MIMO ADAPTIVE CODEBOOK FOR CLOSELY SPACED ANTENNA ARRAYS

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ABSTRACT

We discuss the design of a double structure codebook, in which the final precoding codeword is obtained as the product of codewords from two separate codebooks. The first codeword $\mathbf{W}_1$, representing long-term and wideband channel properties, limits the signal space and the second one $\mathbf{W}_2$, representing short-term and narrowband properties, allows to pick any vector within this space. We study optimization of both codebooks, and discuss low complexity codeword selection algorithms to allow increased quantization granularity for the first codeword. We show that in case of eight transmit antennas at the transmitter our optimized codebooks outperform the codebook in Release 10 3GPP LTE as well as the adaptive codebook of spherical caps.

1. INTRODUCTION

Closed-loop MIMO transmission schemes are currently widely used in wireless industrial standards. For example, single-user MIMO and multi-user MIMO schemes have been already adopted to 3GPP LTE, and coordinated MIMO is being considered for further releases of 3GPP LTE specifications. All these closed-loop MIMO schemes require precise channel state information feedback.

ITU-R has set extremely challenging peak and average spectral efficiency and data rate targets for any future wireless system targeted to become an IMT-Advanced system. To reach these targets, system bandwidth is increasing and new transmission schemes are being developed. Furthermore, the number of transmit antennas is increasing in order to enable high order MIMO schemes required for high spectral efficiencies.

However, the large number of antennas brings new challenges for closed-loop MIMO systems. The precoding space is growing with the increasing number of antennas while feedback remains limited to a small number of bits. Such large antenna arrays are typically implemented with closely spaced antennas, for example as uniform linear arrays (ULA) or cross-polarized (XP) linear antenna arrays, both of which have the advantage of being compact and therefore easy to deploy. The properties of such closely spaced arrays may be taken into account in the feedback design. Specifically, such arrays impose certain kind of spatial correlation effectively limiting the precoding space.

Several techniques have been developed that employ the spatial covariance matrix $\mathbf{R}$ or the long-term channel mean to adjust the codebook according to the correlation. In [1], the square-root of $\mathbf{R}$ is used to transform the codebook. The same approach has been used in [2], but the covariance $\mathbf{R}$ is estimated from the reported codewords instead of using the channel. These techniques produce channel dependent codebooks difficult to be utilized in practical standardized systems. A different approach has been introduced in [3], where a unitary matrix based on long-term channel mean is used to rotate a local codebook.

In this paper, instead of covariance matrix transformation or unitary matrix rotation, we feedback codeword $\mathbf{W}_1$ consisting of M beams such that the beams cover almost the full signal space over the wideband with closely spaced arrays, hence capturing the correlation properties of the channel. Then, the second codeword $\mathbf{W}_2$ combines the beams for each sub-band. This double structure codeword $\mathbf{W} = \mathbf{W}_1\mathbf{W}_2$ is simple and can adapt to angular spread of the channel by covering the instantaneous subspace over the entire band. Our contributions in this paper can be listed as follows:

1. We optimize both codebooks $\mathscr{C}(\mathbf{W}_1)$ and $\mathscr{C}(\mathbf{W}_2)$ in the double structure codebook.
2. We introduce a selection metric for $\mathbf{W}_1$
3. We propose a low-complexity selection algorithm for $\mathbf{W}_1$ to allow increased quantization granularity, i.e. larger number of codewords.
4. We derive optimal combiners $\mathbf{W}_2^{opt}$.
5. We show that the optimized codebooks outperform the current LTE double structure codebook [4] as well as the adaptive codebook [3] in 3GPP LTE single-user MIMO context.

2. SYSTEM MODEL

Consider a precoded OFDM MIMO system with N_t transmit and N_r receive antennas and unitary precoding.

With one subcarrier, the precoder takes as input an R-dimensional vector $\mathbf{x}$ of source symbols and multiplies it with an $N_\mathrm{t} \times R$ unitary matrix $\mathbf{W}$. On each of these R layers an independently modulated and coded data stream may be transmitted. Note that the same power is allocated to all layers due to the unitarity of the precoder.

