THE UNSCENTED KALMAN PARTICLE PHD FILTER FOR JOINT MULTIPLE TARGET TRACKING AND CLASSIFICATION

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ABSTRACT

The probability hypothesis density (PHD) is the first order statistical moment of the multiple target posterior density; the PHD recursion involves multiple integrals that generally have no closed form solutions. A (Sequential Monte Carlo)SMC implementation of the PHD filter has been proposed to tackle the issue of joint estimating the number of targets and their states. However, because the state transition does not take into account the most recent observation, the particles drawn from prior transition may have very low likelihood and their contributions to the posterior estimation become negligible. In this paper, we propose a novel algorithm named Unscented Kalman Particle PHD filter (UK-P-PHD), and associate it with Multiple dynamical Models (MM)method. The algorithm consists of a P-PHD filter that uses an Unscented Kalman filter to generate the importance proposal distribution; the UKF allows the P-PHD filter to incorporate the latest observations into a prior updating routine and thus, generates proposal distributions that match the true posterior more closely. Moreover, The MM solves the problem of tracking manoeuvring targets. Simulation shows that the proposed filter outperforms the P-PHD filter.

1. INTRODUCTION

In single target tracking scenarios, simple methods are usually used to estimate the target trajectory. However, in a multi-target case, where measurements may be generated by clutter and different existing targets at each time step, and where the number of targets changes in time, more efficient filters are needed to tackle these issues. Most multi-target tracking methods involve explicit associations between targets and available measurements. The Joint Probabilistic Data Association (JPDA) filter [1, 2], the Probabilistic MHT (PMHT) [3], and the multi-target particle filter [4] use observations weighted by their corresponding association probabilities. The Random Finite Sets (RFS) approach for multi-target tracking is an alternative to the association-based techniques [5, 6]. It is a Bayesian model for recursively estimating and updating a multi-target density function based on a set of received measurements at each time step. The Probability Hypothesis Density (PHD) filter [5] proposed by Mahler uses RFSs to estimate a multiple target state. However, the implementation of the PHD filter suffers from the problem of multiple integrals involved by the PHD recursion that have no closed form solutions; sequential Monte Carlo techniques [7, 8] have been proposed. The problem is that all these methods use the state transition which does not consider the recent observations to generate particles from. We propose, in this paper, a new algorithm that uses the Unscented Kalman (UK) filter to provide better proposal distributions similar to [9] named Unscented Kalman Particle PHD (UK-P-PHD) filter. A comparison between the proposed algorithm and the simple particle PHD filter is presented in a multiple motion model scenario (MM-UK-P-PHD filter). The single model PHD filter is presented in section 2, section 3 presents the multiple model PHD filter. In section 4, the proposed MM-UK-P-PHD filter is detailed, Simulation results are presented in section 5. Finally, concluding remarks are given in section 6.

2. THE PROBABILITY HYPOTHESIS DENSITY FILTER

Similarly to the single target case where the Kalman filter propagates the first order statistical moment of the single target posterior distribution, the PHD filter considers the same solution applied to the multi-target case. In fact, it propagates the first order statistical moment of the multiple target posterior distribution in time; the number of existing targets is estimated by integrating that moment over the state space. The PHD propagation is computed using a prediction step and an update step. The PHD prediction is given by the following equation [5, 8]

$$D_{k|k-1}(x) = \int \phi_{k|k-1}(x,\zeta)D_{k-1}(\zeta)d\zeta + \gamma_k(x) \qquad (1)$$

where

$$\phi_{k|k-1}(x,\zeta) = P_{S,k}(\zeta)f_{k|k-1}(x|\zeta) + \beta_{k|k-1}(x|\zeta)$$

is obtained from $f_{k|k-1}(x_k|x_{k|k-1})$, the single target transition density; $P_{S,k}(\bullet)$ is the probability of target survival, and $\beta_{k|k-1}(\bullet)$, the PHD for spawned target birth from targets at time k-1. The intensity $\gamma_k(\bullet)$ is the PHD for spontaneous birth of new targets at time k.

