GENERALIZED RECTIFICATION IN THE L_1-NORM WITH APPLICATION TO ROBUST ARRAY PROCESSING

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ABSTRACT

We present in this paper a multiple linear regression scheme in the L_1-norm which operates on the array output covariance matrix. The proposed method rectifies the covariance matrix by exploiting the structural properties resulting from the array geometry. In the case of a defective sensor, it is able to fully restore the covariance which would be observed if all sensors were working correctly. Simulations in Direction Of Arrival estimation with a uniform circular array demonstrate the interest of this L_1 approach compared to conventional L_2 techniques.

1. INTRODUCTION

Most array processing methods are based on the array output covariance matrix: one can cite Direction Of Arrival (DOA) estimation techniques and adaptive detection schemes such as the Adaptive Matched Filter. Performance of these methods depend on the quality of the covariance estimate: they may be heavily impacted by estimation noise or by a failing sensor. A simple way for improving the covariance matrix estimate is to exploit its structural properties. In array processing with M sensors, the $M \times M$ covariance matrix $\mathbf{\Gamma}$ belongs to the M^2-dimensional vector space (over $\mathbb{R}$) of complex Hermitian matrices. However, in the case of a Uniform Linear Array (ULA), $\mathbf{\Gamma}$ has a Toeplitz structure and lies in a vector subspace of much smaller dimension than M^2, namely a $L = 2M-1$-dimensional subspace (over $\mathbb{R}$). As shown in [4], this property of ULA's is shared by other array geometries for an appropriate value of L: this is the case, for instance, for circular arrays. In this framework, when using noisy data, a multiple linear regression over the covariance matrix entries may improve drastically the covariance matrix estimate. Regression in the L_2-norm leads to a conventional rectification procedure which simply amounts, in the ULA case, to averaging the covariance matrix elements along each diagonal. This approach, which was extended to arrays of arbitrary geometry in [4], allows to reduce estimation noise. However, regression in the L_2-norm is well known to be non-robust to outliers: such outliers among the covariance matrix entries are produced by a failing sensor which generates a whole line and a whole column of wrong values. A well-known robust alternative to regression in the L_2-norm is offered by regression in the L_1-norm, and we show in this paper that regression in the L_1-norm applied to the array covariance matrix allows to fix a possible defective sensor in the case of a circular array.

As a very simple illustration in the ULA case, let us consider a $M = 7$ sensors ULA with ideal covariance matrix:

$$\mathbf{\Gamma} = \boldsymbol{T}(1,2,3,4,5,6,7), \tag{1}$$

where $\boldsymbol{T}(a_1, a2, \ldots, a_M)$ is the Hermitian Toeplitz matrix whose first column is $a_1, a_2, \ldots, a_M$. Assume that the third sensor is defective so that the resulting covariance matrix $\mathbf{\Gamma}_d$ at the array output differs from $\mathbf{\Gamma}$: instead of the nominal value $(3,2,1,2,3,4,5)$, the third line and the third column are $(0,0,0.1,0,0,0,0)$ where 0.1 is the noise power at the output of the defective sensor. Regression in the L_2-norm of $\mathbf{\Gamma}_d$ in the space of Toeplitz matrices leads to:

$$\tilde{\mathbf{\Gamma}}_{d,L_2} = \boldsymbol{T}(0.87, 1.33, 1.8, 3, 3.33, 6, 7) \neq \mathbf{\Gamma}, \tag{2}$$

while regression in the L_1-norm **restores the ideal covariance**:

$$\tilde{\mathbf{\Gamma}}_{d,L_1} = \mathbf{\Gamma} \tag{3}$$

This error correcting capability of L_1-regression is somewhat limited in the ULA case: it works only if the non-defective entries are in the majority along each diagonal, and is therefore not fully effective when the failing sensor is close to the ends. *These restrictions do not hold anymore in the case of circular arrays*, as will be seen on simulations: for an infinite number of snapshots, the L_1-norm is fully able to restore the ideal covariance whatever the failing sensor. In the case of a finite number of snapshots and a defective sensor, L_1-regression of the estimated array covariance matrix still outperforms L_2-regression: the MUSIC algorithm applied to the resulting covariance matrix yields the same DOA estimates as if all sensors were running properly.

