ENERGY-EFFICIENT POSITIONING IN SENSOR NETWORKS BY A GAME THEORETIC APPROACH

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ABSTRACT

This paper investigates a game theoretical algorithm for positioning wireless nodes. The particular set-up considers a set of anchor sensor nodes, whose power levels are to be selected according to a twofold criterion: minimum power level and desired positioning quality for users, assessed by the geometric dilution of precision metric. The results of the proposed potential game are compared to those obtained by a centralized algorithm, with complete information and larger computational cost, showing remarkable results and reduced complexity. A distributed implementation of the game, based on local information, is also provided.

1. INTRODUCTION

Localization and positioning is a fundamental aspect in many applications of wireless sensor networks (WSNs). Industrial automation systems, estimation of random fields, or asset tracking are a few examples. Since many WSN applications will be deployed indoors, positioning using Global Navigation Satellite Systems (GNSS) cannot be guaranteed. In addition, the GNSS module is well-known to be power hungry. Positioning methods based on cooperation among sensors are a promising candidate to achieve a high degree of accuracy. Typically, cooperative positioning on WSN is carried out using trilateration, which needs at least three distance measurements (2D) to nodes with known position to obtain an estimate on the node position. To that end, the IEEE 802.15.4a standard has defined a protocol to obtain distance measurements based on time of arrival, which achieves much higher accuracy than received signal power methods [1].

In this paper we assume that a given WSN has a number of anchor nodes with known position and a much larger number of nodes with unknown position. Anchor and target nodes are IEEE 802.15.4 compliant and access the medium in non-beacon mode of operation at the MAC layer [2]. Reference nodes, which are assumed to be static and strategically deployed to provide sufficient distance measurements to all sensors, transmit beacon signals to allow all other nodes to obtain their position based on trilateration. The position of anchor nodes may be obtained via GNSS, or programmed upon deployment. In addition, we assume that nodes obtain distance estimates directly from anchor nodes, not via multi-hop transmission over other sensors. Under this setup, we seek to minimize the transmitted power of anchor nodes in order to maximize battery life. Clearly, a tradeoff exists between transmit power and positioning performance: if an anchor node transmits with lower energy, it will reach a smaller number of sensors, and those left outside coverage will obtain a worse position estimate. We seek to guarantee a minimum positioning performance by specifying a maximum value for the Geometric Dilution of Precision (GDOP) [3]. We then seek to minimize the transmit power of anchor nodes via a non-cooperative game, which belongs to the class of potential games [4]. We use game theory because is an interesting collection of analytic tools that provides distributed decision process. While many applications of game theory for communications exist in the literature [5, 6, 7], the problem of localization [8] and positioning [9] has been less exploited.

The remainder of the paper is organized as follows. We describe the positioning problem addressed in this paper in Section 2. Section 3 introduces some general concepts of game theory as well as the game proposed. In Section 4 the game is modified addressing implementation aspects, as well as obtaining a more distributed solution. Section 5 shows the simulation results and Section 6 concludes the paper.

2. DISTRIBUTED POSITIONING IN WIRELESS SENSOR NETWORKS

The problem under study involves the distributed positioning of nodes in a WSN. The setup we consider in this paper is composed of a set of M nodes, that aim at estimating their position; and a set of N anchor nodes with known locations, emitting ranging signals to allow positioning of the former nodes. Respectively, we define the two-dimensional coordinates of the nodes as

$$\mathbf{x}^{(j)} = [x^{(j)}, y^{(j)}]^T \qquad j = 1, \ldots, M \tag{1}$$
$$\mathbf{x}_a^{(i)} = [x_a^{(i)}, y_a^{(i)}]^T \qquad i = 1, \ldots, N \ . \tag{2}$$

We define the set of anchor nodes that provide coverage to the j-th node as $\mathcal{N}_j$, and its dimension as N_j. Similarly, we define the set of target nodes whose messages are received at the i-th anchor node as $\mathcal{T}_i$, with dimension being T_i.