The PHD update equation is given by

$$\begin{aligned} D_k(x) &= [1-P_{D,k}]D_{k|k-1}(x) \\ &+ \sum_{z\in z_k} \frac{\psi_{k,z}(x)D_{k|k-1}(x)}{k_k(z)+\int \psi_{k,z}(\chi)D_{k|k-1}(\chi)d\chi} \end{aligned} \qquad (2)$$

where $P_{D,k}(\bullet)$ is the probability of detection, $k_k(z) = \lambda_k c_k(z)$ is the intensity of clutter points (λ_k is the Poisson parameter specifying the expected number of false alarms and $c_k(\bullet)$ is the probability distribution over the measurement space) and

$$\psi_{k,z}(x) = P_{D,k}(x)g_k(z|x)$$

where $g(\bullet)$ is the single target likelihood function and $P_{D,k}(\bullet)$ is the probability of detection.
The integration of the PHD over all surveillance region gives an estimation of the number of targets

$$\Gamma_{k|k} = \int_S D_k(x)dx$$

Then, the target states can be estimated by determining the $[\Gamma_{k|k}]$ largest peaks of $D_k(x)$ ($[\Gamma_{k|k}]$ stand for the nearest integer of $\Gamma_{k|k}$) .

3. MULTIPLE MODEL PHD FILTER FRAMEWORK

In the following, we suppose a simple case where each target follows a set of non linear Gaussian dynamical models,

$$x_k = F_{r_k,k-1}x_{k-1} + \upsilon_{r_k,k-1} \tag{3}$$

$$z_k = h_{r_k,k}(x_k, \varepsilon_{r_k,k}) \tag{4}$$

where x_k is the state of the target, $h_{r_k,k}$ is a non linear mode-dependant function (assumed to be known) and $\upsilon_{r_k,k}$ ($\varepsilon_{r_k,k}$) is a zero-mean mode-dependant gaussian process (measurement) noise with covariance matrix Q_{r_k} (R_{r_k}), and r_k is the model index parameter governed by an underlying Markov process with the model transition probability $\pi_k(\bullet|r_{k-1})$.
Furthermore, the survival and detection probabilities are state independent, $P_{S,k}(x) = P_{S,k}$ and $P_{D,k}(x) = P_{D,k}$.

4. MULTIPLE MODEL UK-PARTICLE IMPLEMENTATION OF THE PHD

The procedure of implementing the multiple model UK-P-PHD (MM-UK-P-PHD) algorithm is similar to [10] but modified to integrate the UK correction, and it works as follows.

- $k=0$: Generate N_z particles $\xi_0^{(i)}$ from each measurement (the position components are drawn from the initial measurement and the velocity components are drawn from a uniform distribution $U[-\upsilon_{max}, \upsilon_{max}]$) as in [11, 9]. Let N denote the global number of resulting particles. All these particles are affected with equal weights $\omega_0^{(i)} = K/N$ where K is the initial guess for the number of targets. The particles are characterized by a covariance matrix $P_0^{(i)}$ and an independent and identically distributed (IID) motion model parameter $r_0^{(i)}$ which indicates the particles motion model. The targets model probabilities are proportional to the number of particles representing each model.
- $k = k+1$
  Step 1: Prediction step
  The model is predicted for each particle by importance sampling from a proposal density $\pi_k(\bullet|r_{k-1}^{(i)})$ to get $r_{k|k-1}^{(i)}$ for $i = 1, ..., N$:.
  - $Ind = 0$,
  - For $i = 1, ..., N$:,
    * $\mu = \begin{bmatrix} \xi_{k-1}^{(i)} \\ 0 \\ 0 \end{bmatrix}$ and

$$C = \begin{bmatrix} P_{k-1}^{(i)} & 0 & 0 \\ 0 & Q_{r_{k|k-1}^{(i)}} & 0 \\ 0 & 0 & R_{r_{k|k-1}^{(i)}} \end{bmatrix}$$

