SIMPLIFIED ROBUST FIXED-COMPLEXITY SPHERE DECODER

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ABSTRACT

A simplified robust fixed-complexity sphere decoder (SRFSD) is proposed in this paper. SRFSD reduces the complexity of robust fixed complexity sphere decoder (RFSD) by choosing a detection order minimizing the upper bound of the power of the interference in single expansion (SE) stage. Theoretical proof is given to support the feasibility of SRFSD. Simulation results show that SRFSD retains the robustness of RFSD, and sharply reduces the complexity of RFSD with little sacrifice in bit error rate (BER) performance.

1. INTRODUCTION

Multiple input multiple output (MIMO) has become one of the most promising technology in wireless communication, but the detection of MIMO system is really a time-demanding process. The maximum likelihood (ML) detection has an unaffordable complexity and the suboptimal algorithms with acceptable complexity usually have significant performance degradation. Various algorithms have been proposed to achieve the tradeoff between performance and complexity. The zero-forcing (ZF) [1] detector has an attractive low complexity but with bad performance. Minimum mean square error (MMSE) [2] technique is superior in performance compared to ZF. However, there is still a huge gap between MMSE and ML. Even if MMSE and ZF are improved by MMSE ordered successive interference canceling (MMSE-OSIC) [3] and ZF ordered successive interference canceling (ZF-OSIC) [1], respectively, these gaps still can not be effectively filled. The sphere decoder (SD) [4] does greatly reduce the complexity of ML without significant performance sacrifice. The main drawback of SD lies in its variable complexity and the choice of radius, which depends on the system parameters such as SNR and channel conditions. A lower bound on its average complexity has been shown to be exponential [5]. Fixed-complexity sphere decoder (FSD) [6] can efficiently "fix" the complexity order of SD with a quasi-ML performance. However, FSD is not robust to the antenna configurations (e.g. $N_T > N_R$). The principe of FSD is based on the assumption that $N_T \leq N_R$. For a MIMO configuration with $N_T > N_R$, this method is no more effective since the left pseudo inverse of channel matrix doesn't exist. To overcome this problem, robust fixed-complexity sphere decoder (RFSD), which is robust to all the antenna configurations, is proposed in [7]. Besides its robustness, RFSD has better BER performance than that of FSD. The problem with RFSD is its computational complexity. The purpose of this paper is to develop further the work in [7], and find a simplified RFSD (SRFSD) with reduced complexity but without significant performance degradation.

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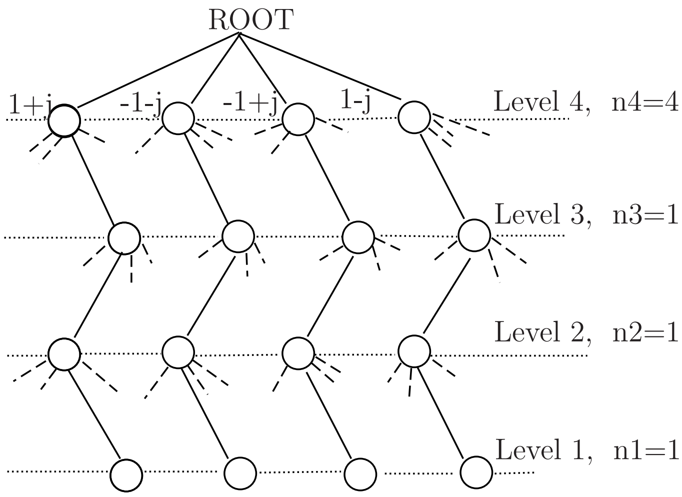

Figure 1: FSD(1, 1, 1, 4) search tree (QPSK, $N_T = 4$, $p = 1$)

Notation: Capital letters of boldface and lowercase letters of boldface are used for matrices, column vectors, respectively. $(\cdot)^H$, $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^\dagger$, $E[\cdot]$ and $\|\cdot\|$ denote operation of Hermitian, transpose, complex conjugate, left pseudo inverse, expectation and Frobenius norm respectively; $\mathbf{A}(:,k)$ represents the k^{th} column of matrix $\mathbf{A}$; $\mathbf{I}_N$ is the $N \times N$ identity matrix; $\mathbf{0}$ represents zero matrix or vector. $tr(\mathbf{A})$ denotes the trace of $\mathbf{A}$.

