MULTIRESOLUTION DECOMPOSITION USING MORPHOLOGICAL FILTERS FOR 3D VOLUME IMAGE DECORRELATION

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ABSTRACT

The efficiency of multiresolution decomposition using morphological filters for 3D volume image decorrelation in lossless compression and computational complexity are evaluated. More efficient decorrelation is performed using morphological wavelets for multiresolution data representation than using Laplacian style pyramidal decomposition with morphological filters. Morphological wavelets are also faster in implementation than morphological pyramids. The computation time of the multiresolution decomposition implemented by morphological wavelets is 3 times shorter than the computation time of the wavelet decomposition with 5/3 filters used in JPEG2000 standard with similar decorrelation efficiency for lossless compression.

1. INTRODUCTION

Compression techniques can be classified as lossless and lossy. Lossless techniques allow exact reconstruction of the original image. Lossy techniques achieve higher compression ratios because they allow some degradation. For medical data, lossless compression is preferred as the image is not degraded and allows accurate diagnosis. Lossless techniques are important for seismic, satellite, biological data. The classical approach to lossless compression is decomposed into two steps: spatial decorrelation and entropy coding of the decorrelated signals. To achieve good compression it is desirable to decorrelate the image as much as possible. Neighbour voxels are highly correlated and a lot of information is redundant. Multiresolution volume representations remove correlation, redundant information is reduced and better compression can be achieved. Multiresolution representation of the image has been used for image coding, computer vision applications and progressive transmission.

Multiresolution representation can be redundant, like Laplacian pyramid and non-redundant, like wavelets [1, 2]. Video coding standards such as H.264 SVC support scalability using Laplacian pyramid [3]. Still-image coding standard JPEG2000 [4] and its extension for 3D volume images, Part10, JP3D [5], use Le Gall 5/3 biorthogonal filters in multiresolution wavelet scheme for lossless compression. Heijmans and Goutsias introduced nonlinear multiresolution transforms with morphological filters, both pyramids and wavelets [1, 2]. Mathematical morphology is non-linear theory for image processing based on set theory [3, 4]. It considers images as sets which permits geometry-oriented transformations of the images. The main advantages of morphological filters are its ability to preserve geometric structure and simplicity in implementation. The shape and the size of the structuring element determine which geometrical features are preserved in the filtered image. The performances of redundant morphological pyramids for 2D image decorrelation are presented in the paper [8]. Morphological wavelets are applied in video coding on residual frames [9]. Their suitability for hardware implementation is shown through FPGA implementation.

In this paper, the suitability of morphological filters in multiresolution schemes, both Laplacian type pyramids and non-redundant wavelets, for 3D volumetric image decorrelation as part of lossless compression is explored. The efficiency of decorrelation is calculated and the computation time of the decomposition is measured. The results are compared with the performance of multiresolution representation with 5/3 filters used in standard JPEG2000.

The paper is organized in 5 sections. Morphological 3D pyramids are presented in Section 2. Morphological wavelet decompositions using the lifting scheme are presented in Section 3. The experimental results illustrating the performances of the morphological pyramids and morphological wavelets for lossless compression are presented in Section 4 and the conclusion is in Section 5.

2. MORPHOLOGICAL PYRAMIDS

The Laplacian pyramid (LP) is one of the earliest examples of multiscale representation of visual data [10], Fig. 1. A coarse approximation of the original volume s is obtained by filtering the higher resolution volume and downsampling by 2 in all spatial dimensions. The detail signal d is calculated as the difference between the original signal x and the prediction p which is the interpolated upsampled version of the coarse signal. The coarse version signal can be decomposed further. The set consisting of the detail signals of all decomposition levels with decreasing spatial resolution and the coarse signal of the last decomposition level is referred to as the detail pyramid. For the linear case, the detail pyramid is called a Laplacian pyramid. The detail signal is highly decorrelated and LP can be represented with fewer bits than