    * Generate, from $\{\mu, C\}$, L sigma points $\{\alpha_{k-1}^l\}_{l=0}^L$ according to the unscented transform [12]

$$\alpha_{k-1}^l = \left[(x_{k-1}^{(l)})^T, (\upsilon_{k-1}^{(l)})^T, (\varepsilon_k^{(l)})^T, (W^{(m,l)})^T, (W^{(c,l)})^T\right]^T$$

    * Compute

$$x_{k|k-1}^{(l)} = F_{r_{k|k-1}^{(i)},k-1} x_{k-1}^{(l)} + \upsilon_{k-1}^{(l)}, l = 0, 1, ..., L$$

$$\bar{x}_{k|k-1}^{(i)} = \sum_{l=0}^{L} W^{(m,l)} x_{k|k-1}^{(l)}$$

$$P_{k|k-1}^{(i)} = \sum_{l=0}^{L} W^{(c,l)} (x_{k|k-1}^{(l)} - \bar{x}_{k|k-1}^{(i)})(x_{k|k-1}^{(l)} - \bar{x}_{k|k-1}^{(i)})^T$$

$$z_{k|k-1}^{(l)} = h_{r_{k|k-1}^{(i)},k}(x_{k|k-1}^{(l)}, \varepsilon_k^{(l)}), l = 0, 1, ..., L$$

$$\eta_{k|k-1}^{(i)} = \sum_{l=0}^{L} W^{(m,l)} z_{k|k-1}^{(l)}$$

$$S_k^{(i)} = \sum_{l=0}^{L} W^{(c,l)} (z_{k|k-1}^{(l)} - \eta_{k|k-1}^{(i)})(z_{k|k-1}^{(l)} - \eta_{k|k-1}^{(i)})^T$$

$$G_k^{(i)} = \sum_{l=0}^{L} W^{(c,l)} (x_{k|k-1}^{(l)} - \bar{x}_{k|k-1}^{(i)})(z_{k|k-1}^{(l)} - \eta_{k|k-1}^{(i)})^T$$

$$K_k^{(i)} = G_k^{(i)} (S_k^{(i)})^{-1}$$

$$P_{k|k}^{(i)} = P_{k|k-1}^{(i)} - G_k^{(i)} (S_k^{(i)})^{-1} (G_k^{(i)})^T$$

    * For each $Y_k^j \in z_k, j = 1, ..., n_{z_k}$ (n_{z_k} is the number of received measurement),

$$d = j + Ind \times n_{z_k}$$

$$\bar{x_k} = \bar{x}_{k|k-1}^{(i)} + K_k^{(i)} (Y_k^j - \eta_{k|k-1}^{(i)})$$

$$\xi_{k|k-1}^{(d)} \sim N(\bar{x_k}, P_{k|k}^{(i)})$$

$$P_{k|k}^{(d)} = P_{k|k}^{(i)}$$

$$\omega_{k|k-1}^{(d)} = \frac{\omega_{k-1}^{(i)}}{n_{z_k}} P_{S,k} N(z; Y_k^j - h_{r_{k|k-1}^{(i)},k}(\xi_{k|k-1}^{(d)}, 0), S_k^{(i)})$$

$$S_k^{(d)} = S_k^{(i)}$$

$$r_{k|k-1}^{(d)} = r_{k|k-1}^{(i)}$$

$$Ind = Ind + 1$$

Step 2: Additional Particle Proposal

- Let $N^{'} = N|z_k|$ be the number of all predicted particles,
- For each $Y_k^j \in z_k, j = 1, ..., n_{z_k}$, generate M_z new particles $\xi_{k|k-1}^{(d)}$ as in the initialisation step. The total number of particles proposed for the "investigation" of newborn targets, is $N_{new} = M_z|z_k|$. These ones are affected with equal weights $\omega_{k|k-1}^{(d)} = \gamma_k(S)/N_{new}$ for $d = 1+N^{'}, ..., N^{'}+N_{new}$ where $\gamma_k(S)$ is the PHD of target birth for the whole surveillance region. Particles are characterized by covariance matrices $P_{k|k-1}^{(d)}$ and IID parameters $r_{k|k-1}^{(d)}$.
- For $d = 1+N^{'}, ..., N^{'}+N_{new}$
  * $\mu = \begin{bmatrix} \xi_{k|k-1}^{(d)} \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} P_{k|k-1}^{(d)} & 0 \\ 0 & R_{r_{k|k-1}^{(d)}} \end{bmatrix}$
  * Generate, from $\{\mu, C\}$, L sigma points $\{\alpha_{k-1}^l\}_{l=0}^L$

