DISTRIBUTED POWER AND ROUTING OPTIMIZATION IN SINGLE-SINK DATA GATHERING WIRELESS SENSOR NETWORKS

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ABSTRACT

This paper addresses a total transmission power minimization problem in single-sink data gathering wireless sensor network. We propose a distributed algorithm for solving the convex problem with partial dual decomposition approach by jointly optimizing the routing and the power allocation. We assume orthogonal multiple access communications under Rayleigh fading. By applying dual decomposition for relaxing the coupling constraint, the optimization problem is decomposed vertically into two independently solvable subproblems: the routing problem in the network layer and the power allocation problem in the physical layer. Furthermore, second-level dual decompositions are performed for distributing the solution process horizontally within each layer. The master dual problem coordinates the whole solution process by introducing the pricing on the link capacities. Gradient projection method is employed to update the primal and dual variables iteratively. Numerical results are provided to show the convergence properties in a static channel and the tracking ability under time-varying Rayleigh channels.

1. INTRODUCTION

The correlated data gathering problem in wireless sensor networks (WSNs) has been under extensive investigation. An objective of data gathering WSN is to achieve energy-efficient communications. Cristescu *et al.* [1] showed that by using Slepian-Wolf (SW) distributed source coding (DSC), the reduction in the total rates have significant influence on the cost function of transporting the data to the sink node. They showed a shortest path tree (SPT) to stand for the optimal routing for data transportation in single-sink data gathering (SSDG) network. However, they ignored the influence of wireless links on the routing by considering only distance-dependent link weights with no interference.

It is crucial to achieve energy-efficient communications with low-complexity infrastructure and autonomous operation of the nodes. In addition to DSC, a cross-layer design is a key enabler to outperform the designs of conventional intra-layer networking systems. Yuen *et al.* [2] proposed a distributed algorithm to minimize the total transmission energy consumption in sensor network by using SW coding for the rate allocation and by finding an optimal transmission structure based on the cost functions of data transportation. The design included congestion control due to the interference on the links but power allocation was not included since the capacities were considered fixed.

Xiao *et al.* [3] investigated simultaneous routing and resource allocation for wireless data networks and derived distributed algorithm based on dual decomposition for finding the solution for the maximum-utility problem. The work of He *et al.* [4] proposed a distributed algorithm that involves the joint optimization of the routing, the random access and the power allocation with an objective to maximize the network lifetime in WSNs. The results showed improvements in network lifetime against the design of minimizing the total energy consumption in the network, like in [2].

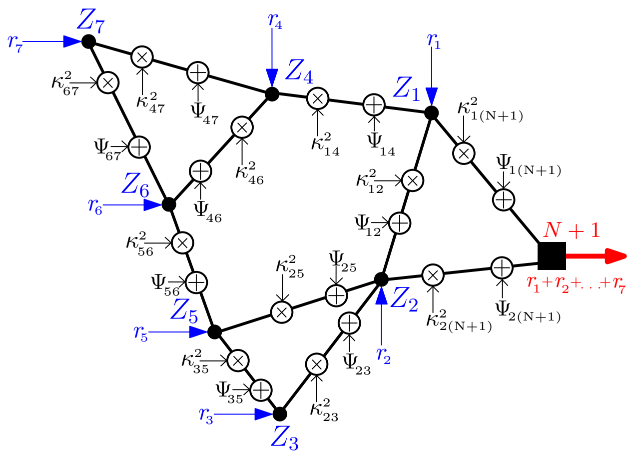


Figure 1: A single-sink data gathering WSN.

Yuan *et al.* [5] addressed a cross-layer optimization framework by jointly optimizing the source quantization, the routing and the power allocation in WSN. They proposed an algorithm to efficiently solve the problem in a modularized way with an objective that introduced the trade-off between minimization of the total transmit power and the distortion incurring in the estimation process. Li *et al.* [6] studied joint coding/routing optimization by introducing tradeoff between the network lifetime and rate-distortion in wireless visual sensor networks for correlated sources.