The paper is organized as follows: section 2 recalls multiple linear regression in the L_1 norm, section 3 presents its application to generalized rectification in array processing and section 4 shows the simulation results. The following convention is adopted: italic indicates a scalar quantity, lower case boldface indicates a vector quantity and upper case boldface a matrix. ^{T} denotes the transpose operator and ^{H} the transpose conjugate. $\mathbf{I}_N$ is the N-th order identity matrix, $\|\ \|_F$ is the Froebenius norm and $\mathbb{E}()$ is the expectation value operator.

2. MULTIPLE LINEAR REGRESSION IN THE L_1 NORM

Let $\mathbf{y}, \mathbf{y}_\varepsilon \in \mathbb{R}^N$ be two real vectors where $\mathbf{y}_\varepsilon$ is assumed to belong to a subspace ε of dimension L. These two vectors are assumed to be linked by the following relation:

$$\mathbf{y} = \mathbf{y}_\varepsilon + \mathbf{e} \tag{4}$$

where $\mathbf{e} \in \mathbb{R}^N$ is an error vector due to noise, mis-modeling, calibration errors, ... Our purpose is to find an estimate in the L_1-norm of $\mathbf{y}_\varepsilon$, denoted $\tilde{\mathbf{y}}$, which belongs to the subspace ε. Let $\{\mathbf{u}_i; i \in [\![1;L]\!]\}$ be a basis of ε, and $\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_L$ the components of $\tilde{\mathbf{y}}$ in this basis:

$$\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_L = \underset{\lambda_1,\lambda_2,\dots,\lambda_L}{\text{Argmin}} \left(\left\| \mathbf{y} - \sum_{i=1}^{L} \lambda_i \mathbf{u}_i \right\|_1 \right) \quad (5)$$

$$= \underset{\lambda_1,\lambda_2,\dots,\lambda_L}{\text{Argmin}} \left(\sum_{t=1}^{N} \left| y_t - \sum_{i=1}^{L} \lambda_i u_{i,t} \right| \right) \quad (6)$$

where y_t and $u_{i,t}$ are respectively the t-th components of $\mathbf{y}$ and $\mathbf{u}_i$.

It is well known that multiple regression in L_1-norm is readily solved by linear programming techniques [2]. Indeed, problem (6) may be rewritten as:

$$\text{Minimize} \sum_{t=1}^{N} v_t, \quad (7)$$

subject to:

$$-v_t \leqslant y_t - \sum_{i=1}^{L} \lambda_i u_{i,t} \leqslant v_t \qquad \forall t \in [\![1;N]\!]$$

This is a linear program with an appropriate variable $\mathbf{z}$, and there exists very fast algorithms for solving it [1, 6]:

$$\underset{\mathbf{z}}{\text{Minimize}}\ \mathbf{c}^T \mathbf{z} \quad (8)$$

subject to:

$$\mathbf{A}\mathbf{z} \leq \mathbf{b}$$

with:

$$\left\{ \begin{array}{l} \mathbf{z} = (\lambda_1\ \dots\ \lambda_L\ v_1\ \dots\ v_N) \\ \mathbf{c} = (\underbrace{0\ \dots\ 0}_{L}\ \underbrace{1\ \dots\ 1}_{N})^T \\ \mathbf{A} = \begin{pmatrix} \mathbf{U} & -\boldsymbol{I}_N \\ -\mathbf{U} & -\boldsymbol{I}_N \end{pmatrix} \\ \mathbf{U} = (\mathbf{u}_1\ \dots\ \mathbf{u}_L) \\ \mathbf{b} = (y_1\ \dots\ y_N\ -y_1\ \dots\ -y_N) \end{array} \right. \quad (9)$$

A well known property of linear regression in the L_1 norm is that the solution $\tilde{\mathbf{y}}$ shares L components with $\mathbf{y}$ [7]. This explains the error-correcting capabilities of regression by the L_1-norm [3]: the solution $\tilde{\mathbf{y}}$ is equal to $\mathbf{y}_\varepsilon$ when the error vector $\mathbf{e}$ in Eq. (4) has a small number of non zero values. This also explains its robustness against outliers compared to classical regression in the L_2-norm [5].

We exploit these properties in an array processing context to elaborate a covariance matrix estimate robust to a defective sensor.