The transmission equation for one subcarrier is given by

$$\underset{N_\mathrm{r}\times 1}{\mathbf{y}} = \sqrt{\frac{\mathscr{E}_s}{R}}\underset{N_\mathrm{r}\times N_\mathrm{t}}{\mathbf{H}}\ \underset{N_\mathrm{t}\times R}{\mathbf{W}}\ \underset{R\times 1}{\mathbf{x}} + \underset{N_\mathrm{r}\times 1,}{\mathbf{n}} \tag{1}$$

where $\mathbf{H}$ is the MIMO channel and $\mathbf{n}$ is the noise vector whose entries are zero-mean complex Gaussian random variables. For convenience, the dimensions of the matrices are marked in Eq.(1).

2.1 Channel model

We consider a calibrated uniform linear array with $\lambda/2$ spacing which is a subset of calibrated cross-polarized uniform linear array of double size. Both antenna settings are preferred by industry due to their compactness.

Closely spaced antennas imply correlation between neighboring antenna array elements. The actual correlation coefficient is a function of channel angular spread. The higher the angular difference between cluster beams coming from the base station, the lower the correlation coefficient.

In this paper we set the channel to ITU-R Urban Macro with no line of sight [5]. This channel has large angular spread and therefore it is the most challenging channel environment for ULA codebook design.

3. DOUBLE CODEBOOK - AN ADAPTIVE CODEBOOK

We design a double codebook for N_{t} transmit antennas with codewords $\mathbf{W}$ separated by product $\mathbf{W} = \mathbf{W}_1\mathbf{W}_2$, where $\mathbf{W}_1 \in \mathscr{C}(\mathbf{W}_1)$ is the coeword of dimension $N_{\mathrm{t}} \times M$ reported long-term wideband with periodicity $P \geq 1$ and $\mathbf{W}_2 \in \mathscr{C}(\mathbf{W}_2)$ is the codeword of dimension $M \times R$ reported short-term narrowband with periodicity $P = 1$. This implies limits on codebook sizes. Whereas $\mathscr{C}(\mathbf{W}_1)$ is bounded by the complexity of codeword selection, the $\mathscr{C}(\mathbf{W}_2)$ is limited by feedback overhead.

In LTE context, periodicity P stands for occurrence of an event in every P-th OFDM frame and sub-band b is typically a b-th continuous group of physical resource blocks PRBs within a system bandwidth.

The codebook defined above is an adaptive one where no long-term covariance matrix needs to be estimated and no codebook transformation is performed in real-time. This makes the double codebook a practical solution. With the first codeword $\mathbf{W}_1$ we cover the sub-space across the entire bandwidth of the system and with the second codeword we pick a proper precoder within this sub-space per each frequency sub-band. In this way, we are able to limit the overhead and keep the quantization error low.

3.1 Optimization of $\mathbf{W}_1$

As already mentioned, the purpose of the first codeword is to cover the signal space across the full band and at the same time limit the null space. With respect to $\lambda/2$ spacing we set the codewords in codebook $\mathscr{C}(\mathbf{W}_1)$ to sub-groups of discrete Fourier transform (DFT) unit vectors $\mathbf{B} = [\mathbf{b}_1, ..., \mathbf{b}_N]$ with

$$\mathbf{b}_n = \frac{1}{\sqrt{N_t}} \begin{bmatrix} 1 & e^{j\frac{2\pi n}{N}} & \dots & e^{j\frac{2\pi(N_t-1)n}{N}} \end{bmatrix}^{\mathrm{T}}, \tag{2}$$

where $N = 8, 16, 32$. We have selected the DFT vectors as basis, because the multi-path time domain channel between ULA array and one receive antenna may be well approximated as

$$\mathbf{h}(t) = \sum_{l=0}^{L-1} c_l \mathbf{b}(\alpha_l)\delta(t - \tau_l), \tag{3}$$

where $\mathbf{b}(\alpha_l) = \begin{bmatrix} 1 & e^{j\alpha_l} & ... & e^{j(N_t-1)\alpha_l} \end{bmatrix}^T$ is a beam of angle $\alpha_l \in (0, 2\pi)$ delayed by τ_l, c_l is a complex beam coefficient, $\delta(t)$ is a Dirac delta function, and L is the number of multipath components. As it will be shown later, the beams are sufficient enough to cover the most of the precoding space within the challenging urban macro channel model and thus as well within channels with lower angular spread.