$$\alpha_{k-1}^l = \left[(\hat{\xi}_{k|k-1}^{(l)})^T, (\varepsilon_k^{(l)})^T, (W^{(m,l)})^T, (W^{(c,l)})^T\right]^T$$

  * Compute

$$z_{k|k-1}^{(l)} = h_{r_{k|k-1}^{(d)},k}(\hat{\xi}_{k|k-1}^{(l)}, \varepsilon_k^{(l)}), l = 0, 1, ..., L$$

$$\eta_{k|k-1}^{(d)} = \sum_{l=0}^{L} W^{(m,l)} z_{k|k-1}^{(l)}$$

$$S_k^{(d)} = \sum_{l=0}^{L} W^{(c,l)}(z_{k|k-1}^{(l)} - \eta_{k|k-1}^{(d)})(z_{k|k-1}^{(l)} - \eta_{k|k-1}^{(d)})^T$$

$$G_k^{(d)} = \sum_{l=0}^{L} W^{(c,l)}(\hat{\xi}_{k|k-1}^{(l)} - \xi_{k|k-1}^{(d)})(z_{k|k-1}^{(l)} - \eta_{k|k-1}^{(d)})^T$$

$$K_k^{(d)} = G_k^{(d)}(S_k^{(d)})^{-1}$$

$$P_{k|k}^{(d)} = P_{k|k-1}^{(d)} - G_k^{(d)}(S_k^{(d)})^{-1}(G_k^{(d)})^T$$

Step 3: Update step

- For each $Y_k^j \in z_k, j = 1, ..., n_{z_k}$, Compute

$$C(Y_k^j) = \sum_{d=1}^{N^{'}+N_{new}} P_{D,k} g_{r_{k|k-1}^{(d)},k}(Y_k^j|\xi_{k|k-1}^{(d)})\omega_{k|k-1}^{(d)}$$

where $g_{r_{k|k-1}^{(d)},k}(Y_k^j|\xi_{k|k-1}^{(d)}) = N(z; Y_k^j - h_{r_{k|k-1}^{(d)},k}(\xi_{k|k-1}^{(d)}, 0), S_k^{(d)})$
- For $d = 1, ..., N^{'}+N_{new}$ (All particles), Compute

$$\omega_k^{(d)} = \left[1 - P_{D,k} + \sum_{Y_k^j \in z_k} \frac{P_{D,k} g_{r_{k|k-1}^{(d)},k}(Y_k^j|\xi_{k|k-1}^{(d)})}{\lambda_k c_k(Y_k^j) + C(Y_k^j)}\right] \omega_{k|k-1}^{(d)}$$

$$\xi_k^{(d)} = \xi_{k|k-1}^{(d)}, r_k^{(d)} = r_{k|k-1}^{(d)}$$

Step 4: Peak Extraction

- The updated PHD is approximated by a Gaussian Mixture using the expectation-maximization (EM) algorithm [13], The EM algorithm steps are as follow [11]:

$$\delta_{dm} \sim k_m \frac{1}{\sqrt{|2\pi\Omega_m|}} e^{-(\frac{1}{2}(\xi_k^{(d)}-\mu_m)^T \Omega_m^{-1}(\xi_k^{(d)}-\mu_m))} \quad (5)$$