3. APPLICATION TO ARRAY PROCESSING

Let us consider an array of M sensors which receives the signals emitted by P zero-mean uncorrelated sources. Let $\mathbf{a}(\theta)$ be the normalized steering vector where θ is the source DOA. The k-th snapshot impinging the array, denoted by $\mathbf{x}(k) \in \mathbb{C}^M$, is given by:

$$\mathbf{x}(k) = \sum_{p=1}^{P} s_p(k)\mathbf{a}(\theta_p) + \mathbf{b}(k) \qquad \forall k \in [\![1;K]\!] \quad (10)$$

where $s_p(k)$ is the p-th source signal, $\mathbf{b}(k) \in \mathbb{C}^M$ denotes the complex noise vector and K is the total number of snapshots.

Let

$$\begin{aligned} \boldsymbol{\Gamma} &= \mathbb{E}\left(\mathbf{x}(k)\mathbf{x}(k)^H\right) \\ &= \sum_{p=1}^{P} \gamma_p \mathbf{a}(\theta_p)\mathbf{a}^H(\theta_p) + \gamma_n \boldsymbol{I}_M \end{aligned} \quad (11)$$

be the ideal covariance matrix where γ_p and γ_n are respectively the p-th source and the noise powers. Applying array processing algorithms to this ideal covariance matrix leads to optimal results. Unfortunately, $\boldsymbol{\Gamma}$ is generally unknown and we have to use a degraded version $\check{\boldsymbol{\Gamma}}$. In this paper, we consider that this degradation may have two origins:

1. an estimation error due to a finite number of snapshots;
2. a mis-modeling due to a defective sensor which delivers only noise at its output.

In this context, array processing algorithms may perform poorly. Therefore, we propose to elaborate a robust covariance matrix estimator dealing with both degradations by using a rectification approach in the L_1-norm.

The principle of matrix rectification is to improve the estimation quality of $\check{\boldsymbol{\Gamma}}$ by exploiting structural properties of $\boldsymbol{\Gamma}$ which result from the array symmetries (as for the ULA and the uniform circular array). More precisely, it was shown in [4] that $\boldsymbol{\Gamma}$ generally belongs to a low L-dimensional subspace ε. The rectification process consists in constraining $\check{\boldsymbol{\Gamma}}$ to belong to this subspace ε. This rectification was developed in the L_2-norm in [4] and we propose in this paper to extend this method to the L_1-norm for a better robustness to a defective sensor. The resulting rectified matrix is denoted by $\tilde{\check{\boldsymbol{\Gamma}}}$.

3.1 Determination of a basis of ε

The rectification process first requires the determination of the subspace dimension L as well as a basis of ε. These steps are briefly described below and we refer to [4] for more details and in particular for the choice of L.

Let Θ of cardinal I be a dense sampling of all possible DOA's. Let $\mathbf{d}(\theta_i) \in \mathbb{R}^{2M^2}$, $\theta_i \in \Theta$:

$$\mathbf{d}(\theta_i) = vec\left(\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i) - \frac{\boldsymbol{I}_M}{M}\right) \quad (12)$$

where $vec(.)$ operates on a complex matrix of dimension M to provide a real vector containing the real and imaginary components in an arbitrary order. The $unvec(.)$ operation is defined such that $unvec(vec(\mathbf{H})) = \mathbf{H}$. Now, let $\mathbf{R} \in \mathbb{R}^{2M^2 \times 2M^2}$ be the matrix defined by:

$$\mathbf{R} = \sum_{i=1}^{I} \mathbf{d}(\theta_i)\mathbf{d}^T(\theta_i) \quad (13)$$

$\mathbf{R}$ has $2M^2$ eigenvalues and eigenvectors but only $L-1$ eigenvalues are significant. Let $\mathbf{u}_1, \dots, \mathbf{u}_{L-1}$ be the $L-1$ eigenvectors of $\mathbf{R}$ associated to the $L-1$ greatest eigenvalues of $\mathbf{R}$. The set of L hermitian's matrices in $\mathbb{C}^{M \times M}$

$$\left\{ \begin{array}{ll} \mathbf{U}_i = unvec(\mathbf{u}_i), & i \in [\![1;L-1]\!] \\ \mathbf{U}_L = \dfrac{\boldsymbol{I}_M}{\sqrt{M}} & \end{array} \right. \quad (14)$$

is an orthonormal basis of ε [4].