2. MIMO COMMUNICATION SYSTEM MODEL

The MIMO system model considered in this paper is a V-BLAST [1] system with N_R antennas at the receiver and N_T antennas at the transmitter. N_R is not necessarily equal or greater than N_T. The symbols of transmitted vector $\mathbf{x} = [x_1, x_2, \cdots, x_{N_T}]^T$ are independently drawn from M-QAM constellation of M points. The received vector $\mathbf{y}$ is given by equation (1):

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{w}$ is an additive white Gaussian noise vector, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_{N_T}]$ represents the frequency-flat channel. In the next sections, the following assumptions are made: 1. $E[\mathbf{x}\mathbf{x}^H] = P\mathbf{I}_{N_T}$, 2. $E[\mathbf{w}\mathbf{w}^H] = \sigma^2\mathbf{I}_{N_R}$, 3. $E[\mathbf{w}\mathbf{x}^H] = \mathbf{0}$.

3. FSD AND RFSD

3.1 FSD

FSD proposed by [6] efficiently "fixes" the complexity of SD by searching a fixed number of candidate vectors.

FSD follows a search order which is determined by a preprocessing algorithm dividing the searching into two stages: full expansion (FE) stage in the first p levels (level $N_T,\ N_T-1,\ \cdots,\ N_T-p+1$) and single expansion (SE) stage in the remaining N_T-p levels (level $N_T-p, N_T-p-1, \cdots, 1$), as shown in Fig. 1. In FE stage, FSD, similar to SD, is usually considered as a tree search: M branches are generated by each node at the first p levels. In SE stage, unlike SD which should calculate all possible children according to the information of its current node, FSD only needs to calculate one child for each node. The preprocessing algorithm proposed in [6] can be expressed as follows:

Preordering of FSD
$\mathbf{num} = [n_1,\ n_2,\ \cdots,\ n_{N_T}]$ is preset
$\mathbf{index} = [\ ]$
$\mathbf{H}_{N_T} = \mathbf{H}$
$for\ i = N_T : -1 : 1$
$\mathbf{H}_i{}^{\dagger} = (\mathbf{H}_i^H\mathbf{H}_i)^{-1}\mathbf{H}_i^H$
$define\ \boldsymbol{\gamma}_i = [\|(\mathbf{H}_i^{\dagger})_1\|^2,\ \|(\mathbf{H}_i^{\dagger})_2\|^2,\ \cdots,\ \|(\mathbf{H}_i^{\dagger})_{N_T}\|^2]^T$
$if\ n_i = M$
$l_i = arg\{max_j\ (\boldsymbol{\gamma}_i)\},\ j \in \{1,\ 2,\ \cdots,\ N_T\} - \{\mathbf{index}\}$
$else$
$l_i = arg\{min_j\ (\boldsymbol{\gamma}_i)\},\ j \in \{1,\ 2,\ \cdots,\ N_T\} - \{\mathbf{index}\}$
end
$\mathbf{H}_i(:,l_i) = \mathbf{0}, \mathbf{H}_{i-1} = \mathbf{H}_i, \mathbf{index} = [l_i,\ \mathbf{index}]$
end

where $\mathbf{H}_i$ is the channel matrix at level i, it preserves i columns of $\mathbf{H}$, with other N_T-i columns zeroed. $\mathbf{H}_i$ is obtained by zeroing the l_{i+1}^{th} column of $\mathbf{H}_{i+1}$. $(\mathbf{H}_i^{\dagger})_j$ is the j^{th} row of $\mathbf{H}_i^{\dagger}$. $\boldsymbol{\gamma}_i$ is a reference vector indicating the post-processing noise amplification of $\mathbf{H}_i^{\dagger}$. The searching order is totally determined by the vectors $\mathbf{num}$ and $\mathbf{index}$. Vector $\mathbf{num}$ divides the searching into FE stage and SE stage, and vector $\mathbf{index}$ indicates the order of FE and SE.

