OVERSTEERING OF END-FIRE ARRAYS WITH FREQUENCY-INVARIANT BEAM PATTERNS

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ABSTRACT

In this paper we propose and assess a technique to tune the trade-off between the directivity and the robustness of a frequency-invariant beam pattern, without the need to modify the designed FIR filters. Starting from a filter-and-sum beamformer over a superdirective array, synthesized by a stochastic and analytic method, which possesses the maximum directivity that the array may achieve and taking into account the probability density functions of the transducers' characteristics, we introduce a post-synthesis optimization that allows to choose a given system performance, after the investigation of a wide space of realistic possibilities. The a posteriori performance tuning will be referred to as oversteering and is effective only for end-fire array. By comparing the results of the proposed technique with those of the optimum synthesis, it is possible to evaluate the proposed method effectiveness in producing frequency-invariant beam patterns with an end-fire looking direction and a number of interesting trade-offs between directivity and robustness.

1. INTRODUCTION

Systems using sensor arrays are often involved in processing broadband signals. Often the performance of the array processor, i.e. the beamformer [1], should be adequately constant over the entire frequency band of the signals. Since the beamformer performance is mainly measured by the array response, a frequency-invariant beam pattern (FIBP) is required. A FIBP allows one to receive the broadband signals without any frequency distortion. Some papers have addressed the general structure of a broadband filter-and-sum beamformer to optimise the FIR filters in such a way that a FIBP is obtained [1-5].

However, in various kinds of applications [6-9] strong constraints are present on the maximum aperture of the array. As a consequence, the condition in which the array aperture, D, is shorter than some of the involved wavelengths, λ, is frequently unavoidable. In this case, the design of a superdirective array, achieved by synthesizing specific apodization functions [9], is essential, and the robustness to array imperfections and random errors becomes a very crucial point. A few approaches have been proposed [10-13] that can be used to synthesize the filters' coefficients necessary to yield a superdirective beamformer and assuring a sufficient robustness against errors in the array characteristics.

Recently, in [14] has been designed a synthesis to produce a data-independent frequency-invariant beamforming with the maximum directivity that the array may achieve, considering the effect of array imperfections. In this paper, adopting the algorithm described in [14], we introduce a technique to change *a posteriori* the beam pattern performance, in terms of directivity and white-noise gain (WNG), without modify the synthesized FIR filters coefficients. In fact, our technique can be seen as a post-synthesis optimization that allows to choose a given array response within a wide space of realistic possibilities. This method can be employed only to array with main response axis (MRA) at end-fire and will be referred to as *oversteering*. The idea is to steer the beam pattern past end-fire (i.e., to steer the beam pattern outside the visible region) and to achieve the desired performance without actually change the look direction of the array. The *oversteering* is obtained by applying opportune delays to the signals received by the array transducers, and for each steering direction past end-fire a different trade-off between directivity and robustness (measured by means of the white-noise gain) occurs.

The chance of obtaining near optimum performance by employing *oversteering* on a narrowband filter-and-sum beamformer is discussed in [16]. Unfortunately, for a broadband beamformer the beam pattern period changes with frequency, thus, in case of FIBP, the *oversteering* needs to be employed carefully. In particular, the period of the beam pattern for an aperture short with respect to λ exceeds the visible region. In such a case, while the beam pattern shifts gradually push the main lobe outside the visible region, on the other side, they introduce inside the visible region parts of the beam pattern previously placed outside this region. For this reason, in order to strictly keep the frequency-invariant profile of the array response, the FIR filters coefficients should be synthesized considering the beam pattern over an extended domain and not just the visible region. This requires to modify the synthesis method described in [14] and to accept a *starting solution*, i.e. the array response before the *oversteering*, with a sub-optimal performance.

This paper shows that a set of interesting performance trade-off, including the solution synthesized by [14], can be obtained by a given amount of *oversteering*.

This paper is organized as follows. Section II describes the method proposed to synthesize a superdirective frequency-invariant beamformer and the use of the oversteering technique to tune the array performance. Section III reports the results obtained on a very short microphone array working over a bandwidth equivalent to four octaves; finally, in Section IV, some conclusions are drawn.