Unfortunately, not all beam groups/codewords are orthogonal and it would be difficult to keep the final codeword $\mathbf{W}_1\mathbf{W}_2$ unitary, especially for all ranks $R > 1$. For this reason we extract the spatial basis using Singular value decomposition (SVD)

$$\mathbf{W}_1 = [\mathbf{U}(1)\mathbf{U}(0)]\Lambda\mathbf{V}^{\dagger} \tag{4}$$

and we set the codeword $\mathbf{W}_1(1)$ to its non-zero basis $\mathbf{W}_1(1) = \mathbf{U}(1)$. This procedure, further referred to as *orthogonalization*, guarantees semi-unitarity of the final precoder and projects $\mathbf{W}_2$ to the space of semi-unitary matrices.

The other option would be to restrict $\mathbf{W}_2$ codewords to e.g. beam selection vectors, which would guarantee the final precoder to be semi-unitary as in LTE codebook. However, we find the limiting of $\mathbf{W}_2$ as counter-productive and thus we will orthogonalize $\mathbf{W}_1$ codewords throughout this study.

3.2 Selection metric for $\mathbf{W}_1$

We select the optimal codeword $\hat{\mathbf{W}}_1$ as

$$\begin{aligned} \hat{\mathbf{W}}_1 &= \underset{\mathbf{W}_1(1)\in\mathscr{C}(\mathbf{W}_1(1)))}{\arg\max} \mathbf{t}_{\mathbf{W}_1} \\ \mathbf{t}_{\mathbf{W}_1} &= \textstyle\sum_b \mathrm{Tr}[(\mathbf{W}_1(1)\mathbf{W}_{2,b}^{opt})^{\dagger}\mathbf{R}_b\mathbf{W}_1(1)\mathbf{W}_{2,b}^{opt}] \end{aligned}, \tag{5}$$

where $\mathbf{R}_b = E[\mathbf{H}_b^{\dagger}\mathbf{H}_b]$ is a covariance matrix averaged over the instantaneous channel $\mathbf{H}_b$ corresponding to the entire sub-band b and $\mathbf{W}_{2,b}^{opt}$ is sub-band b and codeword $\mathbf{W}_1$ specific optimal combining vector. Having orthogonalized $\mathbf{W}_1$, the optimal $\mathbf{W}_{2,b}^{opt}$ is found as the subspace maximizing the power of $\mathbf{W}_1{}^{\dagger}(1)\mathbf{R}_b\mathbf{W}_1(1)$. In other words, setting $\mathbf{W}_{2,b}^{opt}$ to the first rank R singular vectors of the effective channel $\mathbf{H}_b\mathbf{W}_1(1)$. The metric in Eq. (5) maximizes the sum of squared singular values of the equivalent channel and such maximizes the power received by the user.

3.3 Channel coverage

Armed with the selection metric, we have measured the ULA ITU UMa NLOS channel rank $R = 1$ coverage for different 8Tx codebooks $\mathscr{C}(\mathbf{W}_1)$.These codebooks are sets of M-beam combinations from a DFT vectors of size N. The coverage C is defined as the gain using $\mathscr{C}(\mathbf{W}_1)$ codebook with optimal $\mathbf{W}_{2,b}^{opt}$ relative to the gain obtained by SVD precoding

$$C = \frac{\sum_b \mathrm{Tr}[(\hat{\mathbf{W}}_1\mathbf{W}_{2,b}^{opt})^{\dagger}\mathbf{R}_b\hat{\mathbf{W}}_1\mathbf{W}_{2,b}^{opt}]}{\sum_b \sum_{k=1}^{R} \lambda_k}, \tag{6}$$

where λ_k are ordered eigenvalues of $\mathbf{R}_b$ and R is rank of the codebook.