Remarks 1:
One should note that the definition of $(\cdot)^{\dagger}$ is exactly the same as that in [1], [6]. It is necessary to explain the left pseudo inverse of a matrix in our case, we assume a $n \times m$ $(n \geq m)$ matrix Λ with $a_1^{th},\ a_2^{th},\ \cdots, a_k^{th}$ columns zeroed $k < m$, $\Lambda^{\dagger}$, the left pseudo inverse of Λ should meet the following conditions: $\Lambda^{\dagger}\Lambda = diag(\lambda_1,\ \lambda_2,\ \cdots,\ \lambda_m)$, where $\lambda_i = 1$, if $i \notin \{a_1,\ a_2,\ \cdots, a_k\}$, $\lambda_i = 0$ if $i \in \{a_1,\ a_2,\ \cdots, a_k\}$. It is possible that $\Lambda^{\dagger}$ is not unique, the one with the smallest Frobenius norm is selected for V-BLAST. Obviously, for the optimal $\Lambda^{\dagger}$, the $a_1^{th},\ a_2^{th},\ \cdots, a_k^{th}$ rows of $\Lambda^{\dagger}$ are zero. We assume Λ_1 is the matrix obtained by removing $a_1^{th},\ a_2^{th},\ \cdots, a_k^{th}$ zero columns from Λ and we also assume that Λ_2 is the matrix obtained by removing $a_1^{th},\ a_2^{th},\ \cdots, a_k^{th}$ zero rows from $\Lambda^{\dagger}$, then we have $\Lambda_1^{\dagger} = \Lambda_2$.

3.2 RFSD

RFSD [7] generalizes the FSD problem as follows: choosing a suitable $\mathbf{H}_{N_T-p}$ with the minimal post-processing noise amplification in following SE stages by zeroing suitable p column(s) of $\mathbf{H}$. There are $C_{N_T}^{N_T-p} = C_{N_T}^{p}$ possible channel matrices $\mathbf{H}_{N_T-p}$ at level N_T-p, we denote $\mathbf{H}_{N_T-p}^r$, $r = 1,\ 2,\ \cdots,\ C_{N_T}^p$ to distinguish them, different value r corresponds to different detection order O_r, $O_r(s)$ denotes the index of the detected signal at level s. The corresponding channel matrix in SE stage at level s, represented by $\mathbf{H}_s^r$, $s = 1,\ 2,\ \cdots,\ N_T-p-1$, can be obtained by performing OSIC [1] on $\mathbf{H}_{N_T-p}^r$. We denote $\boldsymbol{\alpha}_s^r$ as a reference vector to indicate the post-processing noise amplification as follows:

$$\boldsymbol{\alpha}_s^r = [\|(\mathbf{H}_s^r)_1^{\dagger}\|^2,\ \|(\mathbf{H}_s^r)_2^{\dagger}\|^2,\ \cdots,\ \|(\mathbf{H}_s^r)_{N_T}^{\dagger}\|^2]^T \tag{2}$$

where $(\mathbf{H}_s^r)_j^{\dagger}$ is the j^{th} row of $(\mathbf{H}_s^r)^{\dagger}$. $\boldsymbol{\alpha}_s^r(O_r(s+1)), \cdots,$ $\boldsymbol{\alpha}_s^r(O_r(N_T))$ are ignored and set to 0 since signal components $x_{O_r(s+1)},\ \cdots,\ x_{O_r(N_T)}$ are detected and canceled in previous levels $s+1,\ \cdots,\ N_T$. Denoting

$$\mathbf{c}_{rs}^H = (\mathbf{H}_s^r)_k^{\dagger},\ k = arg(min_j\boldsymbol{\alpha}_s^r), j \neq O_r(s+1), ..., O_r(N_T) \tag{3}$$

the order which minimizes the post-processing impact on SE stage being chosen, the process can be written as:

$$\arg\min_r \sum_{s=1}^{N_T-p} \|\mathbf{c}_{rs}\|^2 \tag{4}$$

The optimal order can be recorded by $\mathbf{index}$, RFSD is performed according to p, $\mathbf{index}$ and $\mathbf{c}_{rs}$. The ordering process of RFSD with $p = 2$ can be expressed as follows:

Preordering of RFSD
$\mathbf{num} = [n_1,\ n_2,\ \cdots,\ n_{N_T}]$ is preset
$\mathbf{index} = [\ ], \mathbf{H}_{N_T} = \mathbf{H}, r = 0, \overline{sum} = +\infty$
$for\ m = 1 : 1 : N_T$ ························ $loop\ 1$
$for\ l = m+1 : 1 : N_T$ ······················ $loop\ 2$
$r = r+1,\ S_r = [m,\ l],\ sum = 0$
$O_r = S_r, \mathbf{H} = \mathbf{H}_{N_T}, \mathbf{H}(:,l) = \mathbf{0}, \mathbf{H}(:,m) = \mathbf{0}, \mathbf{H}_{N_T-p}^r = \mathbf{H}$
$for\ i = N_T-p : -1 : 1$
$(\mathbf{H}_i^r)^{\dagger} = ((\mathbf{H}_i^r)^H\mathbf{H}_i^r)^{-1}(\mathbf{H}_i^r)^H$
$define\ \boldsymbol{\alpha}_i^r = [\|(\mathbf{H}_i^r)_1^{\dagger}\|^2,\ \|(\mathbf{H}_i^r)_2^{\dagger}\|^2,\ ...,\ \|(\mathbf{H}_i^r)_{N_T}^{\dagger}\|^2]^T$
$\mathbf{c}_{ri}^H = (\mathbf{H}_i^r)_k^{\dagger}, k = arg(min_j\boldsymbol{\alpha}_i^r), j \in \{1,\ 2, ..., N_T\} - O_r$
$\mathbf{H}_i^r(:,k) = \mathbf{0}, \mathbf{H}_{i-1}^r = \mathbf{H}_i^r, O_r = [k,\ O_r]$
$sum = sum + \|\mathbf{c}_{ri}\|^2$
end
$if\ \ sum < \overline{sum}$
$\overline{sum} = sum, \mathbf{index} = O_r$
end
end
end

4. SRFSD

RFSD does have both excellent performance and robustness. However, one notes that we need calculate $(N_T-p)C_{N_T}^p$ left pseudo inverses. To reduce the complexity, we consider the reference vector at level N_T-p only. Reference vector $\|\boldsymbol{\alpha}_{N_T-p}^r\|^2$ can be expressed as:

$$\|\boldsymbol{\alpha}_{N_T-p}^r\|^2 = tr[(\mathbf{H}_{N_T-p}^r)^{\dagger}((\mathbf{H}_{N_T-p}^r)^{\dagger})^H] \tag{5}$$

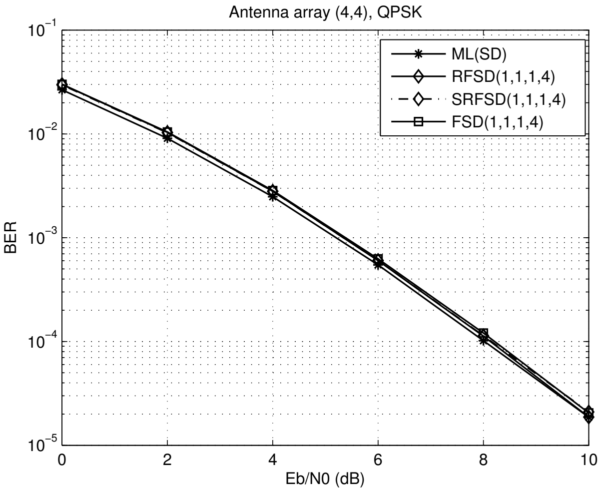

Figure 2: Performance of SRFSD (QPSK, $N_T = 4, N_R = 4$)

Lemma: $\|\boldsymbol{\alpha}^r_{N_T-p}\|^2$ is the upper bound of $\sum_{s=1}^{N_T-p}\|\mathbf{c}_{rs}\|^2$, namely

$$\sum_{s=1}^{N_T-p}\|\mathbf{c}_{rs}\|^2 \le tr[(\mathbf{H}^r_{N_T-p})^\dagger((\mathbf{H}^r_{N_T-p})^\dagger)^H] \tag{6}$$

□

Proof: Assuming O_r is an arbitrary detection order, $O_r(s)$ represents the index of the detected signal component at level s, $s = N_T,\ N_T-1,\ \cdots,\ 1$. It is natural that $\mathbf{H}^r_s(:,O_r(i)) = \mathbf{0}$ for $i = N_T,\ N_T-1,\ \cdots,\ s+1$. We consider the following two optimization problems.