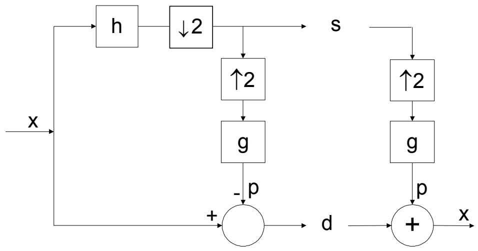

Figure 1 – One-level of Laplacian pyramid decomposition (left part) and reconstruction (right part) with linear h and g filters. If morphological filters h and g are used morphological pyramid is created

the original image. The original signal can be perfectly reconstructed summing the coarse signal prediction *p* and the detail signal *d*.

A drawback of the LP is implicit oversampling and the advantage is that each pyramid level generates only one detail image.

Morphological pyramid contains morphological reduce filter h and morphological expand filter g. The detailed study of the morphological pyramids is made by Heijmans and Goutsias [1]. The pyramid condition for morphological pyramid schemes states that synthesis of a signal followed by analysis returns the original signal and perfect reconstruction holds. Morphological pyramids are applied in multiresolution visualization of large volume data sets [11]. Multiresolution approach allow fast visualization of coarse version of the data in the preview mode which can be progressively refined.

For the signal *f* with domain *F*, $F \subseteq Z^d$ and structuring element $S \subseteq Z^d$, morphological operators dilation and erosion are defined:

$$D: \quad dilation_S(f)(x) = \max_{y \in S, x-y \in F} f(x-y)$$

$$E: \quad erosion_S(f)(x) = \min_{y \in S, x+y \in F} f(x+y)$$

Dilation and erosion are calculated as the maximum or minimum in the neighbourhood defined by structuring element. Morphological filters *opening, O* and *closing, C,* are defined as :

$$O: \quad open_S(f)(x) = dilation_S(erosion_S(f))(x)$$

$$C: \quad close_S(f)(x) = erosion_S(dilation_S(f))(x)$$

2.1 Morphological Haar pyramid

Morphological adjunction pyramid E/D involves the morphological operators erosion and dilation as reduce and expand filters. For the E/D pyramid the detail signals contain only nonnegative values as a consequence of the fact that the analysis and synthesis operators are adjunctions, advantageous for image compression [1].

When the structuring element used for morphological filtering is the cube of size 2x2x2 the morphological Haar pyramid is obtained. 1D and 2D morphological Haar pyramid is illustrated in the paper [1]. For the 3D volume data, coarse version signal *s* and prediction *p* are calculated as:

$$s(m,n,q) = \min\{x(2m+k, 2n+l, 2q+r), \mid 0 \le k,l,r \le 1\}$$

$$\begin{aligned} & p(2m,2n,2q) = p(2m,2n,2q+1) = p(2m,2n+1,2q+1) \\ & = p(2m+1,2n,2q+1) = p(2m+1,2n+1,2q+1) \\ & = p(2m,2n+1,2q) = p(2m+1,2n,2q) = p(2m+1,2n+1,2q) \\ & = s(m,n,q) \end{aligned}$$

2.2 Symmetrized Morphological Haar pyramid

When the structuring element used for morphological filtering in the adjunction E/D pyramid is the cube of size 3x3x3, symmetrized version of the morphological Haar pyramid is obtained. 1D and 2D symmetrized morphological Haar pyramid is presented in the paper [1]. Coarse *s* and prediction signal *p* of 3-dimensional non-separable symmetrized morphological Haar pyramid is calculated using the equations:

$$s(m,n,q) = \min\{x(2m+k, 2n+l, 2q+r), \mid -1 \le k,l,r \le 1\}$$

$$\begin{aligned} p(2m,2n,2q) &= s(m,n,q) \\ p(2m,2n+1,2q) &= \max(s(m,n,q), s(m,n+1,q)) \\ p(2m+1,2n,2q) &= \max(s(m,n,q), s(m+1,n,q)) \\ p(2m+1,2n+1,2q) &= \max(s(m,n,q), s(m,n+1,q), \\ & \quad s(m+1,n,q), s(m+1,n+1,q)) \\ p(2m,2n,2q+1) &= \max(s(m,n,q), s(m,n,q+1)) \\ p(2m,2n+1,2q+1) &= \max(s(m,n,q), s(m,n+1,q), \\ & \quad s(m,n,q+1), s(m,n+1,q+1)) \\ p(2m+1,2n,2q+1) &= \max(s(m,n,q), s(m+1,n,q), \\ & \quad s(m,n,q+1), s(m+1,n,q+1)) \\ p(2m+1,2n+1,2q+1) &= \max(s(m,n,q), s(m+1,n,q), \\ & \quad s(m,n+1,q), s(m+1,n+1,q), \\ & \quad s(m,n,q+1), s(m+1,n,q+1) \\ & \quad s(m,n+1,q+1), s(m+1,n+1,q+1)) \end{aligned} \tag{1}$$

2.3 O/D pyramid

In the O/D pyramid the analysis filter is the opening and the synthesis filter is the dilation. The cubic structuring elements of size 2x2x2 and 3x3x3 are used in the experiments. Sun and Maragos used O/D pyramid with square structuring element of size 3x3 for 2D images compression [12] .

2.4 Trivial morphological /D and /C pyramids

Trivial, reduced, pyramid is obtained by omitting analysis filter from the decomposition scheme. The coarse version signal is only downsampled higher resolution signal. This pyramidal representation contains the same number of samples as the original signal. The scheme of the reduced pyramid with linear filters for 2D image lossless compression is presented in [13].

Trivial morphological /D pyramid contains only synthesis filter dilation. From the equation (1) one out of 8 samples of the prediction signal *p* is equal to the sample of coarse version signal *s* and that sample is equal to the sample of higher resolution signal *x*. So the adequate sample of the difference signal is 0 and it is only needed to transmit 7/8 of the difference signals for 3D reduced morphological pyramid. This pyramidal representation is complete.

Trivial morphological /C pyramids contain only synthesis filters closing.

3. MORPHOLOGICAL WAVELETS

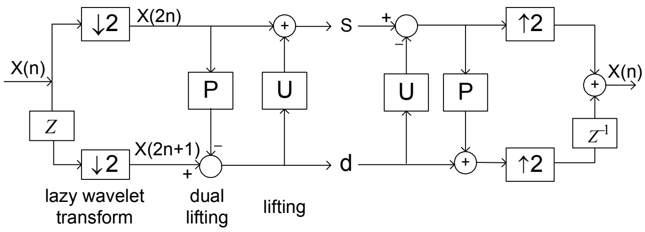

Figure 2 – The forward (left part) and inverse (right part) wavelet transform using lifting. P and U stand for prediction and update, respectively. For reversible morphological wavelet transformation quantizers aren't needed. For the integer to integer wavelet transformation quantizers are needed after prediction and update.

A major impulse to the development of nonlinear wavelet transforms has been given by the introduction of the lifting scheme. Daubechies and Sweldens showed that any DWT can be computed by the lifting scheme with reduced computational complexity compared with the standard filtering algorithm [14]. Lifting allows an in-place implementation of the wavelet transform meaning that the wavelet transform can be calculated without allocating auxiliary memory. In the lifting scheme, Fig. 2, trivial wavelet transform, lazy transform, is computed first. The input signal is split in two arrays, one containing even indexed samples and the other containing odd indexed samples. These two arrays are closely correlated and one array is used as the predictor for the other. In the dual lifting step the prediction and the detail signal *d* is calculated. At lifting step a filter is applied to the detail signal and the aproximation signal *s* is calculated. The built-in features of lifting scheme is that it is always invertible and leads to critically sampled perfect reconstruction filter bank. Inverse transform is calculated by reversing the operations and flipping the signs.

In the lifting schemes implementing integer wavelet transforms for lossless reconstruction the signal is rounded-off after prediction and update filtering and quantizers are needed. The decorrelation efficiency of integer wavelet decomposition of 2D images are presented in [15].