Table 1 shows that the coverage C_{full} grows as expected with increasing number of beams N as well as with the number of beams per codeword/group M. Unfortunately, increasing the number of beams per precoder M increases the precoding space for $\mathbf{W}_2$ and thus we find three beams per codeword $M = 3$ as a good compromise for further optimization of $\mathbf{W}_2$. Another issue is the number of groups which increases significantly the complexity of the selection. For this reason, we suggest not to loop over all groups, but rather scan for each of M beams in a group separately. One sub-optimal solution is the following

Algorithm (*in Matlab notation*):

- Find the grid of N DFT vectors $\mathbf{B}(:, 1 : N)$

Table 1: Coverage of the ITU UMa NLOS channel

M	N	Groups	Cov. C_{full}	Cov. C_{sub}
2	8	28	74.8%	75.0%
2	16	120	81.6%	81.4%
2	32	496	84.2%	82.7%
3	8	56	85.7%	85.7%
3	16	560	91.4%	91.1%
3	32	4960	93.1%	92.9%
4	8	70	92.1%	92.0%
4	16	1820	96.6%	96.3%
4	32	35960	97.5%	96.8%

- Set the first column of selector matrix $\mathbf{I}(:,1)$ to a vector of indexes $[1,...,N]'$.
- Loop precoder $\mathbf{W}_1$ vector index m from 1 to M.
  - Loop $\mathbf{B}$ vector index i from 1 to N.
    - Set i-th precoding codeword $\mathbf{W}_1(1)$ to *orthogonalized* set of vectors $\mathbf{B}(:,\mathbf{I}(i,:))$, selected by row vector $\mathbf{I}(i,:)$.
    - Compute selector metric $\mathbf{t}(i)$ using Eq.(5) for i-th codeword $\mathbf{W}_1(1)$ over all considered sub-bands.
    - Jump back to i loop.
  - Null the selector metric for all previously selected vector indexes $\mathbf{i}_{max}$, $\mathbf{t}(\mathbf{i}_{max}((m-1):1)) = 0$. With nulling we avoid selecting already selected vector.
  - Find the index $\mathbf{i}_{max}(m) = i_{max}$ maximizing metric $\max_i(\mathbf{t}(:))$.
  - Set m-th column of $\mathbf{I}(:,m)$ to $\mathbf{i}_{max}(m)$ and $\mathbf{I}(:,m+1)$ column to vector of indexes $[1,...,N]'$.
  - Jump back to m loop.
- Select precoder as $\hat{\mathbf{W}}_1 = \mathbf{B}(:,i_{max}(1:M))$ and orthogonalize it.

The rank $R=1$ coverage for the sub-optimal selection algorithm is shown in Table 1 as *Cov. C_{sub}*. The coverage is close to full selection coverage C_{full}. The complexity of $\mathbf{W}_1$ selection with exhaustive search is $\frac{N!}{(N-M)!M!}$ computations of Eq. (5), whereas with above algorithm the complexity can be decreased to MN computations being only $2M$ times more than Release 10 double codebook.

3.4 Optimization of $\mathbf{W}_2$

The $\mathbf{W}_2$ precoders are semi-unitary matrices of dimension (M,R) where R is a transmission rank and these semi-unitary matrices are points on the Grassmanian manifold $\mathscr{G}(M,R)$. Manifold can be displayed in three dimensions only if $M=2$. However for $R=1$ and arbitrary M, manifold can be displayed in 3D as $M-1$ sub-points on $M-1$ sub-manifolds $\mathscr{G}(2,1)$. The cross product of two codewords/points on $\mathscr{G}(M,R)$ manifold is equal to the product of cross products between sub-points on $M-1$ $\mathscr{G}(2,1)$ manifolds. Therefore the chordal distance between two codewords is as well proportional to a product of cross-products at the sub-manifolds.