3.2 Rectification in the L_1-norm

The rectified matrix $\tilde{\check{\Gamma}}$ of $\check{\Gamma}$ is determined by estimating its coordinates $\{\tilde{\lambda}_i; i \in [\![1;L]\!]\}$ in ε. This problem is addressed by a multiple linear regression in the L_1-norm as presented in section 2. This reduces to solve the linear program of Eq. (8) with the following correspondences:

$$\begin{cases} N = 2M^2 \\ \mathbf{y} = vec(\check{\Gamma}) \\ \mathbf{y}_\varepsilon = vec(\Gamma) \\ \mathbf{u}_i = vec(\mathbf{U}_i) \\ \tilde{\mathbf{y}} = vec(\tilde{\check{\Gamma}}) \end{cases} \quad (15)$$

The resulting rectified matrix is given by:

$$\tilde{\check{\Gamma}}_{L_1} = \sum_{i=1}^{L} \tilde{\lambda}_i \mathbf{U}_i \quad (16)$$

3.3 Discussion

By using the L_2-norm instead of the proposed L_1-norm, we are also able to obtain a rectified matrix $\tilde{\check{\Gamma}}_{L_2}$ as presented in [4]. In the ideal case (i.e. with an infinity of samples), by considering the properties of regression in the L_1-norm presented at the end of section 2, $\tilde{\Gamma}_{L_1}$ will be close to the ideal one even if the array has a defective sensor while $\tilde{\Gamma}_{L_2}$ will be strongly degraded by the defective sensor. In practical cases (i.e. with a limited number of samples) with a defective sensor, the estimated rectified covariance matrix $\tilde{\check{\Gamma}}_{L_1}$ will be close to the one obtained without the defective sensor while $\tilde{\check{\Gamma}}_{L_2}$ will be still degraded.

4. SIMULATIONS

4.1 Simulations setting

In this section, we consider a uniform circular array composed of $M = 10$ isotropic sensors with a radius $r = 0.7\lambda$ (λ is the wavelength of the transmitted signal). We assume $P = 2$ zero-mean uncorrelated gaussian far-field sources located at $-10°$ and $10°$. The dimension L of the subspace ε was determined in [4] and is equal to 21. Finally, the noise is assumed to be zero-mean complex gaussian and the signal to noise ratio is 10 dB.

Several degraded covariance matrices are considered:

- $\Gamma_d = \sum_{p=1}^{P} \gamma_p \mathbf{a}_d(\theta_p)\mathbf{a}_d^H(\theta_p) + \gamma_n \mathbf{I}_M$ where $\mathbf{a}_d(\theta)$ is equal to $\mathbf{a}(\theta)$ except for the defective sensor component which is 0. Γ_d is thus the array covariance matrix with a defective sensor;
- $\hat{\Gamma} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k)$ is the usual Sample Covariance Matrix (SCM) based on K snapshots without defective sensor;
- $\hat{\Gamma}_d = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}_d(k)\mathbf{x}_d^H(k)$ is the SCM with a defective sensor based on K snapshots $\mathbf{x}_d(k) = \sum_{p=1}^{P} s_p(k)\mathbf{a}_d(\theta_p) + \mathbf{b}(k)$.

By using the method previously described in section 3, these three matrices are rectified yielding $\tilde{\Gamma}_{d,L_1}$, $\tilde{\check{\Gamma}}_{L_1}$ and $\tilde{\check{\Gamma}}_{d,L_1}$. We will compare rectification in the L_1-norm as proposed in this paper with the conventional one performed in the L_2-norm [4].

4.2 Results with Γ_d

In this subsection, we consider the ideal case (i.e. with an infinity of sample) in order to evaluate easily the contribution of the L_1-norm in the rectification process when a defective sensor is present.

The first simulation checks that the rectification processes in L_1-norm and L_2-norm yield close results without defective sensor. The MUSIC algorithm based on $\tilde{\Gamma}_{L_1}$ and $\tilde{\Gamma}_{L_2}$ is used to estimate the sources DOA. The MUSIC pseudo-spectra resulting from $\tilde{\Gamma}_{L_1}$ and $\tilde{\Gamma}_{L_2}$ are plotted in Fig. 1: they are very similar and peaks are close to infinity as expected.