In the lifting schemes implementing morphological wavelet transforms, morphological filters are used in the lifting steps without round-off operation, Fig.2. These morphological lifting schemes don't need quantizers, suitable property for lossless compression.

The Haar wavelet transform is one of the simplest transformations from the space to the local frequency domain. An integer version of the Haar transform, the S transform, is the nonlinear modification of the Haar wavelet that maps integer onto integer-valued signals preserving the property of perfect reconstruction. In the lifting scheme the approximation signal *s* is calculated using the rounding operator $\lfloor\ \rfloor$:

$$d[n] = x[2n+1] - x[2n]$$

$$s[n] = x[2n] + \left\lfloor \frac{1}{2} d[n] \right\rfloor$$

3.1 Morphological Haar min wavelet transformation

One of the simplest example of nonlinear morphological wavelets is the morphological Haar wavelet which is very similar in structure to the linear Haar but uses nonlinear maximum and minimum operators rather than linear ones.

An example illustrating analysis and synthesis of morphological Haar min wavelet transform is presented in the papers [2, 16, 17]. The morphological Haar wavelet decomposition scheme preserves edges better as compared to linear case. The morphological Haar min wavelet decomposition is calculated as:

$$d[n] = x[2n+1] - x[2n]$$

$$s[n] = x[2n] + \min(0, d[n])$$

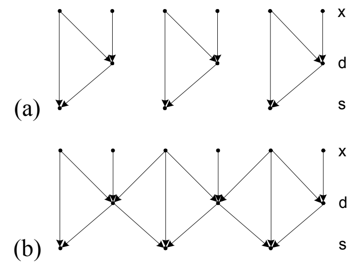

Figure 3 – The calculation of the detail signal *d* and the aproximation signal *s* from signal *x* using the lifting scheme for (a) Haar linear and Haar morphological wavelet transform (b) integer 5/3 and morphological min and max lifting wavelet transform.

3.2 Morphological Haar max wavelet transformation

Similarly, the dual morphological Haar max wavelet decomposition is calculated using the operator maximum in the lifting scheme:

$$d[n] = x[2n+1] - x[2n]$$

$$s[n] = x[2n] + \max(0, d[n])$$

The detail signal *d* is calculated in the same way for the classical integer Haar and the morphological Haar wavelet as the difference between odd and even samples of the signal *x*, Fig.3a. The sample of the aproximation signal *s* is calculated updating even sample from the signal *x* with the sample from the difference signal *d*, but using different operators: rounding the half of the sample from signal *d* for the integer Haar and calculating the minimum of the sample *d* and 0 for the morphological Haar wavelet transformation.

For lossless compression in JPEG2000 standard LeGall 5/3 filters are used in the lifting scheme. The forward equations for the reversible 5/3 wavelet transform are given by:

$$d[n] = x[2n+1] - \left\lfloor \frac{1}{2}(x[2n] + x[2n+2]) \right\rfloor$$

$$s[n] = x[2n] + \left\lfloor \frac{1}{4}(d[n-1] + d[n]) + \frac{1}{2} \right\rfloor$$

3.3 Morphological min lifting wavelet transformation

The detail *d* and aproximation signal *s* of the morphological wavelet transform by the min lifting scheme is calculated as:

$$d[n] = x[2n+1] - \min(x[2n], x[2n+2])$$

$$s[n] = x[2n] + \min(0, d[n], d[n-1])$$

3.4 Morphological max lifting wavelet transformation

The dual morphological wavelet transform is calculated using max lifting scheme:

$$d[n] = x[2n+1] - \max(x[2n], x[2n+2])\,.$$

$$s[n] = x[2n] + \max(0, d[n], d[n-1])\,.$$

An example illustrating analysis of morphological min lifting wavelet is presented in the papers [2, 17]. Min and max lifting schemes have the nice property that preserving local minimum and maximum of a signal respectively over several scales.