2. PROPOSED METHOD

2.1 Filter-and-sum beamforming

In filter-and-sum beamforming, tapped delay line architectures are typically exploited to design a broadband spatial filter [10]. Usually the beamformer design lies on the hypothesis that the transducers characteristics are perfectly known. However, once the real array is considered, the performance is limited by the errors affecting the actual sensors.

Let us consider a uniform linear array composed of N omnidirectional, point-like sensors, each connected to an FIR filter composed of L taps. The far-field array response is a function of the direction of arrival (DOA) and of the frequency. The DOA is often expressed in the sine domain, by adopting the variable u, $u = \sin(\theta)$, $u \in [-1, 1]$. The interval $u \in [-1, 1]$ is referred to as the visible region of the beam pattern or the array response.

The beam pattern [1] can be written as function of u and expressed as follows:

$$BP(u, f, \mathbf{w}) = \sum_{n=0}^{N-1}\sum_{l=0}^{L-1} \mathrm{w}_{n,l} A_n \exp\left[-j2\pi \cdot f \cdot \left(\frac{n \cdot d \cdot u}{c} + l \cdot T_c\right)\right] \quad (1)$$

where f is the frequency, c is the speed of the acoustic waves in the medium, T_c is the sampling interval of the FIR filters, d is the inter-element spacing, and $\mathrm{w}_{n,l}$ represents the l-th tap coefficient of the n-th filter. The n-th actual transducer characteristic is represented by $A_n = a_n\exp(-\gamma_n)$, that includes the actual sensor gain a_n and phase γ_n, both of them supposed to be frequency-invariant.

In (1) the array response coincides with the beam pattern for $u \in [-1, 1]$, i.e. visible region. However, allowing the variable u to span over a wider interval, as shown in Fig. 1, one can observe that the beam pattern is a periodic function. The portions of the u axis that are outside the visible region are called virtual region, and the portions of the virtual region that are contained inside the beam pattern period centered in zero are called invisible region.

The value of the beam pattern period is equal to the ratio between the wavelength λ and the array inter-element spacing d; if the array has a short aperture, i.e. $D < \lambda$, the period of the beam pattern overcomes the visible region. Since the beam pattern period increase with λ, considering a broadband FIBP, the extended u region can be limited to the minimum period value, i.e. $\lambda_{\min}/d$. This avoids to overlap beam pattern replicas at different frequencies, and allows to employ the oversteering only in the optimized section of the broadband beam pattern.

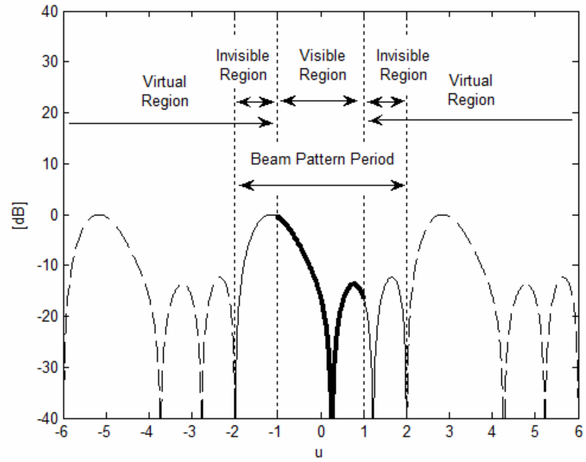

Figure 1 - Example of beam pattern (solid line) for an end-fire array with $d = \lambda/4$. The array response (bold solid line) corresponds to the visible part of the beam pattern. The central beam pattern period includes the invisible region and the beam pattern replicas (dashed line) are in the virtual region.

2.1.1 Synthesis

Our method adopts the synthesis proposed in [14]. The analytic solution of this cost function, produces a beamformer with the maximum directivity that the array may achieve and takes into account the statistics of the transducers' characteristics. We refer to this solution as to the *optimal solution*.