This work has been partially supported by the Spanish Science and Technology Commission: TEC2008-02685/TEC (NARRA), TEC2010-21100 (SOFOCLES), by the European Commission in the framework of the COST Action IC0803 (RFCSET), FP7-251557 (SWAP) and by Generalitat de Catalunya (2009-SGR-940).

2.1 Geometric Dilution of Precision

The GDOP is a dimensionless value that is used to assess the *goodness* of a certain geometry for positioning purposes [3]. Notice that GDOP is a value associated to a given node that observes a spatial disposition of anchor nodes. The so-called *visibility matrix* of the j-th node can be formulated as

$$\mathbf{H}_j = \begin{pmatrix} \frac{x^{(j)}-x_a^{(1)}}{\rho_{j,1}} & \frac{y^{(j)}-y_a^{(1)}}{\rho_{j,1}} \\ \vdots & \vdots \\ \frac{x^{(j)}-x_a^{(N_j)}}{\rho_{j,N_j}} & \frac{y^{(j)}-y_a^{(N_j)}}{\rho_{j,N_j}} \end{pmatrix} , \qquad (3)$$

where $\rho_{j,i} = \sqrt{(x^{(j)} - x_a^{(i)})^2 + (y^{(j)} - y_a^{(i)})^2}$ is the geometrical distance between the j-th node and the i-th anchor. The corresponding GDOP is then calculated as

$$\text{GDOP}_j = \sqrt{\text{trace}\left\{(\mathbf{H}_j^T\mathbf{H}_j)^{-1}\right\}} \qquad (4)$$

and can be thought of as a value that measures the effect of network geometry on the position solution. At a glance, larger GDOP values imply worse positioning solutions, and viceversa. Actually, there is a clear relation between GDOP and the theoretical lower bound on the variance of a position estimator [10].

Notice that anchor nodes in (3) are those of the set $\mathcal{N}_j$, and thus $\mathbf{H}_j \in \mathbb{R}^{N_j\times 2}$. $\mathcal{N}_j$ depends on the number of anchor nodes whose beacons are received with enough power by node j. Therefore, it depends on the transmit powers of the anchor nodes. For a given receiver sensitivity s and distance-dependent path loss function $f_L(d)$, an anchor node with transmit power p_a at distance d_a from node j belongs to set $\mathcal{N}_j$ if $p_a > s/f_l(d_a)$. Therefore, given the dependence of GDOP_j on $\mathcal{N}_j$, we may in turn express the GDOP as a function of the power vector of the anchor nodes $\mathbf{p}$ explicitly as $\text{GDOP} \triangleq \text{GDOP}(\mathbf{p})$. Moreover, such dependence is monotonous, since if we decrease any component in $\mathbf{p}$ the resulting set $\mathcal{N}_j$ is equal or smaller and, in turn, a smaller set $\mathcal{N}_j$ results in larger or equal GDOP, as stated in the following proposition.

Proposition 1. *If p_i and q_i are power levels such that $p_i < q_i$, then* $\text{GDOP}_j(p_i, \mathbf{p}_{-i}) \geq \text{GDOP}_j(q_i, \mathbf{p}_{-i})$.

Proof. It suffices to show that GDOP is a non-increasing function with respect to the power of an anchor node. The rationale is as follows: it was proved that GDOP is a monotonically decreasing function with the number of anchor nodes [3]. In other words, no matter the resulting geometry seen by the node, including a new anchor node in matrix $\mathbf{H}_j$ will always reduce (or at most left unaltered) the GDOP value. The proposition is proved since increasing the power level of a given anchor node increases its coverage and, potentially, can result into a larger number of anchor nodes seen by a non-empty set of nodes. □

Let us define the mean GDOP over the entire network as

$$\overline{\text{GDOP}}(\mathbf{p}) = \frac{1}{M}\sum_{j=1}^{M} \text{GDOP}_j(\mathbf{p}) , \qquad (5)$$

and notice that Proposition 1 also holds for the mean. As a guide, the minimum GDOP value in two-dimensional scenarios was shown to be $2/\sqrt{N}$, with N being the number of anchor nodes considered [11].