The samples of detail signal *d* are calculated from the same 3 samples of the signal *x* in the min or max lifting scheme and in the 5/3 lifting scheme, Fig. 3b, but using different operators. In the min or max lifting schemes, the sample of the detail signal *d* is calculated as the difference of the odd sample from signal *x* and the minimum or maximum of its 2 nearby even samples from signal *x* while in the integer 5/3 wavelet lifting scheme it is calculated as the difference between odd sample from signal *x* and the rounded average of its two nearby even samples. The sample of the aproximation signal is calculated updating the same even sample from the signal *x* with the same 2 samples from the detail signal in the min or max lifting scheme and in the 5/3 lifting scheme but applying different operators on the detail signal samples.

For the volumetric 3D data set of slices pixels are correlated in all three dimensions. 3D DWT is implemented in separable fashion, employing 1D transforms separately in the row, column and slice directions producing 7 detail subbands and aproximation signal. After one scale of decomposition along each direction, aproximation subband is decomposed further, leading to the dyadic decomposition. Wavelet decomposition is complete, producing the same number of samples in the subbands as in the original finest resolution signal. In this paper, the performances of morphological wavelet transforms for the 3D volume image decorrelation for lossless compression are presented.

4. EXPERIMENTAL RESULTS

Volume data sets used in the experiments are 8 bpp monochromatic 3D volume images: ‘Bonsai’, ‘Foot’, ‘Skull’ [18] and ‘Tooth’ [19]. The data sets are computed tomography data. For all data sets, the voxel samples are spaced 1 mm within each slice and the slices are 1 mm apart. Visualization images are calculated using software [20] which implement direct volume rendering technique, Fig.4. 3D data sets are decomposed using morphological pyramids and morphological wavelet transformations. The decorrelation efficiency of the multiresolution representation for lossless compression is calculated and the computation complexity is measured.

4.1 The decorrelation efficiency

The effeciency of the multiresolution representation for 3D data decorrelation is measured by the entropy, a measure of the achievable data compression.

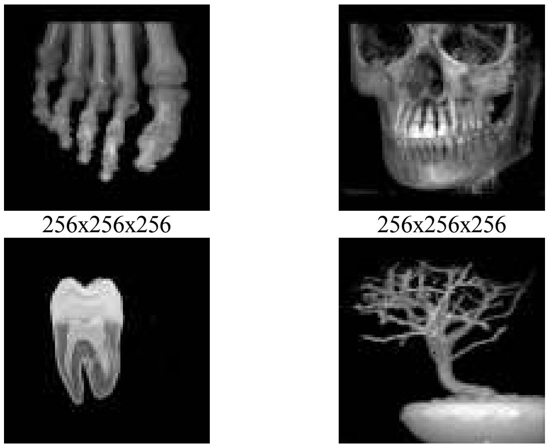

Figure 4 – Visualization images of 3D volume data sets: 'Foot', 'Skull', 'Tooth', 'Bonsai'.

The entropy of 2-level morphological pyramidal representation of 3D data sets are shown in Table I. Morphological pyramids E/D, O/D, /D, /C with cubic structuring elements S of size 2x2x2 and 3x3x3 are implemented. The best 3D data decorrelation is performed by E/D pyramid with structuring element of size 2x2x2 for data sets Foot and Tooth. For the data set Skull the best decorrelation is performed by trivial /C pyramid with S=3x3x3. For the data set Bonsai E/D and /C pyramids decorrelate the data equally. The lowest lossless decorrelation efficiency is performed by O/D pyramid with S=3x3x3.

The weighted entropies of two-level 3D DWT decompositions with 4 morphological and 2 integer wavelet transforms are shown in Table II. The most efficient morphological decorrelator for lossless reconstruction is min lifting. For data sets 'Foot' and 'Bonsai' min lifting is more efficient than 5/3 decorrelator 4% and 15% but for the data sets 'Skull', and 'Tooth' its efficiency is lower 9%, and 5% respectively.

