$ can be obtained by minimizing with respect to γ the L^2-norm of the difference between $\widehat{S}^{\widehat{s}\widehat{\alpha}_1}_{r_2r_2^*}(f)$ and $\gamma\widehat{S}^{\widehat{\alpha}_1}_{r_1r_1^*}((f-\widehat{\nu})/\widehat{s})$. In the ideal case of perfect measurements ($\widehat{S}^{\widehat{\alpha}}_{r_2r_2^*}(f) = S^{\alpha}_{r_2r_2^*}(f)$ and $\widehat{S}^{\widehat{\alpha}}_{r_1r_1^*}(f) = S^{\alpha}_{r_1r_1^*}(f)$ w.p.1), according to (13) with $S^{\alpha}_{n_in_i^*}(f) \equiv 0$, the solution of the minimization problem is

$$\gamma^{(\mathrm{opt})} = |b|^2\, e^{-j2\pi s\alpha_1\bar{\tau}}/|s| \tag{20}$$

which corresponds to a zero L^2-norm. In the real case of finite data-record length, the L^2-norm can be minimized by locating its stationary point with respect to γ, with γ and γ^* considered as independent variables. This leads to

$$\gamma^{(\mathrm{opt})} \;=\; S_0\int_{\mathbb{R}} \widehat{S}^{\widehat{s}\widehat{\alpha}_1}_{r_2r_2^*}(f)\,\widehat{S}^{\widehat{\alpha}_1}_{r_1r_1^*}\left(\frac{f-\widehat{\nu}}{\widehat{s}}\right)^*\mathrm{d}f \tag{21}$$

with $S_0 > 0$, which can be shown to be a global minimum. Consequently, due to (20), the estimate of the TDOA is obtained as

$$\widehat{s\tau} = -\angle\left[\gamma^{(\mathrm{opt})}\right]/(2\pi\widehat{\alpha}_1)\,. \tag{22}$$

In addition, according to (12), an estimate for the difference $d_2 - d_1$ is given by $\widehat{s_2}\widehat{\tau}$.

5. NUMERICAL RESULTS

In this section, numerical experiments are conducted aimed at selecting waveform parameters and investigating the range-Doppler estimation performance of the proposed method.

The parameters of pulse $q(t)$ of the PAM signal $x_0(t)$, unlike otherwise specified, are $T_p = 64T_s$, $T_d = 12T_s$, and $\gamma_c = 0.01/T_s^2$, where T_s is the sampling period, and the a_k assume values ± 1 with equal probability. The propagation channels have parameters $s_1 = 1$, $\nu_1 = 0$, $\tau_1 = 0$, $s_2 = 1.01$, $\nu_2 = 0.0025/T_s$, and $\tau_2 = 3.4T_s/s_1$. Thus the true values of TSR, FDOA, and TDOA are $s = s_2$, $\nu = \nu_2$, and $d = s\bar{\tau} = s_2\tau_2$. On both sensors the disturbance is constituted by circular AWGN, independent on the two sensors, and an interfering PAM signal coming from a moving source. The SNR of the Gaussian noise in the bandwidth $[-1/2T_s, 1/2T_s]$ is 25 dB on both sensors. The interfering PAM signal on the first sensor has pulse $q(t)$, pulse-repetition period $T_{pI} = 30T_s$, pulse width $T_{dI} = 12T_s$, chirp rate $\gamma_{cI} = 0.01/T_s^2$, and has pulse amplitudes assuming values ± 1 with equal probability. The interfering PAM signal on the second sensor is the same as that on the first sensor except for TSR $s_I = 1.005$, FDOA $\nu_I = 0.00125/T_s$, and TDOA $s_I\bar{\tau}_I = 10T_s$. On both sensors, Gaussian noise and interference signal have power spectral density completely overlapped with that of the useful signal. However, by taking $\alpha_0 = \beta_0 = 1/T_p$, the useful received signals on the two sensors exhibit cycle frequencies $\alpha_i = s_i/T_p$ and conjugate cycle frequencies $\beta_i = s_i/T_p + 2\nu_i$ which are not shared with the disturbances. In the experiments, unlike otherwise specified, $\Delta\alpha = \Delta\beta = 1/(2T_p)$, $N_b = 2^7$ and $\mathrm{SIR} = -3$dB on both sensors.

The normalized sample root mean-square error (RMSE), computed by 100 Monte Carlo runs, of TSR estimate $\widehat{s}$, FDOA estimate $\widehat{\nu}$, and TDOA estimate $\widehat{d} = \widehat{s\tau}$ is evaluated. The performance of the proposed method is compared with that of the estimation methods based on the NB-CAF and WB-CAF. For SIR $<$ 0dB the proposed cyclostationarity-based method outperforms the classical method consisting in locating the maximum of magnitude of the WB-CAF. In fact, in the considered scenario, two peaks are present in the magnitude of the WB-CAF: the first due to the useful signal and the second due to the interference. When SIR $<$ 0dB, the peak due to the interference is higher than that of the useful signal so that the estimate based on the WB-CAF is biased. In contrast, when SIR $>$ 0dB the method based on the WB-CAF, which under some circumstances has almost-optimality properties [12], has better performance. Due to the wideband scenario, the performance of the method based on the NB-CAF is very poor.