2.1.1 Extended domain of synthesis

With the aim of optimize *a posteriori* the array performance, we modify the synthesis in [14] as following. The FIR filters coefficients are synthesized considering the FIBP over an extended u domain, the borders of this domain are fixed by the higher considered frequency, i.e., by the minimum value of the beam pattern period. The cost function term that accounts for the directivity maximization is removed, and only the adherence term, expressing the difference between the actual beam pattern (ABP) and the desired beam pattern (DBP), is preserved. It follows that the synthesized beam pattern will have sub-optimal performance, with initial unbalanced trade-off between the directivity and the WNG. We refer to this beam pattern as to the *starting solution.*

Let P be the odd number of points used in discretizing the extended domain in the u axis, Q the number of points used in discretizing the frequency axis over the desired bandwidth, $BP_{pq}(\mathbf{w})$ the value of the ABP in u_p and f_q, computed by (1), applying the tap coefficients contained in the vector $\mathbf{w}$, and BPd_p the value of the DBP calculated in u_p for an arbitrary frequency (since the DBP is supposed to be frequency invariant it doesn't depend on the index q). Let be u_S the u value corresponding to the end-fire look direction, and let u_S organize the values of BPd_p, for $p = 1, 2, \ldots, S\text{-}1, S\text{+}1, \ldots, P$ into the vector $\mathbf{BP}d$ of length P–1. The DBP at the MRA is kept fixed at the normalized value 1. The adopted cost function is the following:

$$J(\mathbf{w}, \mathbf{BP}d) = \sum_{p=1}^{P}\sum_{q=1}^{Q}\left|BP_{pq}(\mathbf{w}) - BPd_p\right|^2 \cdot \quad (2)$$

Such a cost function expresses the adherence between ABP, calculated as in (1), and DBP, in a least squares sense, for all the frequencies and the u of interest. The minimization of this cost function produces both the DBP with end-fire MRA that best fit the extended u domain, and the filters' coefficients which assure the best adherence of the ABP to the DBP over the whole frequency band. Arranging the complex exponentials in (1), multiplied by the sensor characteristics A_n, in a row vector $\mathbf{g}(f, u)$ of length $M = NL$, and developing (2), one obtains:

$$J(\mathbf{w}, \mathbf{BP}d) = \sum_{p=1}^{P}\sum_{q=1}^{Q} \mathbf{w}\cdot\mathbf{g}_{p,q}^{\mathrm{T}}\mathbf{g}_{p,q}^{*}\cdot\mathbf{w}^{\mathrm{T}} + BPd_p^{\ 2} + -2BPd_p\mathbf{w}\cdot\mathrm{Re}\left\{\mathbf{g}_{p,q}^{\mathrm{T}}\right\}_p^2, \quad (3)$$

where the superscript $^{\mathrm{T}}$ denotes the transposed and the superscript * denotes the complex conjugate. Fixing the DBP value for u_{S} and considering that the second term in (3) accounts for the minimization of the DBP energy, the possibility of obtaining a flat beam pattern is avoided.

Considering small-size sensor arrays, the resulting beamformers are known to be highly sensitive to errors in the array characteristics, especially the sensor gain and phase. To overcome this drawback the strategy presented in [10] has been adopted. The idea is to optimise the mean performance using the probability density functions (PDFs) of the sensors' characteristics as weights in a weighted sum of cost functions in (3). To this end a total robust cost function $J^{tot}(\mathbf{w}, \mathbf{BP}d)$ can be defined as:

$$J^{tot}(\mathbf{w}, \mathbf{BP}d) = \int_{A_0}\dots\int_{A_{N-1}} J(\mathbf{w}, \mathbf{BP}d, A_0, \dots, A_{N-1}) f_A(A_0)\cdot \dots f_A(A_{N-1})dA_0\dots dA_{N-1}, \quad (4)$$

where $J(\mathbf{w}, \mathbf{BP}d, A_0, \dots, A_{N\text{-}1})$ is the cost function for a specific sensor characteristic set $\{A_0 \dots A_{N\text{-}1}\}$ and $f_A(A)$ is the probability density function of the stochastic variable $A = a\exp(-\gamma)$, i.e., the joint PDF of the stochastic variables a (gain) and γ (phase) related to a single sensor. Adopting the matrix notation introduced in [14] it is possible to express the robust cost function in (4) as a quadratic form and the global minimum can be found analytically.

2.2 Oversteering

The action of the *oversteering* in the proposed method takes the place of the directivity term in the cost function adopted in [14]. Actually both the synthesis are able to produce the desired system performance: the sythesis in [14] automatically produces an *optimal solution*, the proposed synthesis allows to select *a posteriori* the solution that best fit the performance requirements.