The main contribution of this paper is to propose a distributed algorithm for total transmit power minimization in SSDG WSNs by jointly optimizing the routing and the power allocation with the given source rates. The joint optimization is done combined with the SW coding of sources in order to achieve energy-efficient data transportation for the data gathering. The functionality of the proposed algorithm is studied under static and time-varying Rayleigh channels.

2. SYSTEM MODEL

2.1 Network Topology

Consider a SSDG WSN consisting of N sensing nodes and a sink node $N+1$. The WSN can be modeled as directed graph $G = (\mathcal{A}, \mathcal{L})$, where set $\mathcal{A} = \{1, 2, \ldots, N, N+1\}$, determines the set of $N+1$ nodes with indices $i \in \mathcal{A}$ and set $\mathcal{L} = \{1, 2, \ldots, L\}$ represents the set of L directed wireless links between the sensor nodes with indices $l \in \mathcal{L}$. The set of N source nodes is defined as $\mathcal{S} = \{1, 2, \ldots, N\}$, such that $\mathcal{A} = \mathcal{S} \cup \{N+1\}$. A SSDG WSN is illustrated in Fig. 1.

Network topology w.r.t. the interactions between the nodes and links can be compactly described with node-link incidence matrix $\boldsymbol{A} \in \mathbb{Z}^{(N+1)\times L}$. An entry a_{il} of the matrix, associated with node i and link l, is of the following form: [3]

$$a_{il} = \begin{cases} 1, & \text{if node } i \text{ is the start node of link } l \\ -1, & \text{if node } i \text{ is the end node of link } l \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

2.2 Multi-path Routing Model

The routing of data packets is assumed to follow a braided multi-path routing model, that is discussed in further details in [7]. Let $f_l \geq 0$ denote the amount of total flow on the link $l \in \mathcal{L}$, when the corresponding vector for the network is $\boldsymbol{f} = [f_1, f_2, \ldots, f_L]^T \in \mathbb{R}^L_+$. In addition, each source node $i \in \mathcal{S}$ is associated with an external flow $r_i > 0$, that is the source rate of node i. The sink node is associated with the sink rate $r_{N+1} < 0$. Thus, the rate vector for the whole network is $\boldsymbol{r} = [r_1, r_2, \ldots, r_N, r_{N+1}]^T \in \mathbb{R}^{N+1}$.

The concept of lossless data gathering in the network involves, that the flow conservation law has to hold at each node $i \in \mathcal{A}$. The flow conservation law at each node $i \in \mathcal{A}$ can be expressed as follows [6]

$$\sum_{l\in\mathcal{O}(i)} f_l - \sum_{l\in\mathcal{I}(i)} f_l = \begin{cases} r_i, & \text{if } i \in \mathcal{S} \\ -\sum_{i\in\mathcal{S}} r_i, & \text{if } i = N+1, \end{cases} \tag{2}$$

where $\mathcal{O}(i)$ denotes the outgoing links of node i and $\mathcal{I}(i)$ the incoming links. The flow conservation law introduces also a quality of service requirement, since all the source rates has to be delivered to the sink node, that is $r_{N+1} = -\sum_{i\in\mathcal{S}} r_i$.

The compact expression for the flow conservation law in the whole network can be written as [3]

$$\boldsymbol{A}\boldsymbol{f} = \boldsymbol{r}. \tag{3}$$

2.3 Communication Model

We assume frequency division multiple access (FDMA) to be used in the system leading to a non-interfering communications across the links. Each sensor node $i \in \mathcal{S}$ is capable of transmitting, receiving and relaying data. Sensor nodes operate in a full-duplex mode with frequency division duplexing. The sink node receives the data originated from each source and has capability of performing the joint decoding of data. Each sensor node $i \in \mathcal{S}$ has a fixed transmission range d_i^t. The distance between nodes i and j is denoted with d_{ij}, $i, j \in \mathcal{A}$, $i \neq j$. Thus, a wireless link from node i to j, denoted with (i, j), exists, if $d_{ij} \leq d_i^t$. [6]