$$\delta_{dm} = \frac{\delta_{dm}\omega_k^{(d)}}{\sum_{l=1}^{M} \delta_{dl}}, \mu_m = \frac{\sum_{d=1}^{N^{'}+N_{new}} \xi_k^{(d)} \delta_{dm}}{\sum_{d=1}^{N^{'}+N_{new}} \delta_{dm}} \quad (6)$$

$$K_m = \frac{\sum_{d=1}^{N^{'}+N_{new}} \delta_{dm}}{\sum_{d=1}^{N^{'}+N_{new}} \sum_{m=1}^{M} \delta_{dm}} \quad (7)$$

$$\Omega_m = \frac{\sum_{d=1}^{N^{'}+N_{new}} \delta_{dm}(\xi_k^{(d)} - \mu_m)(\xi_k^{(d)} - \mu_m)^T}{\sum_{d=1}^{N^{'}+N_{new}} \delta_{dm}} \quad (8)$$

where K_m denotes the weight, Ω_m the covariance and μ_m the mean of the m^{th} component of the mixture, for $m = 1, ..., M$ where $M = |z_k| + [\Gamma_{k|k}]$ is the number of Gaussians of the resulting mixture. For the EM algorithm initialisation, mean values are chosen as the measurement locations z_k and at the peaks of the previous scan, then, the algorithm repeats steps (5) through (8) until it converges.

Step 5: Resampling

- Particles are resampled using a Monte Carlo technique to get $\{\omega_k^{(j)}, \xi_k^{(j)}, P_{k|k}^{(j)}, r_k^{(j)}\}_{j=1}^{N_p}$, maintaining the weights's sum equal to $\Gamma_{k|k}$ given by:

$$\Gamma_{k|k} = \sum_{d=1}^{N^{'}+N_{new}} \omega_k^{(d)}$$

Furthemore, the same number of resampled particles is allocated to each model. After resampling, equal weight is affected to particles.
- Set $N = N_p$

5. SIMULATION

The MM-UK-P-PHD filter is used to track multiple and time varying number of targets evolving in cluttered environment. Two modes are considered, characterized by two models ($M = 2$), the constant velocity model (CVM) represented by its matrix $F_{1,k}$ and the constant turn rate model (CTRM) represented by its matrix $F_{2,k}$. Targets 1 and 2 are born at the same time but at two different locations. They follow the first model during $120s$ (their tracks cross at $k = 100s$) and then switch to the CTRM for $40s$. The model transition matrix is given by

$$\pi_{ij} = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}$$

Each target, with its state $x_k = [P_{x,k}, \dot{P}_{x,k}, P_{y,k}, \dot{P}_{y,k}]^T$, follows the state model given in (3) and (4) where

$$F_{1,k} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F_{2,k} = \begin{bmatrix} 1 & \frac{\sin(wT)}{w} & 0 & -\frac{(1-\cos(wT))}{w} \\ 0 & \cos(wT) & 0 & -\sin(wT) \\ 0 & \frac{1-\cos(wT)}{w} & 1 & \frac{\sin(wT)}{w} \\ 0 & \sin(wT) & 0 & \cos(wT) \end{bmatrix}$$

$$Q_1 = Q_2 = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_{v_x}^2 & 0 & 0 \\ 0 & 0 & \sigma_y^2 & 0 \\ 0 & 0 & 0 & \sigma_{v_y}^2 \end{bmatrix}$$

$$h_{r_k,k}(x_k, \varepsilon_{r_k,k}) = \begin{bmatrix} \sqrt{P_{x,k}^2 + P_{y,k}^2} \\ arctan(\frac{P_{x,k}}{P_{y,k}}) \end{bmatrix} + \varepsilon_{r_k,k}\,; r_k = 1,2$$

where $w = 0.2rd/s, T = 1s$ is the sampling period, $\sigma_x = \sigma_y = 5m$ and $\sigma_{v_x} = \sigma_{v_y} = 4(m/s)$ are the standard deviations of the process noise. The additive noises $\varepsilon_{1,k}$ and $\varepsilon_{2,k}$ are characterized by