The actual performance of an array (i.e., the directivity and the white-noise gain) are evaluated by taking into account only the array response. The most important element of the array response is the main lobe, i.e., the lobe witch has the maximum value within the visible region. This peak value identifies the MRA of the array. In case of broadside array the main lobe is steered in the centre of the visible region, i.e. in $u = 0$, and the maximum of the beam pattern coincide with the MRA. Instead, an end-fire array has the MRA in $u = -1$ or $u = 1$, and only a section of the main lobe is visible in the array response, i.e., the maximum of the beam pattern could occurs outside the array response. The *oversteering* aim is to modify the beamformer performance by steering the beam pattern without change the MRA. It follows that the *oversteering* is efficient only if employed over an end-fire array.

The FIR filters of an end-fire beamformer centre the MRA in $u = -1$ or $u = 1$, instead, the *oversteering* technique inserts additional delays to gently move the main lobe outside the visible region, i.e., past end-fire.

Oversteering the beam pattern in (1) corresponds to subtract a value equal to u_{OS} to the variable u, as following:

$$BP(u, f, \mathbf{w}) = \sum_{n=0}^{N-1}\sum_{l=0}^{L-1} \mathrm{w}_{n,l} A_n \exp\left[-j2\pi\cdot f\cdot\left(\frac{n\cdot d\cdot(u-u_{OS})}{c} + l\cdot T_c\right)\right], \quad (5)$$

i.e., to apply delays to the signals received by the array transducers. For the n-th sensor the delay τ_n is:

$$\tau_n = n\frac{u_{\mathrm{OS}}\cdot d}{c}. \quad (6)$$

The possible delays in (6) are those that allow to keep at end-fire the maximum of the array response. Moreover, not all u values are suitable to obtain a frequency-invariant array response with sufficient robustness to array imperfection and enough directivity for efficient spatial filtering: shifting the main lobe, parts of the beam pattern, previously located in the invisible region (as shown in Fig. 1), are moved inside the visible region, and can deteriorate the array performance.

2.3.1 Array performance tuning

The beamformer performance can be derived from the directivity and the WNG. Both the system gains indicates the improvement in the signal-to-noise ratio provided by the array, as a single omnidirectional sensor: the directivity for an isotropic noise field and plane waves [9,15], and the WNG for sensor self-noise assumed to be spatially white [15]. The inverse of the WNG is called "sensitivity factor" and corresponds to the sensitivity of the beam pattern to the array imperfections (e.g., element response errors). Consequently, an excessive decrease in the WNG value cannot be accepted. For the beam pattern expressed in (5) the directivity versus frequency $G(f)$ can be expressed as follows [15]:

$$G(f) = \frac{\left|\sum_{n=0}^{N-1} H_n(f)\cdot S_1(f)\right|^2}{\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} H_n(f)\,H_m^*(f)\,\mathrm{sinc}\left[(n-m)\cdot 2\pi\cdot\frac{d}{\lambda}\right]\cdot S_{n-m}(f)}, \quad (7)$$

where $H_n(f)$ is the frequency response of the nth FIR filter, λ is the wavelength of the monochromatic wave, sinc(α) is defined as sin(α)/α, and the term $S_t(f)$ is defined as:

$$S_t(f) = \exp\left[-j2\pi f\cdot\frac{t\,d\cdot u_{OS}}{c}\right]. \quad (8)$$

In accordance with Eq.(5) the WNG versus the frequency $G_w(f)$ can be defined [16] as follows:

$$G_w(f) = \frac{\left|\sum_{n=0}^{N-1} H_n(f)\cdot S_1(f)\right|^2}{\sum_{n=0}^{N-1}\left|H_n(f)\right|^2}. \quad (9)$$