Problem I: Find a suitable $N_R \times 1$ vector $\mathbf{v}_{s-1}$, $(2 \le s \le N_T - p)$ which minimizing the following criterion for a given q, $1 \le q \le s-1$.

$$\begin{cases} \min J_1 = \mathbf{v}^H_{s-1}\mathbf{v}_{s-1} \\ s.t.\ 1.\ for\ all\ k \ne q, 1 \le k \le s-1 \\ \mathbf{v}^H_{s-1}\mathbf{H}^r_{s-1}(:,O_r(k)) = 0 \\ 2.\quad \mathbf{v}^H_{s-1}\mathbf{H}^r_{s-1}(:,O_r(q)) = 1 \end{cases} \tag{7}$$

Problem II: Find a suitable $N_R \times 1$ vector $\mathbf{v}_s$, $(2 \le s \le N_T - p)$ which minimizing the following criterion for a given q, $1 \le q \le s-1$.

$$\begin{cases} \min J_2 = \mathbf{v}^H_s\mathbf{v}_s \\ s.t.\ \ 1.\ \ for\ all\ k' \ne q, 1 \le k' \le s \\ \mathbf{v}^H_s\mathbf{H}^r_s(:,O_r(k')) = 0 \\ 2.\ \ \mathbf{v}^H_s\mathbf{H}^r_s(:,O_r(q)) = 1 \end{cases} \tag{8}$$

Problem II is equivalent to

$$\begin{cases} \min J_2 = \mathbf{v}^H_s\mathbf{v}_s \\ s.t.\ 1.\ for\ all\ k \ne q, 1 \le k \le s-1 \\ \mathbf{v}^H_s\mathbf{H}^r_{s-1}(:,O_r(k)) = 0 \\ 2.\ \ \mathbf{v}^H_s\mathbf{H}^r_{s-1}(:,O_r(q)) = 1 \\ 3.\ \ \mathbf{v}^H_s\mathbf{H}^r_s(:,O_r(s)) = 0 \end{cases} \tag{9}$$

The two problems are exactly the same except that problem II has one more constraint (Constraint 3) than problem I, which means that the optimal solution of problem II is just a suboptimal solution to problem I, namely *min* $(\mathbf{v}^H_{s-1}\mathbf{v}_{s-1}) \le$ *min* $(\mathbf{v}^H_s\mathbf{v}_s)$. Actually, the optimal $\mathbf{v}^H_{s-1}$ and $\mathbf{v}^H_s$ are given by $(\mathbf{H}^r_{s-1})^\dagger_{O_r(q)}$ and $(\mathbf{H}^r_s)^\dagger_{O_r(q)}$ respectively. Thus, we have the following inequality:

$$\|(\mathbf{H}^r_{s-1})^\dagger_{O_r(q)}\|^2 \le \|(\mathbf{H}^r_s)^\dagger_{O_r(q)}\|^2 \le ... \le \|(\mathbf{H}^r_{N_T-p})^\dagger_{O_r(q)}\|^2 \tag{10}$$

The signal component with index $O_r(s-1)$ is detected at level $s-1$ if detection order O_r is adopted, which means that $q = s-1$, we have

$$\|(\mathbf{H}^r_{s-1})^\dagger_{O_r(s-1)}\|^2 \le \|(\mathbf{H}^r_{N_T-p})^\dagger_{O_r(s-1)}\|^2 \tag{11}$$

which in turn leads to

$$\sum_{s=2}^{N_T-p}\|(\mathbf{H}^r_{s-1})^\dagger_{O_r(s-1)}\|^2 \le \sum_{s=2}^{N_T-p}\|(\mathbf{H}^r_{N_T-p})^\dagger_{O_r(s-1)}\|^2 \tag{12}$$

Add $\|(\mathbf{H}^r_{N_T-p})^\dagger_{O_r(N_T-p)}\|^2$ to both sides of inequality (12), the right side becomes $tr[(\mathbf{H}^r_{N_T-p})^\dagger((\mathbf{H}^r_{N_T-p})^\dagger)^H$ and the left side becomes:

$$\sum_{s=2}^{N_T-p}\|(\mathbf{H}^r_{s-1})^\dagger_{O_r(s-1)}\|^2 + \|(\mathbf{H}^r_{N_T-p})^\dagger_{O_r(N_T-p)}\|^2 \tag{13}$$

O_r is just an arbitrary detection order, for the order proposed by RFSD, formula (13) is $\sum_{s=1}^{N_T-p}\|\mathbf{c}_{rs}\|^2$, finally, $\sum_{s=1}^{N_T-p}\|\mathbf{c}_{rs}\|^2 \le tr[(\mathbf{H}^r_{N_T-p})^\dagger((\mathbf{H}^r_{N_T-p})^\dagger)^H]$ is derived.