3. GAME THEORETICAL APPROACH TO POSITIONING WIRELESS SENSORS

Game Theory is a collection of models and analytic tools used to study interactive decision processes [12, 5]. We limit our discussion to non-cooperative models that address the interaction among individual decision makers. Such models are called *games* and the decision makers are referred to as *players* which are assumed to be rational in this work. A strategic non-cooperative game $\Gamma(\Omega, \mathcal{A}, u)$ has three main components: *i)* Ω is the set of N players; *ii)* $\mathcal{A}$ is the set of pure strategies and $\mathbf{a} = [a_1, \ldots, a_N]^T \in \mathcal{A} \subseteq \mathbb{R}^N$ the chosen strategies, where $a_i \in \mathcal{A}_i$ represents the strategy of the i-th player over the set of its possible strategies $\mathcal{A}_i$. Thus, $\mathcal{A} = \times_{i=1}^N \mathcal{A}_i$ and $\mathbf{a}_{-i} \in \mathcal{A}_{-i} = \times_{j\neq i}^N \mathcal{A}_j$ represents the strategies of all players but the i-th; *iii)* $u_i : \mathcal{A} \mapsto \mathbb{R}$ is the utility function of the i-th player. The utility function (or payoff) quantifies the preferences of each player to a given strategy, provided the knowledge of other's strategies. Then, $u \triangleq \{u_i\}_{i\in\Omega}$ is the set of all N utility functions.

Then, a non-cooperative game is a procedure where players choose the strategy that maximizes their utility function. The *Nash equilibrium* (NE) is a stable solution of the game in which no player may improve its utility function by unilaterally deviating from it.

Definition 1 (Nash Equilibrium)**.** *A strategy profile $\mathbf{a}^*$ is a Nash equilibrium if, $\forall i \in \Omega$ and $\forall a_i \in \mathcal{A}$, $u_i(\mathbf{a}^*) \geq u_i(a_i, \mathbf{a}^*_{-i})$.*

In general, games may have a large number of NE or may not have any. Thus, it is of interest to design the utility function in a way such that the game has at least one equilibrium point. It is proved in [13] that under certain conditions of the utility function, the existence and uniqueness of a NE is ensured. However, the utility function may be designed according to a criteria which could eventually yield to non-convex functions. In those cases, there is another way for deriving sufficient conditions for existence and uniqueness of the NE in a game based on the so-called *potential games* [4]. This type of games is given when the incentive of all players to change their strategy can be expressed by a global utility function $V(\mathbf{a})$. We use the name *exact potential game* (EPG) when the game admits an exact potential function, i.e., a player-independent real valued function that measures the marginal payoff when any player deviates unilaterally.

Definition 2 (EPG)**.** *A strategic game $\Gamma(\Omega, \mathcal{A}, u)$ is an exact potential game if there exist an exact potential function $V : \mathcal{A} \to \mathbb{R}$ s.t. $\forall i \in \Omega, \forall \mathbf{a}_{-i} \in \mathcal{A}_{-i}$ and $\forall a_i, b_i \in \mathcal{A}_i$*

$$V(a_i, \mathbf{a}_{-i}) - V(b_i, \mathbf{a}_{-i}) = u_i(a_i, \mathbf{a}_{-i}) - u_i(b_i, \mathbf{a}_{-i}) \ . \qquad (6)$$

An important result due [4] is that the optima of the potential function of an EPG correspond to the Nash equilibria of the game.