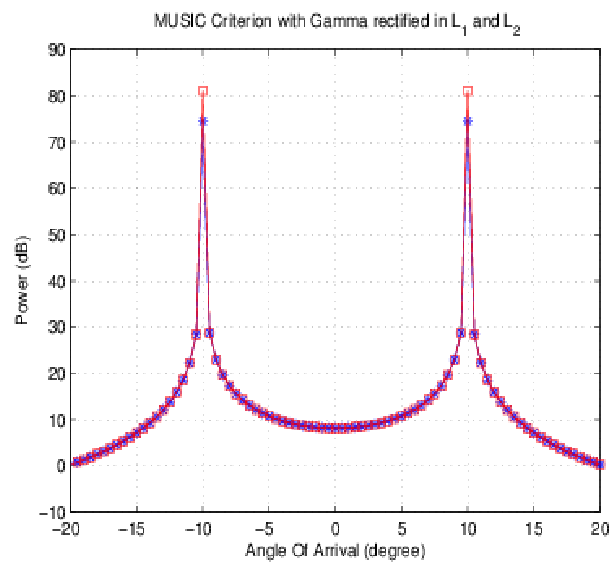

Figure 1: MUSIC pseudo-spectra for $\tilde{\Gamma}_{L_1}$ (blue star) and $\tilde{\Gamma}_{L_2}$ (red square).

In the second simulation, we are interested in the behaviour of both rectification schemes in front of a defective sensor. The MUSIC algorithm based on Γ_d (with defective sensor and without rectification), $\tilde{\Gamma}_{d,L_2}$ (with defective sensor and with rectification in L_2-norm) and $\tilde{\Gamma}_{d,L_1}$ (with defective sensor and with rectification in L_1-norm) is also used to estimate the sources DOA. The MUSIC pseudo-spectra resulting from Γ_d, $\tilde{\Gamma}_{d,L_2}$ and $\tilde{\Gamma}_{d,L_1}$ are plotted in Fig. 2. As a reference, we also plot the MUSIC pseudo-spectrum obtained with $\tilde{\Gamma}_{L_1}$. With a defective sensor, we notice that the peak height is strongly reduced to 0 dB without rectification. If Γ_d is rectified in the L_2-norm, the peaks go up to 10 dB and become more distinct. Finally, if Γ_d is rectified in the L_1-norm, both peaks are very close to those obtained without defective sensor. This illustrates that rectification in the L_1-norm allows to correct the defective sensor and to obtain almost ideal results when using an infinite number of snapshots.

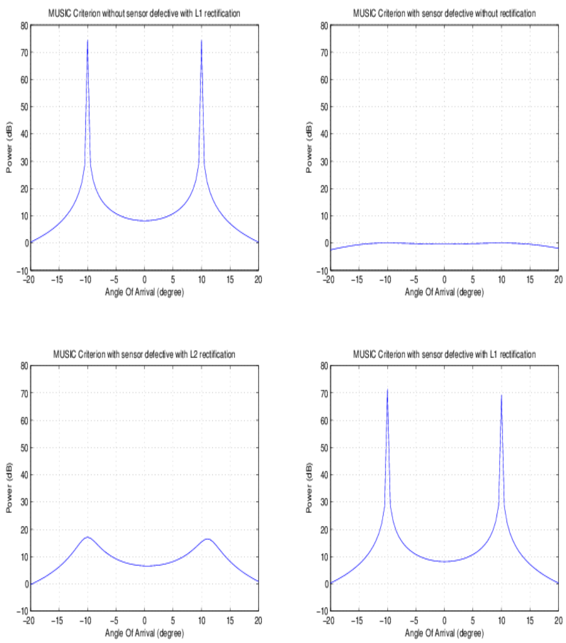

Figure 2: MUSIC pseudo-spectra for $\tilde{\mathbf{\Gamma}}_{L_1}$ (top left), $\mathbf{\Gamma}_d$ (top right), $\tilde{\mathbf{\Gamma}}_{d,L_2}$ (bottom left) and $\tilde{\mathbf{\Gamma}}_{d,L_1}$ (bottom right). One sensor is defective for $\mathbf{\Gamma}_d$, $\tilde{\mathbf{\Gamma}}_{d,L_1}$ and $\tilde{\mathbf{\Gamma}}_{d,L_2}$.