□

The purpose of SRFSD is to minimize the upper bound of $\sum_{s=1}^{N_T-p}\|\mathbf{c}_{rs}\|^2$ by choosing a proper detection order O_r that

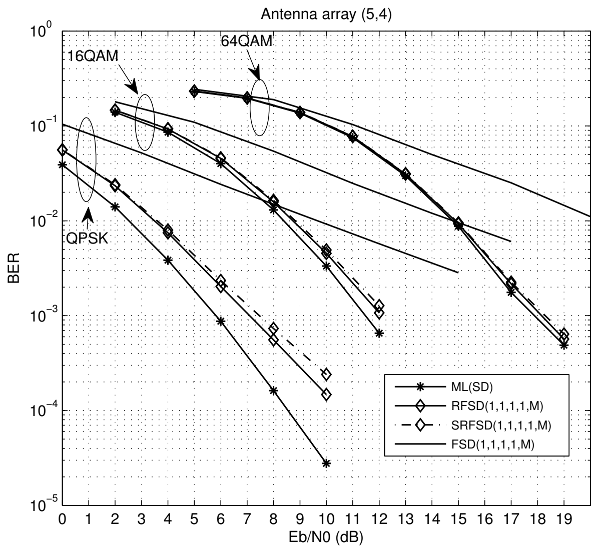

Figure 3: Performance of SRFSD ($N_T = 5, N_R = 4$)

determines which layers need a FE. It can be described as follows:

Preordering of SRFSD
$\mathbf{num} = [n_1, \, n_2, \, \cdots, \, n_{N_T}]$ is preset
$\mathbf{index} = [\,], \mathbf{H}_{N_T} = \mathbf{H}, \, r = 0$
$for \, m = 1:1:N_T \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \, loop \, 1$
$\quad for \, l = m+1:1:N_T \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \, loop \, 2$
$\quad r = r+1, \, S_r = [m, \, l]$
$\quad \mathbf{H} = \mathbf{H}_{N_T}, \, \mathbf{H}(:,l) = \mathbf{0}, \, \mathbf{H}(:,m) = \mathbf{0}, \, \mathbf{H}^r_{N_T-p} = \mathbf{H}$
$\quad (\mathbf{H}^r_{N_T-p})^{\dagger} = ((\mathbf{H}^r_{N_T-p})^H \mathbf{H}^r_{N_T-p})^{-1} (\mathbf{H}^r_{N_T-p})^H$
$\quad end$
end
$\quad k = \arg\min_r tr[(\mathbf{H}^r_{N_T-p})^{\dagger}((\mathbf{H}^r_{N_T-p})^{\dagger})^H]$

Remarks 2:
1. SRFSD listed above can find an optimal set S_k which contains the indexes of layers needing to be fully expanded. The detection order of other layers can be obtained exactly like SE in traditional FSD.
2. We can learn that only $C^p_{N_T} + N_T - p$ left pseudo inverses need to be calculated. The number is $2N_T - 1$ when $p = 1$, a little more than traditional FSD which needs to calculate N_T left pseudo inverses.

5. SIMULATION RESULTS

The BER performance of SRFSD has been examined by means of Monte-Carlo simulation. The simulated MIMO systems are V-Blast systems with different configurations of antennas. In addition, E_b/N_0 is defined as $E_b/N_0 = \frac{P}{\sigma^2 log_2 M}$, E_b/N_0 is represented in dB in the figures of this paper. Groups of simulation have been done to compare SRFSD with RFSD, FSD, and ML(SD) in terms of BER and flops/symbol, the results are shown in Fig.2-5. Fig.3-5 simulate a 5×4 system with difference modulation schemes (where the left pseudo inverse $\mathbf{H}^{\dagger}$ is still calculated by the MATLAB function "$pinv(\cdot)$", we just use it as a demonstration although the left pseudo inverse of $\mathbf{H}$ dose not exit.), it is shown by Fig.3-5 that, compared with RFSD, SRFSD can still achieve excellent BER performance with greatly reduced complexity, even through $N_T > N_R$. In addition, Fig.4-5 shows that almost both RFSD and SRFSD have lower complexity than ML(SD). Fig. 2 is obtained based on a 4×4 QPSK system. The BER curves of FSD, RFSD, SRFSD and ML, in Fig. 2, are overlapped by one another. In conclusion, SRFSD, not only retains the robustness of RFSD, but also sharply reduces the complexity of RFSD with little performance degradation.