For all data sets the morphological wavelets min lifting and morphological Haar min are more efficient for lossless representation than morphological pyramids. However, for the data sets Foot and Bonsai morphological pyramid E/D with S=3x3x3 is more efficient than the wavelet representation with 5/3 filters.

4.2 The comutation complexity

The morphological pyramids and wavelets have very low computational complexity as only minimum, maximum (comparisions) and addition of integers is involved in their computation.

The computation times of 2-level morphological pyramids using S=2x2x2 are shown in Table III. The fastest is the calculation of trivial /D pyramid and the slowest is O/D. The computation times of E/D and /C are similar.

The computation times of 2-level 3D DWT using for 4 morphological and 2 integer wavelet transforms are shown in Table IV. The shortest computation time is measured using morphological Haar wavelet decompositions and the slowest is integer 5/3 wavelet decomposition.

TABLE I

3D MORPHOLOGICAL PYRAMIDAL REPRESENTATION'S ENTROPY

anal/synth. filter		Foot	Skull	Tooth	Bonsai
E/D	S=2x2x2	**2.51**	4.27	**3.35**	**1.75**
	S=3x3x3	2.64	4.43	3.47	1.82
/C	S=2x2x2	2.56	4.24	3.4	**1.75**
	S=3x3x3	2.56	**4.21**	3.37	**1.75**
/D	S=2x2x2	2.68	4.5	3.5	1.98
	S=3x3x3	2.74	4.35	3.42	2.2
O/D	S=2x2x2	2.73	4.69	3.59	1.99
	S=3x3x3	2.85	4.85	3.7	2.1

TABLE II

3D WAVELET REPRESENTATION'S ENTROPY

DWT	Foot	Skull	Tooth	Bonsai
original data set	2.81	5.16	3.99	2.17
integer Haar	2.57	3.83	3.14	1.89
morph. Haar min	2.5	3.91	3.24	1.62
morph. Haar max	2.7	4.03	3.27	1.96
min lifting	**2.44**	3.79	3.17	**1.52**
max lifting	2.77	3.93	3.22	1.93
5/3	2.55	**3.4**	**2.99**	1.76

The computation time of 2-level 3D DWT using morphological min and max lifting scheme is 3 times shorter than the time using lifting scheme with 5/3 filters. The computation times of morphological pyramids are 2-2.5 times shorter than the computation time of the wavelet representation using 5/3 filters.

5. CONCLUSION

The performances of 3D volume image multiresolution decomposition, both redundant and non-redundant, using nonlinear morphological filters for lossless compression are evaluated. The implementation of morphological multiresolution decomposition is computationally very efficient as only integer arithmetic is used. More efficient decorrelation of 3D volume image data is accomplished using morphological wavelets than using morphological pyramids. The computation time of 3D morphological wavelet decomposition by morphological min or max lifting schemes is 3 times shorter than using the lifting scheme with 5/3 filters implemented in JPEG2000 standard while the efficiency of the decorrelation implemented by the two schemes are similar.

TABLE III

COMPUTATION TIME [S] OF 2-LEVEL 3D MORPHOLOGICAL PYRAMIDS ON THE SYSTEM WITH 1.83 GHZ AMD ATHLON XP PROCESSOR

analysis/sinthesis	foot	skull	tooth	Bonsai
E / D	2.42	2.8	1.61	2.37
/ C	2.58	2.76	1.64	2.55
/ D	1.42	1.52	0.91	1.48
O / D	3.64	4.22	2.44	3.53

TABLE IV

COMPUTATION TIME [S] OF 2-LEVEL 3D WAVELET REPRESENTATIONS ON THE SYSTEM WITH 1.83 GHZ AMD ATHLON XP PROCESSOR

DWT	foot	skull	tooth	bonsai
iteger Haar	2.84	2.86	1.79	2.91
morph. Haar min	0.91	1.05	0.59	0.91
morph. Haar max	0.91	1.02	0.59	0.87
min lifting	1.76	2.03	1.23	1.76
max lifting	1.86	2.11	1.19	1.87
5/3	5.81	5.86	3.62	5.86

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