Each node $i \in \mathcal{S}$ can allocate different transmit powers to its outgoing links $\mathcal{O}(i)$. The link capacity as a function of transmit power $p_l \geq 0$ for $l \in \mathcal{L}$ with unit bandwidth and with inverse-square path loss model is given by [3]

$$c_l = \log_2\left(1 + \left(\frac{d_0}{d_l}\right)^2 \frac{\kappa_l^2 p_l}{\varsigma_l^2}\right), \quad l = 1, 2, \ldots, L, \tag{4}$$

where $d_0 = \min_{l\in\mathcal{L}} d_l$ is a reference distance, d_l is the length of link l, ς_l^2 is the power spectral density of additive white Gaussian noise (AWGN) present at each receiver and $\kappa_l \geq 0$ is a real-valued, time-varying and input-independent Rayleigh distributed channel coefficient of link l. Noise realizations Ψ_{ij}, $i, j \in \mathcal{A}$, seen in Fig. 1 are assumed to be uniformly distributed across the network, such that $\varsigma_l^2 = \varsigma^2, \forall l \in \mathcal{L}$.

Since we assume the capacity constrained communication links, the total amount of flow on each link $l \in \mathcal{L}$ has to satisfy $f_l \leq c_l$ in order to achieve successful data transmissions across link l. Each sensor node $i \in \mathcal{S}$ is limited with the total amount of transmit power P_i^{tot} that it can allocate to its outgoing links, that is

$$\sum_{l\in\mathcal{O}(i)} p_l \leq P_i^{\text{tot}}, \; i = 1, 2, \ldots, N. \tag{5}$$

The maximum transmit powers of the nodes are expressed with vector $\boldsymbol{g} = [P_1^{\text{tot}}, P_2^{\text{tot}}, \ldots, P_N^{\text{tot}}, 0]^T \in \mathbb{R}^{N+1}_+$, where the last entry corresponds to the sink node. The total power constraints regarding the entire network can be expressed as

$$\boldsymbol{B}\boldsymbol{p} \preceq \boldsymbol{g}, \tag{6}$$

where $\boldsymbol{B} = \boldsymbol{A}_+ \in \mathbb{Z}_+^{(N+1)\times L}$ and $\boldsymbol{p} = [p_1, p_2, \ldots, p_L]^T \in \mathbb{R}^L_+$ is the transmit power vector. An element $(a_{il})_+$ of $\boldsymbol{A}_+$ is given by $(a_{il})_+ = \max\{0, a_{il}\}$, thus $\boldsymbol{B}$ identifies the outgoing links of each node i [3].

3. JOINT OPTIMIZATION OF ROUTING AND POWER ALLOCATION

The objective is to minimize the total transmit power in the SSDG WSN while guaranteeing that all the individual but spatially correlated data can be fully recovered at the destination. SW coding is performed for each information source Z_i, $i \in \mathcal{S}$, to remove all the redundancy in correlated data. The sources are considered to follow a Gaussian random process leading to the global SW rate allocation process described in more detail in [8].

After SW code rates are assigned to the source nodes, the total transmit power minimization problem can be expressed as a joint optimization over the power and flow variables as

$$\begin{aligned} \underset{p_l, f_l}{\text{minimize}} \quad & \sum_{l\in\mathcal{L}} p_l \\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{f} = \boldsymbol{r} \\ & f_l \leq \log_2\left(1 + \left(\frac{d_0}{d_l}\right)^2 \frac{\kappa_l^2 p_l}{\varsigma^2}\right), \;\; \forall l \in \mathcal{L} \\ & \boldsymbol{B}\boldsymbol{p} \preceq \boldsymbol{g} \\ & f_l \geq 0, \; p_l \geq 0, \;\; \forall l \in \mathcal{L}, \end{aligned} \tag{7}$$

where the N first entries of rate vector $\boldsymbol{r}$ belong to the SW rate region $\mathcal{R}_{\text{SW}}$, that is given by [9]

$$\mathcal{R}_{\text{SW}} = \left\{[r_1, r_2, \ldots, r_N]^T : \forall \mathcal{K} \subseteq \mathcal{S}, \sum_{i\in\mathcal{K}} r_i \geq H(\boldsymbol{Y}_{\mathcal{K}}|\boldsymbol{Y}_{\mathcal{K}^c})\right\}, \tag{8}$$

where $\boldsymbol{Y} = [Y_1, Y_2, \ldots, Y_N]^T$ is Gaussian distributed random vector, $\mathcal{K}$ is a subset of source nodes in $\mathcal{S}$ and $\mathcal{K}^c$ denotes the complementary set of $\mathcal{K}$.