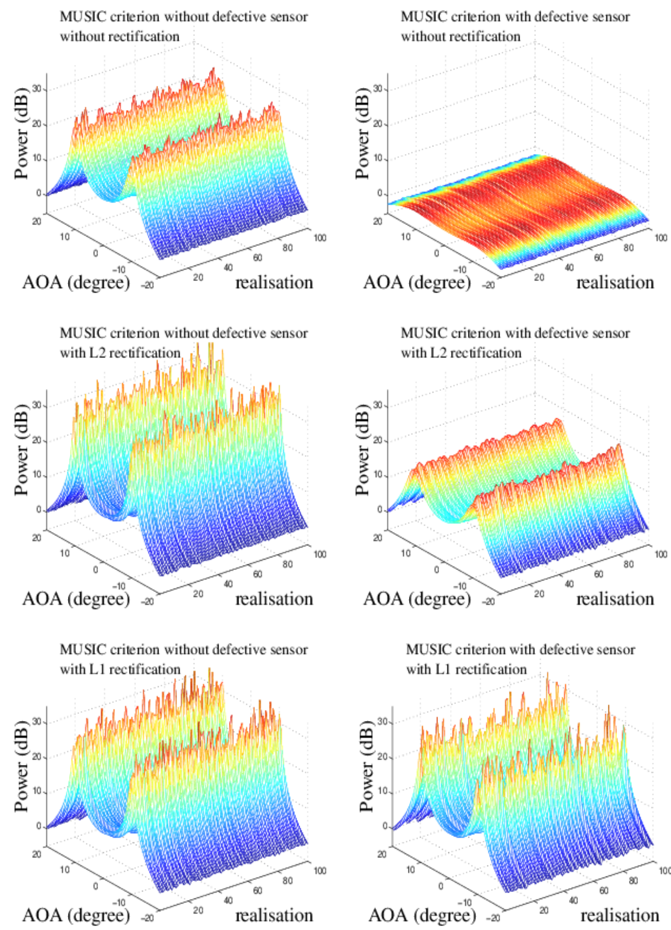

Figure 3: MUSIC pseudo-spectra for $\hat{\mathbf{\Gamma}}$ (top left), $\tilde{\hat{\mathbf{\Gamma}}}_{L_2}$ (middle left), $\tilde{\hat{\mathbf{\Gamma}}}_{L_1}$ (bottom left), $\hat{\mathbf{\Gamma}}_d$ (top right), $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_2}$ (middle right) and $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_1}$ (bottom right). One sensor is defective for $\hat{\mathbf{\Gamma}}_d$, $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_1}$ and $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_2}$.

4.3 Results with $\hat{\mathbf{\Gamma}}$ and $\hat{\mathbf{\Gamma}}_d$

Results with MUSIC

In this subsection, we consider the more realistic case where the number K of snapshots is finite. In all simulations, K is set to 200 for estimating $\hat{\mathbf{\Gamma}}$ and $\hat{\mathbf{\Gamma}}_d$. The MUSIC algorithm based on the SCM $\hat{\mathbf{\Gamma}}$ without defective sensor, the L_2-rectified SCM $\tilde{\hat{\mathbf{\Gamma}}}_{L_2}$ without defective sensor, the L_1-rectified SCM $\tilde{\hat{\mathbf{\Gamma}}}_{L_1}$ without defective sensor,the SCM $\hat{\mathbf{\Gamma}}_d$ with defective sensor, the L_2-rectified SCM $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_2}$ with defective sensor and the L_1-rectified SCM $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_1}$ with defective sensor, is used to estimate the sources DOA. The MUSIC pseudo-spectra resulting from $\hat{\mathbf{\Gamma}}$, $\tilde{\hat{\mathbf{\Gamma}}}_{L_2}$, $\tilde{\hat{\mathbf{\Gamma}}}_{L_1}$, $\hat{\mathbf{\Gamma}}_d$, $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_2}$ and $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_1}$ are plotted in Fig. 3. For each case, we have considered 100 trials.

Plots are organized as follows: the left (resp. right) column refers to results obtained without (resp. with) defective sensor. Plots are obtained on the first line with the SCM's ($\hat{\mathbf{\Gamma}}$ and $\hat{\mathbf{\Gamma}}_d$), on the second line with the L_2-rectified SCM's ($\tilde{\hat{\mathbf{\Gamma}}}_{L_2}$ and $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_2}$) and on the third line with L_1-rectified SCM's ($\tilde{\hat{\mathbf{\Gamma}}}_{L_1}$ and $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_1}$). Without defective sensor, we notice that the rectification process (in L_1 or L_2 norms) improves DOA estimation (a gain of 5 dB can be observed). With a defective sensor, only the L_1-rectification is able to provide a similar result as without defective sensor. This confirms the results previously obtained with an infinite of snapshots.