3.1 Game Theoretical Algorithm

In our problem, players are the anchor nodes and the game is that of finding a NE such that each anchor node is transmitting at a minimal power while maintaining a certain positioning quality for the M target nodes. As a metric to assess such quality we use the GDOP. With this setup, following the nomenclature introduced in Section 2, Ω is the set of anchor nodes in the network. The set of strategies that the i-th reference node can choose are the set of its possible discrete power levels $\mathcal{P}_i$. We define $\mathbf{p} = [p_1, \ldots, p_N]^T \in \mathcal{P} = \times_{i=1}^{N} \mathcal{P}_i$ as the vector containing the strategies of each node. We also assume that, at the beginning of the game, anchor nodes transmit with their maximum power level in order to gather information and allow initial positioning of nodes.

We adopt an iterative *best response* algorithm to achieve a NE of the game defined by $\Gamma(\Omega, \mathcal{P}, u)$. Anchor nodes decide iteratively its power transmission by maximizing its utility function,

$$\hat{p}_i = \arg\max_{p_i \in \mathcal{P}_i} \{u_i(p_i, \hat{\mathbf{p}}_{-i})\} \ . \tag{7}$$

After each iteration, the selected power level may modify the geometry of the network, thus impacting on the maximization of other players' utility.

The design of a utility function and the existence of a potential function is crucial for the task of identifying NE in the game. In our algorithm the goal is to attain a desired positioning quality for the M target nodes, as well as reducing the total power of the N anchor nodes. As presented in Section 2.1, the GDOP provides an appealing metric to assess such quality. Therefore, the algorithm accepts a strategy if condition $\overline{\mathrm{GDOP}}(\mathbf{p}) \leq \gamma$ is fulfilled, with γ being a design parameter. Recall that the initial topology is such that all nodes transmit at maximum power. Following the result in [6], the utility function stated in Proposition 2 is considered.

Proposition 2. *The game* $\Gamma(\Omega, P, u)$ *where the individual utilities are given by*

$$u_i(p_i, \mathbf{p}_{-i}) = \begin{cases} p_{max} - p_i & \text{if} \quad \overline{\mathrm{GDOP}}(p_i, \mathbf{p}_{-i}) \leq \gamma \\ -p_i & \text{otherwise} \end{cases} \tag{8}$$

is an EPG and the exact potential function is

$$V(\mathbf{p}) = \begin{cases} p_{max} - \sum\limits_{i \in \Omega} p_i & \text{if} \quad \overline{\mathrm{GDOP}}(p_i, \mathbf{p}_{-i}) \leq \gamma \\ -\sum\limits_{i \in \Omega} p_i & \text{otherwise,} \end{cases} \tag{9}$$

where p_{max} *is the maximum power of the sensor node.*

Proof. Refer to [7] for a similar result. □

The designed game falls into the category of EPG games, and thus finding the NE point of (8) is equivalent to maximize the potential function in (9). We should notice that GDOP is not a convex function on $\mathbf{p}$. Therefore, we cannot claim that $V(\mathbf{p})$ has a single optimum, and thus the game might have several NE that satisfy $\overline{\mathrm{GDOP}}(\mathbf{p}) \leq \gamma$.

4. DISTRIBUTED IMPLEMENTATION

The game presented above has several challenges when it comes to implementation. A major concern relates to the amount of information exchange required in the networks, as anchor nodes require knowledge of global information of target nodes' in order to calculate $\overline{\mathrm{GDOP}}(\mathbf{p})$. Our goal here is to minimize the information exchange requirements in order to preserve the benefits from power savings, due to reduced transmission power at the reference nodes. To that aim we propose to use other metrics, instead of $\overline{\mathrm{GDOP}}(\mathbf{p})$, that only require transmission of information from in-range target nodes to anchors at each game iteration. This information includes the target's own position estimate and the corresponding set $\mathcal{N}_j$.

We propose to modify the discontinuity condition in (8)-(9) so as to use only local GDOP estimates. Two alternatives are presented. Similarly to the game using global information, we consider that at the beginning of both games players transmit with maximum power in order to allow initial positioning of target nodes and information gathering. The algorithms proceed in an iterative best response fashion until convergence.