The optimization problem in (7) includes a capacity constraint for each link $l \in \mathcal{L}$ that couples optimization variables p_l and f_l. The coupling constraint can be relaxed by applying a partial dual decomposition w.r.t. the constraint. By introducing Lagrange multipliers $\boldsymbol{\nu} = [\nu_1, \nu_2, \ldots, \nu_L]^T \in \mathbb{R}_+^L$ for the links, the relaxed optimization problem appears as

$$\begin{aligned} \underset{p_l, f_l}{\text{minimize}} \quad & \sum_{l \in \mathcal{L}} \left[p_l + \nu_l \left(f_l - \log_2 \left(1 + \left(\frac{d_0}{d_l} \right)^2 \frac{\kappa_l^2 p_l}{\varsigma^2} \right) \right) \right] \\ \text{subject to} \quad & \boldsymbol{Af} = \boldsymbol{r} \\ & \boldsymbol{Bp} \preceq \boldsymbol{g} \\ & f_l \geq 0, \ p_l \geq 0, \ \forall l \in \mathcal{L}. \end{aligned} \tag{9}$$

Due to the relaxation, the optimization problem in (9) is decomposed into two independently solvable subproblems: routing problem in the network layer and power allocation problem in the physical layer. Since this involves cross-layer optimization across the protocol stack, it is referred to a vertical decomposition [4]. The objective function, i.e., the partial Lagrangian in (9) can be decomposed as

$$\mathfrak{L}(\boldsymbol{f}, \boldsymbol{p}, \boldsymbol{\nu}) = \mathfrak{L}_f(\boldsymbol{f}, \boldsymbol{\nu}) + \mathfrak{L}_p(\boldsymbol{p}, \boldsymbol{\nu}), \tag{10}$$

where

$$\mathfrak{L}_f(\boldsymbol{f}, \boldsymbol{\nu}) = \sum_{l \in \mathcal{L}} \nu_l f_l \tag{11}$$

$$\mathfrak{L}_p(\boldsymbol{p}, \boldsymbol{\nu}) = \sum_{l \in \mathcal{L}} \left[p_l - \nu_l \log_2 \left(1 + \left(\frac{d_0}{d_l} \right)^2 \frac{\kappa_l^2 p_l}{\varsigma^2} \right) \right].$$

The associated dual function is of the form

$$\mathfrak{D}(\boldsymbol{\nu}) = \inf_{\boldsymbol{f}} \mathfrak{L}_f(\boldsymbol{f}, \boldsymbol{\nu}) + \inf_{\boldsymbol{p}} \mathfrak{L}_p(\boldsymbol{p}, \boldsymbol{\nu}) \tag{12}$$

with the constraint set in (9).

Let us denote the optimal flow variables with $\boldsymbol{f}^* = [f_1^*, f_2^*, \ldots, f_L^*]^T$ and the optimal power variables with $\boldsymbol{p}^* = [p_1^*, p_2^*, \ldots, p_L^*]^T$ attained from finding the infimum points for the dual function in (12). Finally, the dual problem can be written as

$$\begin{aligned} \underset{\boldsymbol{\nu}}{\text{maximize}} \quad & \mathfrak{D}^*(\boldsymbol{\nu}) \\ \text{subject to} \quad & \boldsymbol{\nu} \succeq 0, \end{aligned} \tag{13}$$

where the objective function is

$$\mathfrak{D}^*(\boldsymbol{\nu}) = \mathfrak{L}_f(\boldsymbol{f}^*, \boldsymbol{\nu}) + \mathfrak{L}_p(\boldsymbol{p}^*, \boldsymbol{\nu}). \tag{14}$$