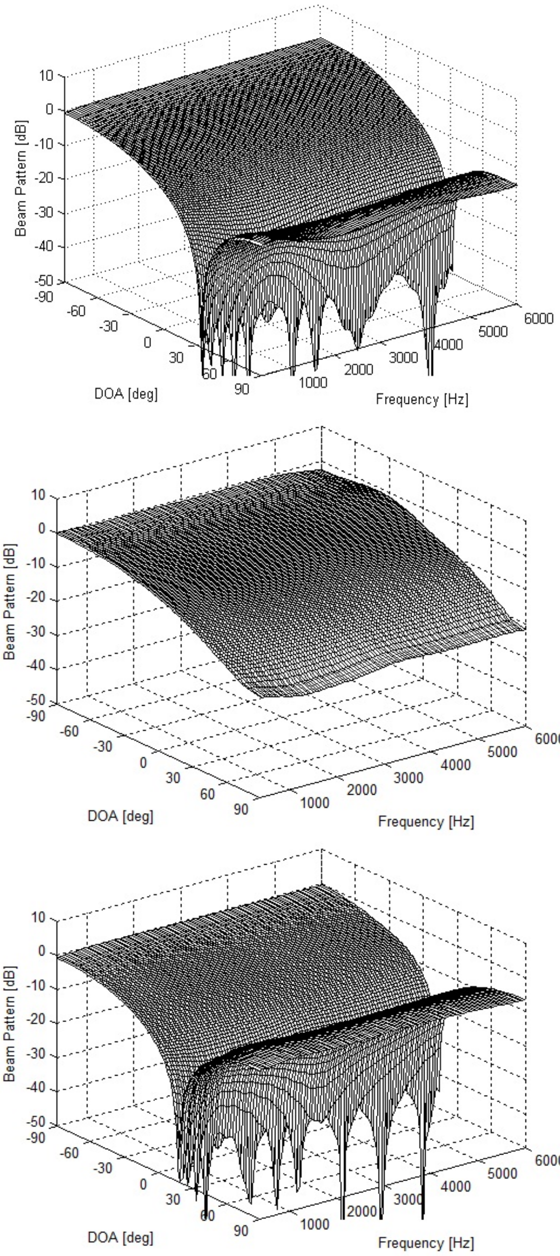

Figure 2. Array response for an uniform linear array with 6 microphones over an aperture of 8 cm. Optimal solution (up) and starting solution before (middle) and after (down) oversteering with $u_{OS} = -0.9$.

The directivity and WMG as defined in (7) and (9) state that the array performance depend on the actual amount of over-steering, i.e., the complex exponential in (8): steering the beam pattern produces a new directivity value, as well as a new WNG value.

3. RESULTS

As example of application of the proposed method, let us consider a uniform linear array, made up of 6 point-like omnidirectional microphones, with a spatial aperture of 8 cm. It has been designed to work in air where the sound speed is c = 340 m/s.

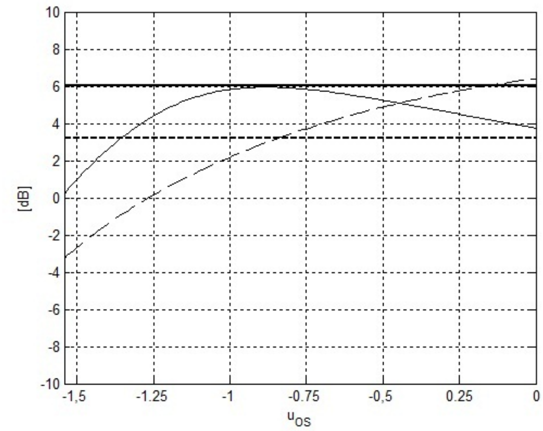

Figure 3. Performance tuning with oversteering. Directivity (solid line) and WNG (dashed line) trade-off for the synthesized solution at 1000 Hz. The corresponding values for the optimal solution are in bold line (without dependence by u_{OS}).