4.1 Local GDOP Average

In this case we consider a local estimate of the average GDOP, defined as $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ in (10) for the i-th anchor node. Recall that $\mathcal{T}_i$ is the set of target nodes from which the i-th anchor nodes receives status information, as they are within its range. Then, each anchor can compute

$$\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p}) = \frac{1}{T_i} \sum_{j \in \mathcal{T}_i} \mathrm{GDOP}_j(\mathbf{p}) \ . \tag{10}$$

The resulting utility function for the i-th player is then modified to take values as $p_{\max} - p_i$ if $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(p_i, \mathbf{p}_{-i}) \leq \gamma$. With this setup, it is possible that the overall $\overline{\mathrm{GDOP}}$ value exceeds the threshold eventually, since the average used by each player is local. In other words, a certain strategy might lead to $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p}) \leq \gamma$ but $\overline{\mathrm{GDOP}}_{\mathcal{T}_{i'}}(\mathbf{p}) > \gamma$, forcing the i'-th node to increase its power in next game iteration.

Notice that this distributed solution approximates the previous game when transmission powers of target nodes are such that one can consider $\overline{\mathrm{GDOP}} \simeq \overline{\mathrm{GDOP}}_{\mathcal{T}_i}$, $\forall i$. Figure 1 shows the Root Mean Square Error (RMSE) between $\overline{\mathrm{GDOP}}$ and $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}$, defined as

$$\xi(\mathrm{GDOP}) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} |\overline{\mathrm{GDOP}} - \overline{\mathrm{GDOP}}_{\mathcal{T}_i}|^2} \ , \tag{11}$$

versus the ratio range of target nodes over the maximum distance in the network (thus being independent of a particular node's power levels). The approximation is valid for increasing target node's power and density.

4.2 Worst Case GDOP

We propose here an alternative design where worst-case is addressed. In this configuration, the condition to

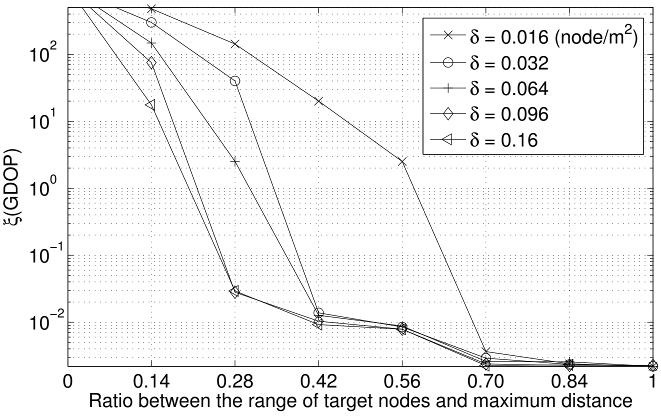

Figure 1: RMSE in (11) for a number of target node densities δ (node/m^2) and 100 Monte Carlo trials.

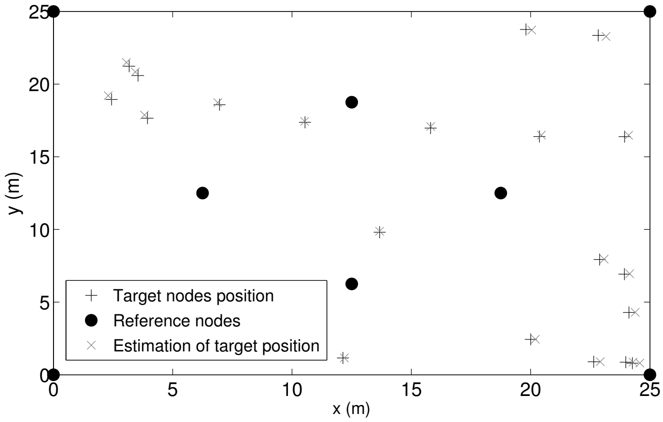

Figure 2: Simulation scenario consisting of a 25×25 meter region where a set of anchor sensor nodes (big dots) are distributed at known positions, whereas the target sensor nodes (black crosses) are placed randomly. Position estimates (grey crosses) are also shown for the last iteration of the game.