Since the primal problem in (7) is convex and Slater's condition is assumed to hold, the duality gap is zero [10, p.226]. Due to the convexity of the primal problem and the differentiability of the objective function of the dual problem, the solution for (13) can be found by using the gradient projection method [11]. The derivative of the objective function in (13) w.r.t. $\nu_l, \ l \in \mathcal{L}$, is

$$\frac{\partial\, \mathfrak{D}^*(\boldsymbol{\nu})}{\partial \nu_l} = f_l^* - \log_2 \left(1 + \left(\frac{d_0}{d_l} \right)^2 \frac{\kappa_l^2 p_l^*}{\varsigma^2} \right). \tag{15}$$

Lagrange variables $\nu_l, \ l \in \mathcal{L}$, are updated at each iteration instance t as

$$\nu_l(t+1) = \left[\nu_l(t) + \beta_\nu(t) \frac{\partial\, \mathfrak{D}^*(\boldsymbol{\nu})}{\partial \nu_l}(t) \right]^+, \tag{16}$$

where $\beta_\nu(t)$ is the step size and $[m]^+$ denotes the projection on to the set of non-negative numbers, $[m]^+ = \max\{0, m\}$.

In order to update dual variables $\nu_l, \ l \in \mathcal{L}$, in (16), the optimal flows and powers have to be attained for a given ν_l. Dual variables ν_l connect the subproblems to each other by acting as coordinators for the solution process. The subproblems can be independently solved in the respective layer while intercommunicating only with the dual variables between the layers.

By means of (12), the routing problem in the network layer involves solving the following convex problem:

$$\begin{aligned} \underset{f_l}{\text{minimize}} \quad & \sum_{l \in \mathcal{L}} \nu_l f_l \\ \text{subject to} \quad & \boldsymbol{Af} = \boldsymbol{r} \\ & f_l \geq 0, \ \forall l \in \mathcal{L} \end{aligned} \tag{17}$$

By performing a second-level dual decomposition for distributing the solution process horizontally in the layer, the relaxed problem appears as

$$\underset{f_l \geq 0}{\text{minimize}} \quad \sum_{l \in \mathcal{L}} \big(\nu_l f_l \big) + \boldsymbol{\lambda}^T \big(\boldsymbol{Af} - \boldsymbol{r} \big), \tag{18}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_N, \lambda_{N+1}]^T \in \mathbb{R}_+^{N+1}$ are the Lagrange multipliers associated with the flow conservation law constraint.

The dual function for minimizing the partial Lagrangian $\mathfrak{L}_f(\boldsymbol{f}, \boldsymbol{\nu}, \boldsymbol{\lambda})$ in (18) is of the following form:

$$\mathfrak{D}_f(\boldsymbol{\nu}, \boldsymbol{\lambda}) = \inf_{f_l} \sum_{l \in \mathcal{L}} \big(\nu_l f_l \big) + \boldsymbol{\lambda}^T \big(\boldsymbol{Af} - \boldsymbol{r} \big) \tag{19}$$

After finding the optimal flow variables $\boldsymbol{f}^*$ for (19), the associated dual problem can be expressed as

$$\begin{aligned} \underset{\boldsymbol{\lambda}}{\text{maximize}} \quad & \mathfrak{D}_f^*(\boldsymbol{\nu}, \boldsymbol{\lambda}) \\ \text{subject to} \quad & \boldsymbol{\lambda} \succeq 0. \end{aligned} \tag{20}$$