We show an example of parametrization for $M=3$ and $R=1$, precoder $\mathbf{W}_2$

$$\begin{bmatrix} \mathbf{W}_2(1,1) \\ \mathbf{W}_2(2,1) \\ \mathbf{W}_2(3,1) \end{bmatrix} = \begin{bmatrix} \cos(\alpha_1) \\ \sin(\alpha_1)\cos(\alpha_2)e^{j\beta_1} \\ \sin(\alpha_1)\sin(\alpha_2)\cos(\alpha_3)e^{j\beta_2} \end{bmatrix} \tag{7}$$

and the two $\mathscr{G}(2,1)$ sub-manifolds are

$$\mathbf{s}_1 = \begin{bmatrix} \cos(\alpha_1) \\ \sin(\alpha_1)e^{j\beta_1} \end{bmatrix}, \mathbf{s}_2 = \begin{bmatrix} \cos(\alpha_2) \\ \sin(\alpha_2)e^{j(\beta_1-\beta_2)} \end{bmatrix}, \tag{8}$$

where the absolute value of cross product of two codewords $\mathbf{W}_2^{(1)}$ and $\mathbf{W}_2^{(2)}$, $\cos\angle(\mathbf{W}_2^{(1)},\mathbf{W}_2^{(2)}) = \cos\angle(\mathbf{s}_1^{(1)},\mathbf{s}_1^{(2)})\cos\angle(\mathbf{s}_2^{(1)},\mathbf{s}_2^{(2)})$. The result can be generalized for arbitrary M.

For completeness, Figures 1(a),1(b),1(c) and 1(d) show the distribution of optimal codeword $\mathbf{W}_2^{opt}$ on the sub-manifold for identical and independently distributed (IID) source and for the used ITU channel source with $\mathscr{C}(\mathbf{W}_1)$ codebook of parameters N=16 and M=3. The polar coordinates are mapped to Cartesian coordinates as

$$\begin{aligned} z &= 1/2\cos 2\alpha \\ x &= 1/2\sin 2\alpha\cos\beta \\ y &= 1/2\sin 2\alpha\sin\beta \end{aligned} . \tag{9}$$

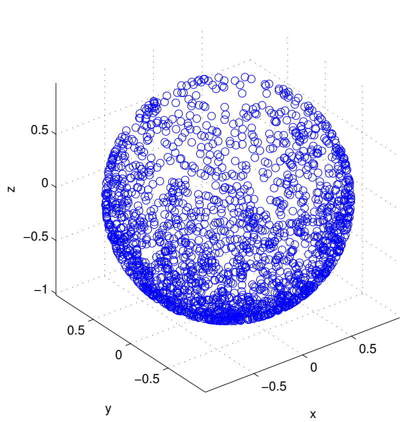
We can conclude from the figures that the phase (longitude) is equally distributed for IID as well as for ITU source, but the codewords of the ITU source are closer to the north pole. That means that the first coordinate of the precoder $\mathbf{W}_2$ is on average higher than the second one, which in turn is higher than the third coordinate in absolute values. This can be proven by measuring covariance matrix $\mathbf{C} = E[\mathbf{W}_2^{opt}\mathbf{W}_2^{opt\dagger}]$ which results in $\mathbf{C} = \text{diag}[0.33\ 0.33\ 0.33]$ for IID and $\mathbf{C} = \text{diag}[0.43\ 0.35\ 0.22]$ for ITU source. The ITU correlation is due to the orthogonalization performed on the $\mathbf{W}_1$ precoders.

Based on the analysis above, we may design optimal IID codebook $\mathscr{C}(\mathbf{W}_2)$ using geodesical line packing from [6] and then move the designed codewords $\mathbf{W}_2^i$ slightly towards the north pole using geodesic $\Gamma[\mathbf{W}_2^i,\mathbf{E}_0,p]$ in [6], where $\mathbf{W}_2^i$ represents i-th codeword, $\mathbf{E}_0 = [1\ 0\ 0]^T$ represents the north pole and p is empirically set to $p=0.15$. In case of rank $R=2$ transmission we set the north pole to $\mathbf{E}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$.