In order to investigate the dependence of RMSE on the transmitted waveform parameters, N_b and T_d are assumed to be variable. The estimates' sample RMSE versus the number N_b of train pulses is considered (Figure 1). From these results it follows that when N_b is sufficiently large, then the signal selectivity property of the cyclostationarity-based technique is effective and the proposed method outperforms both methods based on the NB-CAF and WB-CAF. Results for variable T_d and $N_b = 2^7$ are reported in Figure 2. From these results it follows that for the proposed algorithm $T_d/T_s = 10$ is the best choice. In contrast, in this interference scenario, classical algorithms have performance practically independent of T_d. In fact, bias is the dominating term in their RMSE. Specifically, RMSE depends on the behavior of NB-CAF and WB-CAF in correspondence of the peak relative to interference which does not depend on T_d. Analogous results, not reported here for lack of space, can be found for performance as a function of the chirp rate.

6. CONCLUSION

Cyclostationarity properties of transmitted signals are exploited under the wide-band condition for TSR, TDOA, and FDOA estimation in multistatic radars with two receiver sensors. Simulation results have shown the tolerance of the proposed method to severe noise and interference environments where the disturbance signals overlap in both time and frequency domains with the useful signals. The dependence of the method performance on the transmitted signal parameters is investigated via simulation experiments.

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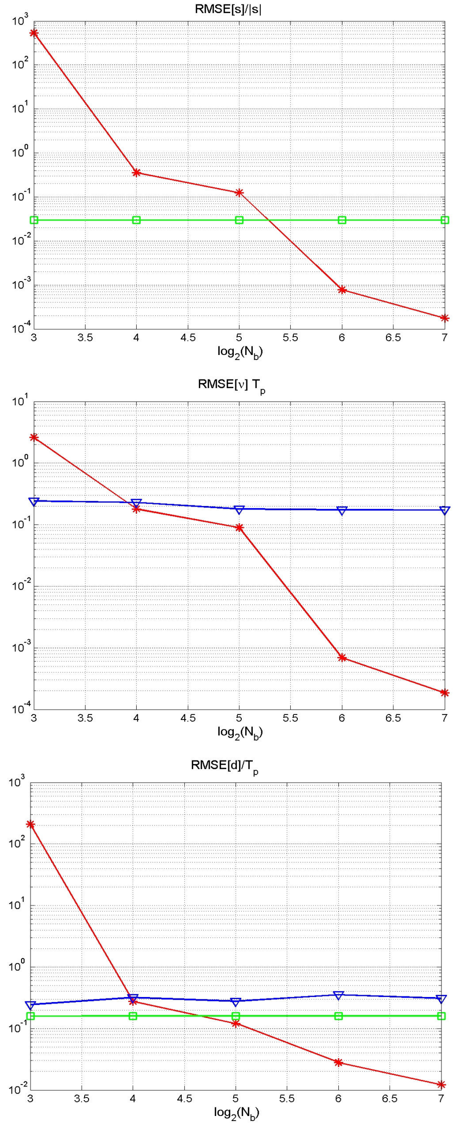

Figure 1: Normalized RMSE of estimated TSR, FDOA, and TDOA versus N_b. $(*)$ estimate by proposed method. $\square$ estimate by the wideband cross-ambiguity function. $\triangledown$ estimate by the narrowband cross-ambiguity function.

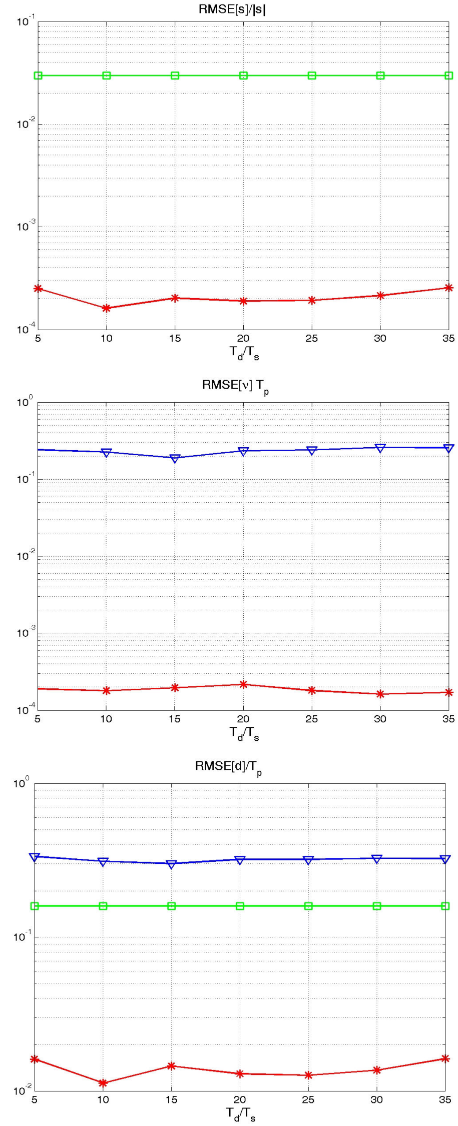

Figure 2: Normalized RMSE of estimated TSR, FDOA, and TDOA versus T_d. $(*)$ estimate by proposed method. $\square$ estimate by the wideband cross-ambiguity function. $\triangledown$ estimate by the narrowband cross-ambiguity function.

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