The dual problem in (20) can be solved with the primal-dual algorithm, which simultaneously updates the primal and dual variables towards the optimum points [4, 6]. The partial derivatives of the objective function in (20) with a given $\boldsymbol{\nu}$ w.r.t. $f_l, \ l \in \mathcal{L}$, and $\lambda_i, \ i \in \mathcal{A}$, are given as

$$\frac{\partial \mathfrak{L}_f(\boldsymbol{f}, \boldsymbol{\nu}, \boldsymbol{\lambda})}{\partial f_l} = \nu_l + \boldsymbol{a}_l^T \boldsymbol{\lambda} \tag{21}$$

$$\frac{\partial \mathfrak{L}_f(\boldsymbol{f}, \boldsymbol{\nu}, \boldsymbol{\lambda})}{\partial \lambda_i} = \boldsymbol{a}_i \boldsymbol{f} - r_i. \tag{22}$$

The variables are updated according to the gradient projection method at each iteration instance t as

$$f_l(t+1) = \left[f_l(t) - \alpha_f(t) \Big(\frac{\partial \mathfrak{L}_f(\boldsymbol{f}, \boldsymbol{\nu}, \boldsymbol{\lambda})}{\partial f_l}(t) \Big) \right]^+ \tag{23}$$

$$\lambda_i(t+1) = \left[\lambda_i(t) + \alpha_\lambda(t) \Big(\frac{\partial \mathfrak{L}_f(\boldsymbol{f}, \boldsymbol{\nu}, \boldsymbol{\lambda})}{\partial \lambda_i}(t) \Big) \right]^+, \tag{24}$$

where $\alpha_f(t)$ and $\alpha_\lambda(t)$ are the step sizes.

According to (12), the power allocation problem in the physical layer can be formulated as a convex problem as

$$\begin{aligned} \underset{p_l}{\text{minimize}} \quad & \sum_{l\in\mathcal{L}}\left[p_l - \nu_l \log_2\left(1+\left(\frac{d_0}{d_l}\right)^2\frac{\kappa_l^2 p_l}{\varsigma^2}\right)\right] \\ \text{subject to} \quad & \boldsymbol{Bp} \preceq \boldsymbol{g} \\ & p_l \geq 0, \;\; \forall l \in \mathcal{L}. \end{aligned} \tag{25}$$

A second-level dual decomposition is applied for distributing the solution process horizontally in the layer. By introducing Lagrange variables $\boldsymbol{\omega} = [\omega_1, \omega_2, \ldots, \omega_N, \omega_{N+1}]^T \in \mathbb{R}_+^{N+1}$ w.r.t. the total power constraint of each node $i,\; i \in \mathcal{A}$, the relaxed power allocation problem can be written as:

$$\begin{aligned} \underset{p_l\geq 0}{\text{minimize}} \quad & \sum_{l\in\mathcal{L}}\left[p_l - \nu_l \log_2\left(1+\left(\frac{d_0}{d_l}\right)^2\frac{\kappa_l^2 p_l}{\varsigma^2}\right)\right] \\ & + \boldsymbol{\omega}^T\big(\boldsymbol{Bp}-\boldsymbol{g}\big) \end{aligned} \tag{26}$$

The associated dual function is to minimize the partial Lagrangian $\mathfrak{L}_p(\boldsymbol{p},\boldsymbol{\nu},\boldsymbol{\omega})$ in (26), thus of the following form:

$$\begin{aligned} \mathfrak{D}_p(\boldsymbol{\nu},\boldsymbol{\omega}) \;=\; \inf_{p_l} & \sum_{l\in\mathcal{L}}\left[p_l - \nu_l \log_2\left(1+\left(\frac{d_0}{d_l}\right)^2\frac{\kappa_l^2 p_l}{\varsigma^2}\right)\right] \\ & + \boldsymbol{\omega}^T\big(\boldsymbol{Bp}-\boldsymbol{g}\big) \end{aligned} \tag{27}$$

The corresponding dual problem after attaining optimal power variables $\boldsymbol{p}*$ for (27) is

$$\begin{aligned} \underset{\boldsymbol{\omega}}{\text{maximize}} \quad & \mathfrak{D}_p^*(\boldsymbol{\nu},\boldsymbol{\omega}) \\ \text{subject to} \quad & \boldsymbol{\omega} \succeq 0. \end{aligned} \tag{28}$$