$$R_1 = R_2 = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}$$

where $\sigma_r = 150m$ and $\sigma_\theta = 1°$. Moreover, the targets can appear with PHD of birth $\gamma_k(x) = 10^{-7}$. Finally, each target has survival probability $P_{S,k} = 0.99$ and is detected with probability $P_{D,k} = 0.98$.
The detected measurements are immersed in clutter that can be modeled as a Poisson RFS K_k with intensity [14]

$$k_k(z) = \lambda_c V u(z)$$

where V is the volume of the surveillance region, $\lambda_c = 12.5\text{x}10^{-6}m^{-2}$ is the average number of clutter returns per unit volume and $u(\bullet)$ is the uniform density over the surveillance region.
The number of particles is $N_z = 50$ for the initialisation and $M_z = 10$ for prospecting new appearing targets; that is a small number compared to [11], used in order to compare the standard MM-P-PHD filter and the proposed filter behaviours in situations where real time application is needed. The true targets trajectories and their MM-P-PHD filter and MM-UK-P-PHD filter estimates are presented in figure 1; figure 5 and figure 6 plot the estimated models for each target obtained with 100 Monte Carlo iterations, the Root Mean Square Errors (RMSE)(in m) of position estimates are given in figures 2 and 3. Results obtained using $N_z = 300$ particles and $M_z = 50$ for target 1 (results for target 2 are globally the same) are presented in figures 4 and 7.

As it can be seen from figure 1, both MM-P-PHD and MM-UK-P-PHD filters are able to track the two crossing targets even after motion model transition. However, figures 2 and 3 show that the proposed filter is better than the MM-P-PHD filter . In addition, the results shown in figures 5 and 6,

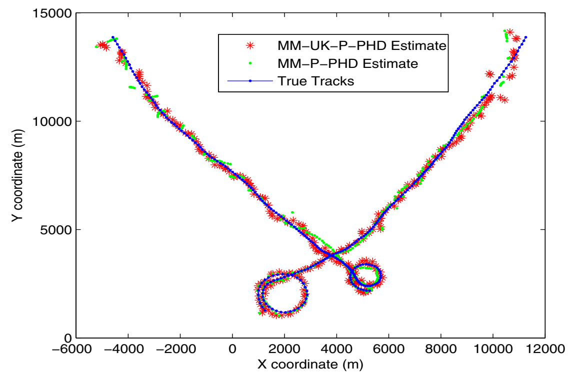

Figure 1: Estimated Trajectories

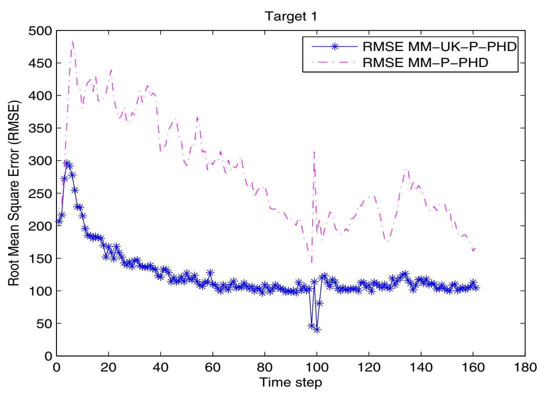

Figure 2: RMSE on Target 1 ($N_z = 50$, $M_z = 10$)

indicates that the new MM-UK-P-PHD filter estimates better the motion model than the simple MM-P-PHD filter. Figures 4 and 7 show that the performances of the proposed filter using $N_z = 50$ and $M_z = 10$ are equivalent to the performances of the standard MM-P-PHD filter using $N_z = 300$ and $M_z = 50$.

6. CONCLUSION

The problem of tracking multiple targets in cluttered environment is tackled in this paper, where the special case of crossing trajectories is considered. Moreover, the proposed UK-P-PHD filter is associated with a multiple model dynamic representation in order to better fit real applications where targets may suddenly change their motion model.The latter filter has also the ability to track multiple targets, correctly estimate the corresponding motion models with only a few number of particles using the integrated UK-correction loop. Simulation results show that it outperforms the standard MM-P-PHD filter which needs a great number of particles to achieve similar performances.