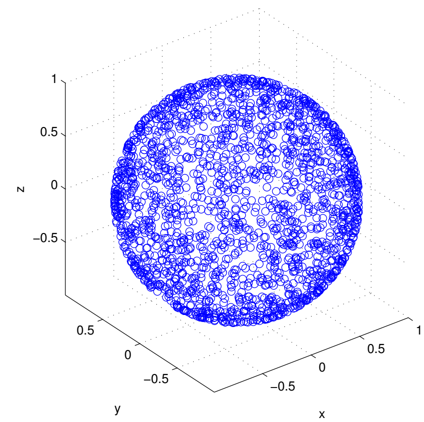
4. SIMULATION RESULTS

We have benchmarked the codebook in 10 MHz bandwidth 3GPP LTE link simulator. We schedule one user to total of 6 PRBs (physical resource blocks) located at both edges of the band and in the middle, each sub-band comprising 2 PRBs. Number of antennas $N_t = 8$. The $\mathbf{W}_1$ and $\mathbf{W}_2$ are both reported with periodicity of $P=5$ subframes being equal to the periodicity of channel estimation CSI-RS pilots.

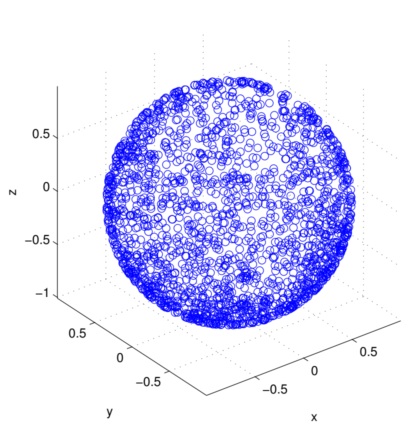
Figure 2 shows the comparison between $M=3$ and $M=4$ rank $R=1$ codebooks. It is clearly visible that $M=3$ codebook performs better with the same amount of feedback information. Codebook $\mathscr{C}(\mathbf{W}_1)$ has 7 bits and the codebook $\mathscr{C}(\mathbf{W}_2)$ has 4 bits for both codebooks. Despite the better coverage of $M=4$ codebook, the performance of $M=3$



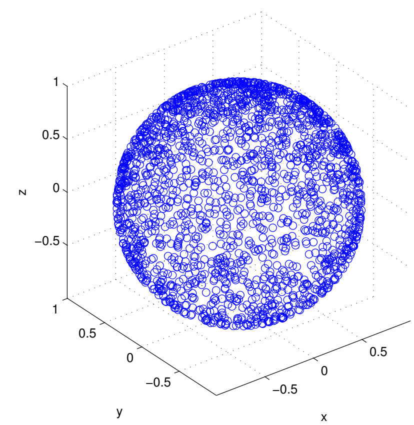
(a) Distribution of points on the 2x1 manifold s_1 for IID source



(b) Distribution of points on the 2x1 manifold s_2 for IID source



(c) Distribution of points on the 2x1 manifold s_1 for ITU channel source



(d) Distribution of points on the 2x1 manifold s_2 for ITU channel source

Figure 1: Optimal codeword $\mathbf{W}_2^{opt}$ distribution for ITU and IID source

codebook is better. This is due to the limited feedback of precoder $\mathbf{W}_2$. The $\mathbf{W}_2$ precoding space of $M = 3$ codebook is smaller and thus better covered with 4 bits than the $\mathbf{W}_2$ precoding space of $M = 4$ codebook.