Due to the strict convexity, the optimal powers $\boldsymbol{p}*$ for a given $\boldsymbol{\nu}$ can be uniquely found by means of the derivative of the partial Lagrangian $\mathfrak{L}_p(\boldsymbol{p},\boldsymbol{\nu},\boldsymbol{\omega})$ w.r.t. $p_l,\; l \in \mathcal{L}$, that is

$$p_l^* = \max\left\{0,\; \frac{\nu_l}{\ln 2\big(1+\boldsymbol{b}_l^T\boldsymbol{\omega}\big)} - \frac{1}{\gamma_l}\right\}, \tag{29}$$

where γ_l stands for the link condition factor, that is

$$\gamma_l = \left(\frac{d_0}{d_l}\right)^2 \frac{\kappa_l^2}{\varsigma^2}. \tag{30}$$

The gradient projection method can be employed to solve the dual problem in (28). The partial derivative w.r.t. each Lagrange multiplier $\omega_i,\; i \in \mathcal{A}$, is expressed as

$$\frac{\partial \mathfrak{L}_p(\boldsymbol{p}^*,\boldsymbol{\nu},\boldsymbol{\omega})}{\partial \omega_i} = \boldsymbol{b}_i\boldsymbol{p}^* - g_i. \tag{31}$$

Lagrange multipliers $\omega_i,\; i \in \mathcal{A}$, are updated at each iteration instance t as

$$\omega_i(t+1) = \left[\omega_i(t) + \alpha_\omega(t)\Big(\frac{\partial \mathfrak{L}_p(\boldsymbol{p}^*,\boldsymbol{\nu},\boldsymbol{\omega})}{\partial \omega_i}(t)\Big)\right]^+, \tag{32}$$

where $\alpha_\omega(t)$ is the step size.

At the last iteration instance, the powers are recovered at each link $l \in \mathcal{L}$ according to the capacity region in (4) as

$$p_l = \frac{2^{f_l^*}-1}{\gamma_l}. \tag{33}$$

The distributed algorithm for joint routing and power optimization is summarized in Algorithm 1. In the beginning, the sink node has to gather the information about the sum rate in the network. It is remarkable that the remainder of the algorithm requires only local information exchange of the variables within the extreme neighborhood of each node $i \in \mathcal{A}$. In addition, channel state information of the outgoing links of each node $i \in \mathcal{S}$ is required.

Algorithm 1 Joint Power and Routing Optimization

1. **Initialization**
   - a) For each node $i \in \mathcal{A}$, choose initial $\lambda_i, \omega_i \geq 0$ and set the total power constraint g_i for $i \in \mathcal{S}$
   - b) For each link $l \in \mathcal{L}$, choose initial $f_l, \nu_l \geq 0$
   - c) For the sink node, collect $r_{N+1} = -\sum_{i\in\mathcal{S}} r_i$
2. **Distributed algorithm** - At each iteration instance t:
   - I. **The routing subproblem**
   - a) For each node $i \in \mathcal{A}$, collect flow variables f_l from the links connected to node i
   - b) For each link $(i,j),\; i,j \in \mathcal{A}$, collect λ_i and λ_j from the end nodes of the link
   - c) Update f_l for each link $l \in \mathcal{L}$ according to (23)
   - d) Update λ_i for each node $i \in \mathcal{A}$ according to (24) by using the updated flow variables $f_l,\; l \in \mathcal{L}$
   - II. **The power allocation subproblem**
   - a) For each node $i \in \mathcal{S}$, collect power variables p_l of the outgoing links of node i
   - b) For each link $l = (i,j),\; i,j \in \mathcal{A}$, collect ω_i from the start node of the link and acquire link condition factor γ_l
   - c) For each link $l \in \mathcal{L}$, set the optimal powers p_l^* according to (29)
   - d) Update ω_i for each node $i \in \mathcal{A}$ according to (32)
   - III. **The master dual problem**
   - a) Update ν_l for each link $l \in \mathcal{L}$ according to (16)
3. **Power recovery** - At the last iteration instance
   - a) Recover p_l for each link $l \in \mathcal{L}$ according to (33)