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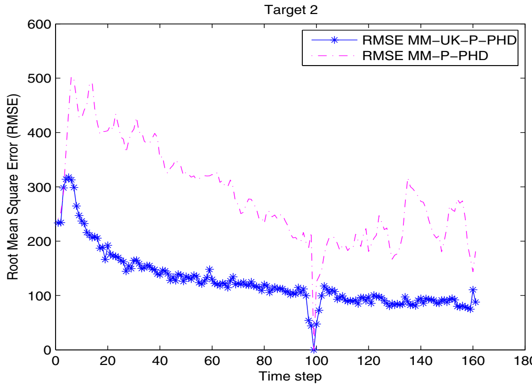

Figure 3: RMSE on Target 2 ($N_z = 50$, $M_z = 10$)

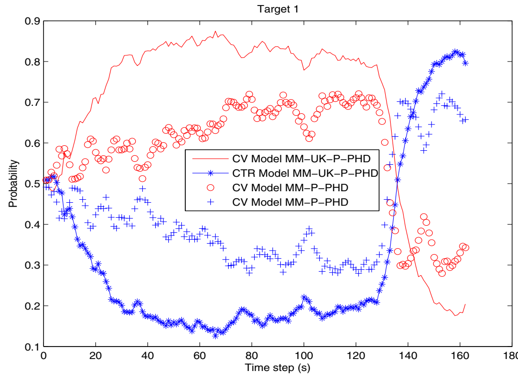

Figure 5: Estimated Models, target 1 ($N_z = 50$, $M_z = 10$)

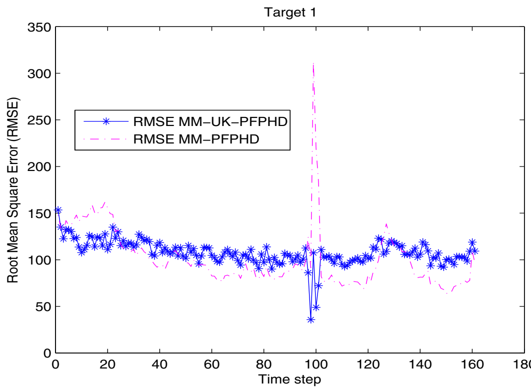

Figure 4: RMSE on Target 1 ($N_z = 300$, $M_z = 50$)

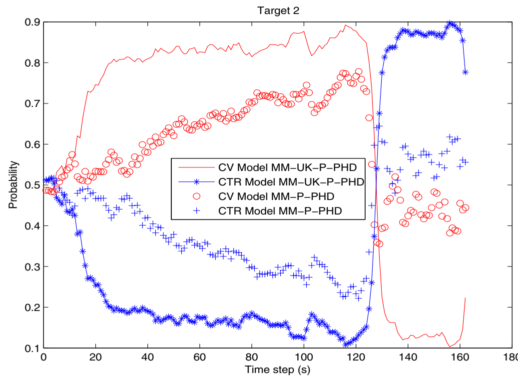

Figure 6: Estimated Models, target 2 ($N_z = 50$, $M_z = 10$)

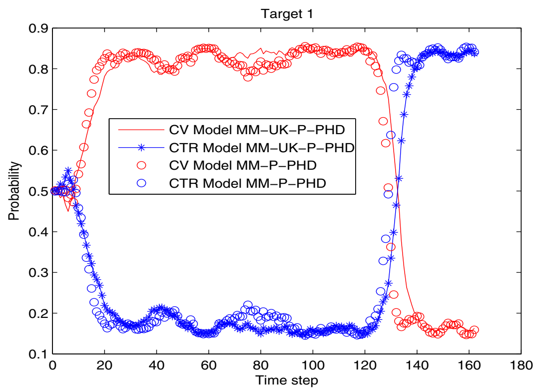

Figure 7: Estimated Models, tar.1 ($N_z = 300$, $M_z = 50$)

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