Figure 3 shows the comparison of five rank $R = 1$ codebooks. The first one in the legend box refers to SVD codebook requiring infinite feedback and being theoretical maximum for single user SU-MIMO. The second one is the beam selection Release 10 8Tx LTE codebook ($\mathbf{W}_1 = 4$ bits,$\mathbf{W}_2 = 4$ bits) described e.g. in [7], the fourth one is a $M = 3$, $N = 32$ codebook using the proposed sub-optimal algorithm in Section 3.3 and the third one is the 7-bit codebook being a subset of $M = 3$, $N = 16$ codebook, where codeword's neighboring beams are at most distance of two from each other. An example of the most sparse codeword consists of 1,4,7-th beams from the DFT vectors $\mathbf{B}$. The fourth as well as the third codebook outperform beam selection Release 10 LTE codebook by 0.75-1dB with the cost of doubled overhead of $\mathscr{C}(\mathbf{W}_1)$. However $\mathbf{W}_1$ is reported only wideband and long-term and thus the feedback overhead is not an issue. The selection metric for the final precoder $\mathbf{W}$ is the chordal distance. The fifth codebook is an adaptive codebook from [3]. The wideband codebook $\mathscr{C}(\mathbf{W}_1)$ is set to N=32 DFT vectors $\mathbf{B}$ and the narrow band codebook $\mathscr{C}(\mathbf{W}_2)$ is a set of 4bit spherical caps obtained from four IID codebooks of size 5, 6, 7 and 8 bits obtained using geodesical packing in [6]. Thus, 2 wideband feedback bits are required to choose the most appropriate spherical cap. The codewords from wideband and narrowband codebooks are selected jointly maximizing the sum capacity across the band. The codebook shows poor performance, because uniformly quantized spherical caps are not able to adapt to correlation of the ULA channel.

Figure 4 shows rank $R = 2$ performance comparison of three codebooks 1,2 and 4 from Figure 3. Here, our codebook outperforms the Release 10 LTE codebook ($\mathbf{W}_1 =$ 4bit,$\mathbf{W}_2 =$ 4bit) by 2-3dB. The advantage compared to Release 10 LTE codebook comes from the better design of $\mathbf{W}_2$ codebook and larger and more flexible $\mathbf{W}_1$ codebook, as well as due to the fact that LTE rank $R = 2$ codebook is designed primarily for XP antenna array. The selection metric for the final precoder $\mathbf{W}$ was the post-processing throughput.

5. CONCLUSION

We have analyzed the double structure codebook and suggested its optimization for challenging high angular spread

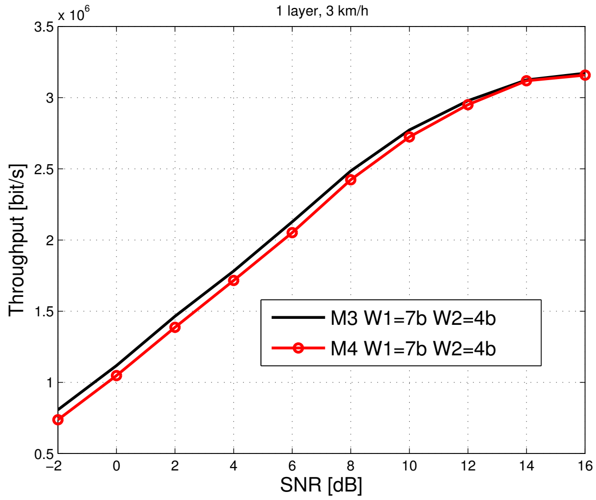


Figure 2: Comparison of $M = 3$ and $M = 4$ codebook with same amount of feedback

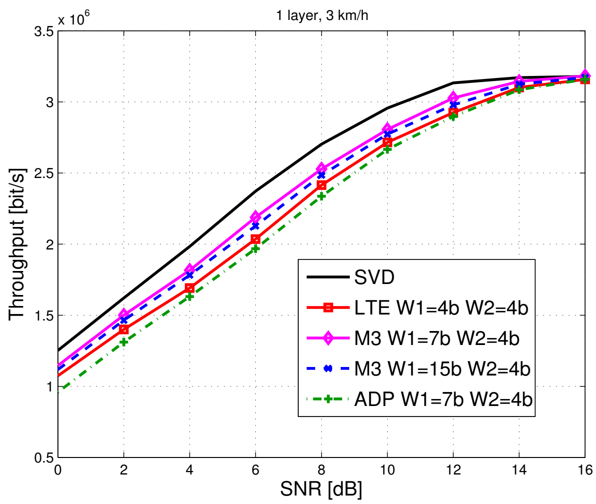


Figure 3: Comparison of $R = 1$ codebooks: 1, Optimal SVD codebook 2, Release 10 codebook 3, Sub-group codebook 4, Full-group, sub-optimal selection codebook 5, Adaptive codebook [3]