4. NUMERICAL RESULTS

The numerical results for the proposed algorithm were generated with networks having square grid form topology. The source nodes were placed 100 units from each other and the sink node was placed in the center of the network. Sources were assigned with SW rates with a fixed correlation. The primal optimal solutions were found in centralized manner with CVX software [12]. The number of iterations was set to 5000 and diminishing step sizes used for gradient updates were $\alpha_f = 4.0/\sqrt{t}$, $\alpha_\lambda = 0.3/\sqrt{t}$, $\alpha_\omega = 0.3/\sqrt{t}$ and $\beta_\nu = 0.3/\sqrt{t}$. The noise variance was set to $\varsigma^2 = 0.01$ and the total transmit power constraint for each node $i \in \mathcal{S}$ to $g(i) = 1.0 \times 10^3$.

The convergence of the proposed algorithm w.r.t. normalized duality gap (duality gap normalized w.r.t. the primal value) is shown in Fig. 2 for one network instance in static

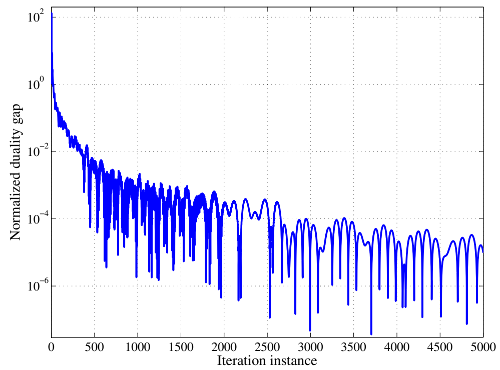
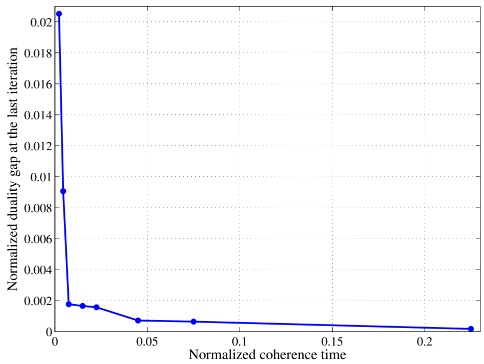
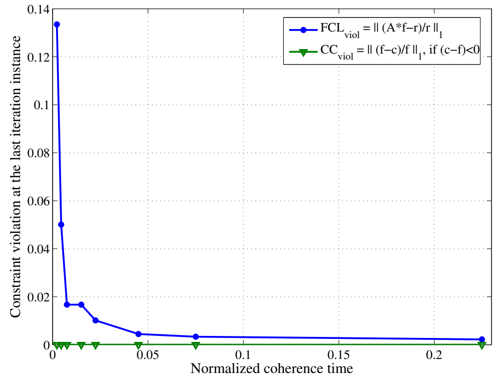


Figure 2: Evolution of normalized duality gap.



(a) Duality gaps.



(b) Constraint violations.

Figure 3: Tracking of the distributed algorithm.

channels for $N = 16$. The tracking of the algorithm under time-varying channels was studied during 5000 iterations with different normalized coherence times (normalized w.r.t. 5000 iteration instances) by averaging over 25 channel initializations for $N = 8$. Fig. 3a shows the normalized duality gaps and Fig. 3b the constraint violations w.r.t. the flow conservation law (FCL_{viol}) and the capacity constraint (CC_{viol}). It can be seen that the algorithm tracks relatively well down to normalized coherence time of 0.0075.

5. CONCLUSIONS

We stated a total transmit power minimization problem in a SSDG WSN and proposed a distributed algorithm which jointly optimizes the power allocation and the routing with the given SW coded source rates. First, the capacity constraint was relaxed leading to independently solvable subproblems, and then, second-level dual decompositions were applied to distribute the solution process across each protocol layer. The primal-dual method combined with the gradient projection method was used to update the variables. Numerical results showed that the algorithm converges in the certain network scenarios in static channels and has also the ability to track the solution under slow fading Rayleigh channels.

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