maximize $u_i(p_i, \mathbf{p}_{-i})$ is to ensure that all target nodes have the specified GDOP. That is, the condition for the i-th player can be formulated as

$$\mathrm{GDOP}_j(\mathbf{p}) \leq \gamma, \forall j \in \mathcal{T}_i \ , \tag{12}$$

and the utility in (8) should be modified accordingly. It can be easily seen that a game implementing such utility yields to a steady state solution. Remember that game starts with all players transmitting with maximum power. Notice that a player has no incentives to decrease its power if it causes at least one target node decrease its GDOP. Same applies to the rest of players when iterating, and thus a stable solution is eventually achieved when no player can modify further its strategy.

Although the achieved solution is not optimal (from an energy-efficient point of view), it provides a strategy set which ensures the specified target GDOP. This might be useful in applications where this is the most restrictive issue, rather than proper power control.

5. SIMULATION RESULTS

The proposed algorithm was tested in a scenario composed of $M = 20$ nodes that aim at locating themselves using the received signal strength indicator (RSSI) to a set of $N = 8$ anchor nodes (Figure 2). Anchor nodes are distributed at known positions in a 25×25 meters region, whereas the M nodes are placed randomly in the space. We perform 5 iterations of the game per player, which play in an ordered sequential fashion. At each iteration, the corresponding anchor node has to compute GDOP values, which depend on the estimated positions of the nodes ($\hat{\mathbf{x}}^{(j)}$ for the j-th node). Such position estimate is performed at target nodes using the set of received ranges $\{\hat{\rho}_{j,i}\}_{i \in \mathcal{N}_j}$ by a least squares procedure. A common model [1] for RSSI-based range estimates is the log-normal model

$$\hat{\rho}_{j,i} = \rho_{j,i} \cdot 10^{\frac{v_{j,i}}{10 \cdot p}} \ , \tag{13}$$

where $v_{j,i} \sim \mathcal{N}(0, \sigma_{j,i}^2)$ and $p = 3$ is the channel path loss exponent. In our setup, $\sigma_{j,i} = 0.1$ dBm for all possible $\{i, j\}$ pairs.

We consider that sensor nodes have the CC2420 transceiver that is IEEE 802.15.4 compliant. The set of power levels for the reference nodes are $\mathcal{P} = \{0, 0.032, 0.1, 0.31, 0.50, 0.79, 1\}$ mW [14] that we approximate to ranges $\{0, 5, 10, 15, 20, 25, 30\}$ m with [15]. We selected a threshold for the mean GDOP value of $\gamma = 1.3$. Recall that initially all nodes transmit at their maximum power. Results of the proposed algorithm were averaged over 100 Monte Carlo independent trials and compared to those obtained by an algorithm that globally optimizes the set of power levels $\mathbf{p}$. That is, the solution of the coordination game that finds the global optima of the potential function $V(\mathbf{p})$. This solution, implemented by exhaustive search, explores all combinations of power levels for the N nodes ($\dim\{\mathcal{P}\}^N$) and obtains the set of strategies with lower mean power ($\bar{\mathbf{p}}_{\min}$) over the network, with the condition on the GDOP holding. In the simulation results, we compare the average results of our method with the GDOP average of all the target nodes $\overline{\mathrm{GDOP}}(\mathbf{p})$, the distributed game with local GDOP average $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ and the distributed game with worst case GDOP, as well as $\bar{\mathbf{p}}_{\min}$.

Figure 3 shows the evolution of the mean power of the network versus the iterations of the game. We can observe that this value decreases and tends to $\bar{\mathbf{p}}_{\min}$. Of interest is the comparison of these results with those in Figure 4, where we can identify that although our algorithm might yield larger mean power values, we experience a tradeoff in the final GDOP achieved. Results of the case with local GDOP average come closer to $\bar{\mathbf{p}}_{\min}$ than for worst case GDOP. This is because worst case GDOP assures that each GDOP is below the threshold.