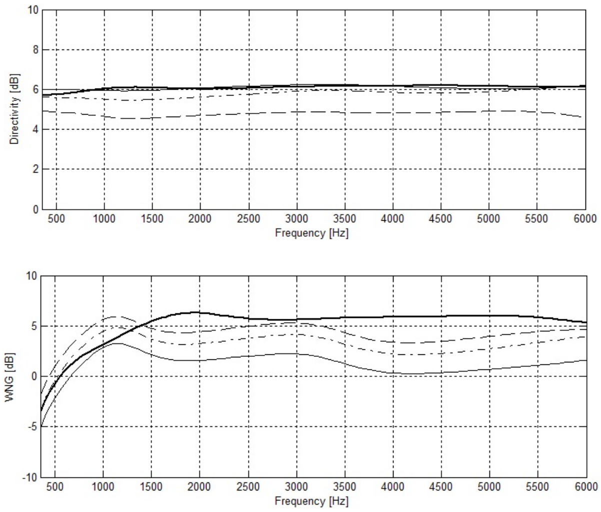

Figure 4. Directivity (up) and WNG (down) for oversteering values $u_{OS} = -0.3$ (dashed line), $u_{OS} = -0.6$ (dash-dotted line), $u_{OS} = -0.9$ (solid line) and for the optimum solution (bold solid line).

Each microphone feeds a 70th-order FIR filter (i.e., having L = 71 taps) with a sampling frequency equal to 12600 Hz. The frequency band considered to design a FIBP ranges from 350 to 6000 Hz (i.e., more than 4 octaves) and is discretized by using Q = 100 equally spaced points.

The MRA has been fixed at end-fire with $u_S = -1$.

The beam pattern period for f_{max} = 6000 Hz is equal to λ_{min}/d = 3.5417 and the extended region of synthesis was set to $u \in [-1, 2.5417]$, discretized by using P = 39 equally spaced points.

The PDFs of the microphone gain and phase are assumed to be Gaussian functions with a mean value respectively equal to 1 and 0 rad, and a standard deviation respectively equal to 0.05 and 0.05 rad. It is important to note that the array aperture is shorter than the wavelengths up to 4250 Hz, i.e., more than 3 octaves.

We have considered also the solution provided by the *optimal synthesis* for the same array, with weighting parameter set to 0.01. The beam pattern is shown in Fig.2 (up), it represents the optimal solution that we want to reproduce employing the *oversteering* over the *staring solution* shown in Fig.2 (middle). As anticipated the *starting solution* has sub-optimal performance: the beam pattern consists just on a broad lobe that cover the whole visible region.

3.1 Performance with oversteering

In order to optimize the performance of the synthesized *starting solution* the *oversteering* was employed. As effect of the *oversteering* the directivity and the WNG changed their values as shown in Fig.3 for f =1000 Hz. The performance of the starting solution in Fig.3 are the values for $u_{\mathrm{OS}} = 0$. By steering the beam pattern the main lobe becomes narrower, and the directivity increase reaching the value of the *optimal solution*. On the other hand, since the beam pattern maximum goes far from the visible region, the absolute value of the array response at the MRA is being reduced and the initial WNG decreases, assuming, in correspondence of the directivity peak, a value close to the *optimum solution*. One notes that a number of interesting trade-off between directivity and WNG can be selected.

As example of solution after oversteering we set $u_{\mathrm{OS}} = -0.9$. The beam pattern of the *starting solution* in Fig. 2 (middle) shifts, and the new array response is plot in Fig.2 (down), maintaining a FIBP. The new main lobe is sharper and a sidelobe, previously outside the visible region, appears on the opposite end-fire (i.e., $u = 1$). In Fig. 4 the directivity (up) and WNG (down) of solutions corresponding to the oversteering values $u_{\mathrm{OS}} = -0.3$, $u_{\mathrm{OS}} = -0.6$ and $u_{\mathrm{OS}} = -0.9$ are displayed and compared to the *optimal solution* performance.

4. CONCLUSIONS

In this paper a technique to tune the trade-off between the directivity and the robustness of a frequency-invariant beam pattern over a superdirective end-fire array, without the need to modify the designed FIR filters, has been described. Such a method is based on a data-independent filter-and-sum beamformer structure and takes into account the probability density functions of the transducers' characteristics. The synthesized beamformer has initial sub-optimal performance and is optimized *a posteriori* by steering the beam pattern within the visible region, i.e., selecting the desired array response. This post-synthesis optimization is referred to *oversteering* and is obtained by adding opportune time delays to the signals received by the array elements. By comparing the results of the proposed technique with the reference solution case, it was possible to evaluate the technique effectiveness in designing a beamformer with the desired performance. Moreover, the performance tuning technique is able to produce a number of interesting frequency-invariant beam patterns with different trade-off between directivity and robustness.

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