6. CONCLUSION

The selection of the signal components to be detected at FE stage is a key factor for FSD, RFSD and SRFSD. RFSD is based on the selection in FE stage with the smallest post processing impact on SE stage. It is better than original FSD in terms of BER and robustness, but it introduces substantial computational complexity. To alleviate the complexity of RFSD, a simplified RFSD is proposed in this paper by selecting the signal components in FE stage to minimize the upper bound of the power of the interference in SE stage. Simulation shows that SRFSD not only retains the robustness of RFSD, but also sharply reduces the complexity of RFSD

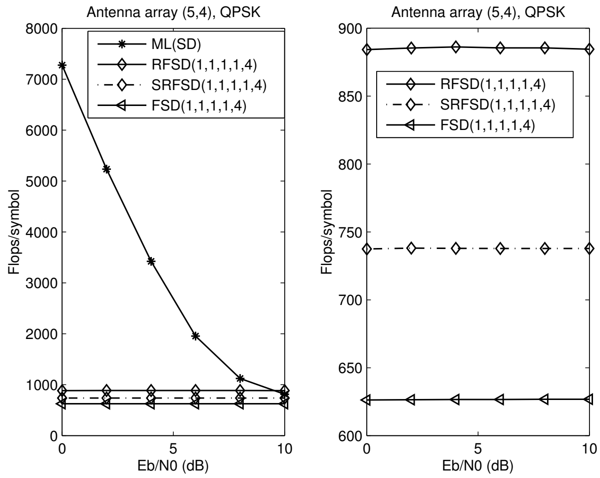

Figure 4: Complexity comparison (QPSK, $N_T = 5, N_R = 4$)

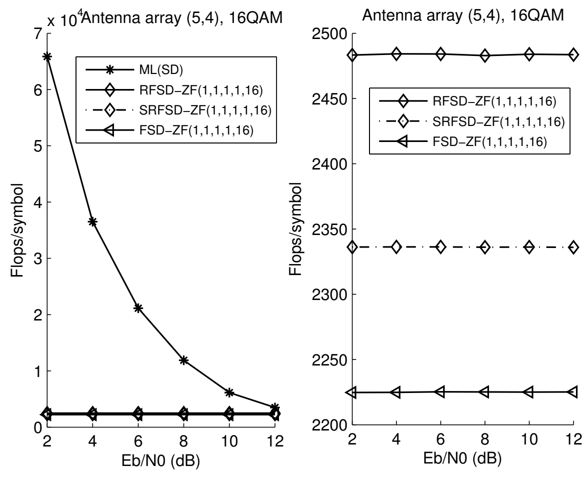

Figure 5: Complexity comparison (16QAM, $N_T = 5, N_R = 4$)

with little performance degradation.

REFERENCES

[1] P. W. Wolniansky, G. J. Foschini and G. D. Golden et al., “V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless channel,” in Proc. ISSSE, Pisa, Italy, pp. 295-300, 29 Sept.-2 Oct. 1998.

[2] V. M. Bogachev and I. G. Kiselev, “Optimum combining of signals in space-diversity reception,” Telecommun. Radio Eng., Vol. 34-35, pp. 83-85, 1980.

[3] David Tse, Fundmentals of wireless communication, Cambridge, pp. 357-362, 2007.

[4] M.O. Damen, H.E. Gamal, and G. Caire, ”On maximumlikelihood detection and the search for the closest lattice point,” IEEE Trans. Inf. Theory, Vol.49, pp.2389-2402, 2003.

[5] J. Jalden and B. Ottersten, On the complexity of sphere decoding in digital communications.” IEEE Transactions on Signal Processing, Vol. 53, pp. 1474-1484, 2005.

[6] L.G. Barbero, and J.S. Thompson, ”A Fixed-Complexity MIMO Detector Based on the Complex Sphere Decoder,” in 7th IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC ’06), Cannes, France, July 2006, pp. 1-5.

[7] Y. Ding, Y. Wang, J. F .Diouris, “Robust fixed complexity sphere decoder”, IEEE GLOBECOM 2010, Miami, Florida, USA, 2010.