For the sake of completeness, we also plot the resulting RMSE on the positioning solution after the power control game was executed, showing bounded results. Notice that for each iteration of the game the RMSE decreases in this phase of refinement of the error.

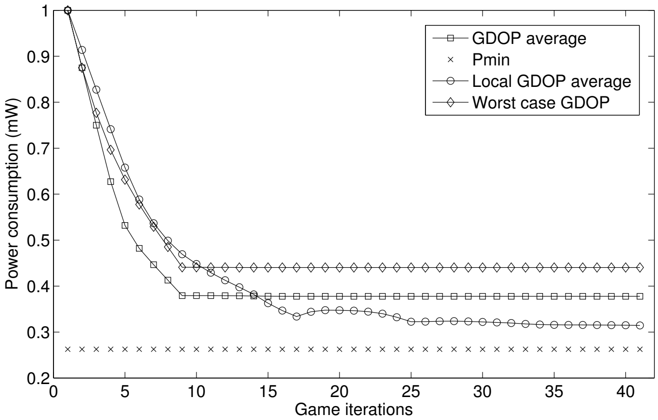

Figure 3: Mean power of anchor nodes versus iterations of the game.

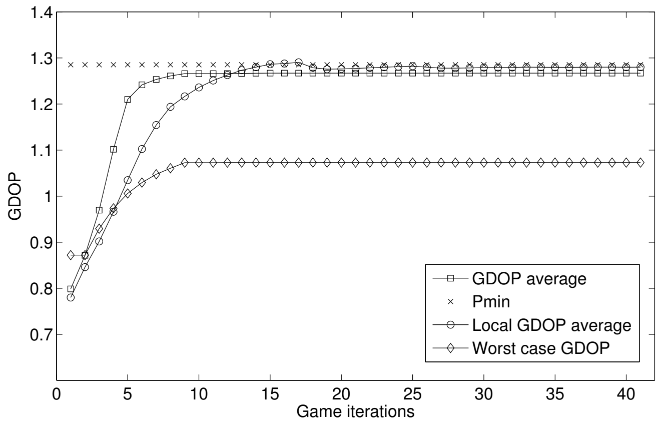

Figure 4: $\overline{\text{GDOP}}$ versus iterations of the game.

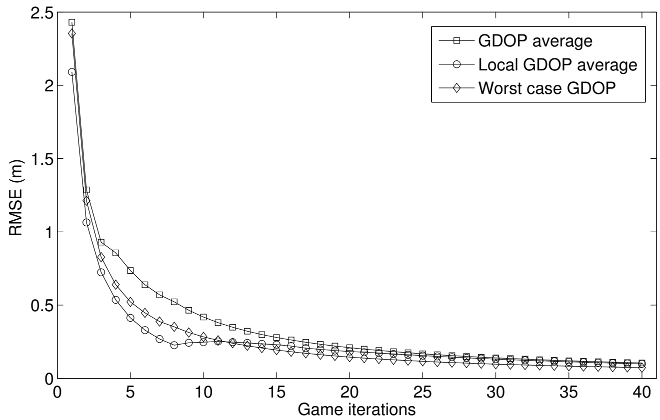

Figure 5: Average RMSE of the positioning solution (target nodes) over the network versus iterations.

6. CONCLUSIONS

In this paper we proposed an algorithm for positioning nodes in WSNs using the framework provided by potential games. The proposed solution provides a distributed approach to select the power levels of anchor nodes such that a predefined positioning quality is ensured, as quantified by the GDOP parameter. From computer simulations we observed that the algorithm obtains results which are comparable to a global approach, as well as requiring much less computational resources. The complexity is on the order of $\mathcal{O}(n_p^N)$ and $\mathcal{O}(n_p)$ for the global and proposed solutions, respectively, with n_p being the number of available power levels. Also we present a fully-distributed implementation of this game where $\overline{\text{GDOP}}$ is estimated using merely the local information available at each anchor node.

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