macro-urban channel and closely spaced antenna array. We showed that for rank one the three-beam codebooks perform better than the four-beam codebooks when only finite feedback is available. Moreover, we proposed a low complexity algorithm to employ wideband $\mathscr{C}(\mathbf{W}_1)$ codebooks covering 96.8% of the available gain. With realistic feedback we were able to provide performance 0.75-1dB below the SVD precoding and above the standardized LTE codebook for rank one, and 2-3dB below the SVD precoding and above the standardized LTE codebook for rank two. The advantage compared to Release 10 LTE codebook comes from the better design of narrowband $\mathscr{C}(\mathbf{W}_2)$ codebook and larger and more flexible $\mathscr{C}(\mathbf{W}_1)$ codebook. The increased size of the $\mathscr{C}(\mathbf{W}_1)$ codebook is not an issue, because its codewords are reported only wideband and with low periodicity. The benchmarked

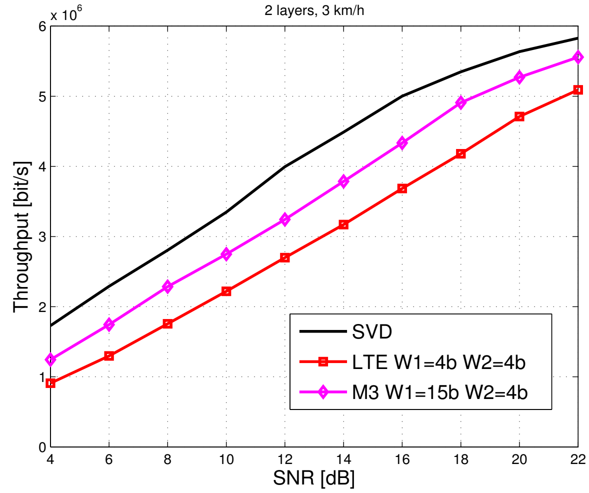


Figure 4: Comparison of $R = 2$ codebooks: 1, Optimal SVD codebook 2, Release 10 codebook 3, Full-group, sub-optimal selection codebook

adaptive codebook of spherical caps performed even worse than Release 10 LTE codebook, because spherical caps are not able to adjust to the correlation of ULA antenna array.

Acknowledgment

The authors acknowledge the support of Renesas Mobile Corporation and Foundation of Nokia Corporation, Finland.

REFERENCES

[1] D. J. Love and R. W. Heath Jr., "Limited feedback diversity techniques for correlated channels," *IEEE Trans. Veh. Tech.*, vol. 55(2), pp. 718 – 722, 2006.

[2] K. Amiri, D. Shamsi, B. Aazhang, and J.-R. Cavallaro, "Adaptive codebook for beamforming in limited feedback MIMO systems," in *Proc. Conf. Inf. Sci. Sys.*, Mar. 2008, pp. 994–998.

[3] V. Raghavan and R. W. Heath Jr., "Systematic codebook design for quantized beamforming in correlated MIMO channels," *IEEE J. Sel. Areas Comm.*, vol. 25(7), pp. 1298 – 1310, Sept. 2007.

[4] 3rd Generation Partnership Project; TSG RAN, "Physical channels and modulation," Technical Specification TS 36.211, Dec ver. 10.0.0 2010, available at `http://www.3gpp.org/`.

[5] Internation Telecommunication Union, "Guidelines for evaluation of radio technologies for IMT-advanced," Technical Specification ITU-R M.2135, Nov 2008, available at `http://www.itu.int/publ/R-REP-M.2135-2008/en`.

[6] K. Schober, P. Jänis, and R. Wichman, "Geodesical codebook design for precoded MIMO systems," *IEEE Comm. Lett.*, vol. 13(10), pp. 773 – 775, Oct. 2009.

[7] T. Shuang, T. Koivisto, H.-L. Määttänen, K. Pietikäinen, T. Roman, and M. Enescu, "Design and evaluation of LTE-advanced double codebook," in *Proc. IEEE VTC*, Apr. 2011, pp. 1–5.