Mean Square Errors

In this paragraph, we give the normalized Root Mean Square Errors (RMSE) between the ideal covariance matrix $\mathbf{\Gamma}$ and:

- $\hat{\mathbf{\Gamma}}_d$, the SCM with the defective sensor;
- $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_2}$ the L_2-rectified SCM with a defective sensor;
- $\tilde{\hat{\mathbf{\Gamma}}}_{d,L_1}$ the L_1-rectified SCM with a defective sensor.

The corresponding RMSE's are defined as:

$$\begin{aligned} RMSE_d &= \sqrt{\mathbb{E}\left[\frac{\left|\left|\mathbf{\Gamma}-\hat{\mathbf{\Gamma}}_d\right|\right|_F^2}{||\mathbf{\Gamma}||_F^2}\right]} \\ RMSE_{d,L_2} &= \sqrt{\mathbb{E}\left[\frac{\left|\left|\mathbf{\Gamma}-\tilde{\hat{\mathbf{\Gamma}}}_{d,L_2}\right|\right|_F^2}{||\mathbf{\Gamma}||_F^2}\right]} \\ RMSE_{d,L_1} &= \sqrt{\mathbb{E}\left[\frac{\left|\left|\mathbf{\Gamma}-\tilde{\hat{\mathbf{\Gamma}}}_{d,L_1}\right|\right|_F^2}{||\mathbf{\Gamma}||_F^2}\right]} \end{aligned} \quad (17)$$

1000 trials are used to estimate these quantities.

Resulting RMSE's are plotted in Fig. 4 as a function of the number K of snapshots (SNR is 10 dB in this case). We notice that the RMSE computed from $\hat{\boldsymbol{\Gamma}}_d$ has a value of 40% for large K due to the defective sensor. When $\hat{\boldsymbol{\Gamma}}_d$ is rectified in the L_2-norm, the RMSE reaches 20%. But when $\hat{\boldsymbol{\Gamma}}_d$ is rectified in the L_1-norm, the RMSE converges to 0 when K grows as would be observed without defective sensor. Resulting RMSE's as a function of SNR ($K = 200$ in this case) are presented in Fig. 5. At low SNR, we observe that L_2 and L_1 rectifications perform equally because all sensors (defective or not defective) behave almost similarly: they deliver mainly noise. However at high SNR, we note the improvement brought by L_1-rectification.

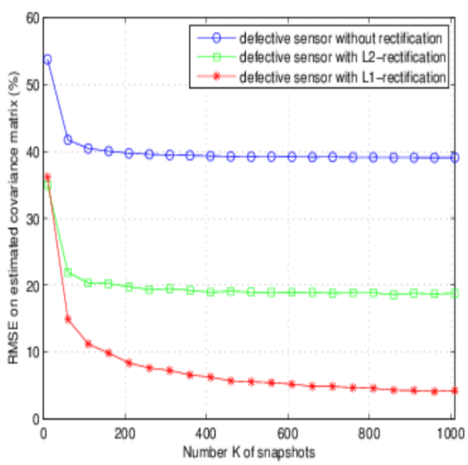

Figure 4: RMSE between $\boldsymbol{\Gamma}$ and: $\hat{\boldsymbol{\Gamma}}_d$ (blue circle), $\tilde{\hat{\boldsymbol{\Gamma}}}_{d,L_2}$ (green square), $\tilde{\hat{\boldsymbol{\Gamma}}}_{d,L_1}$ (red star). One sensor is defective for the three cases.

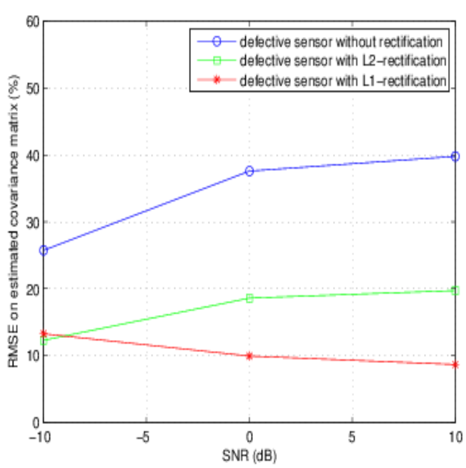

Figure 5: RMSE between $\boldsymbol{\Gamma}$ and: $\hat{\boldsymbol{\Gamma}}_d$ (blue circle), $\tilde{\hat{\boldsymbol{\Gamma}}}_{d,L_2}$ (green square), $\tilde{\hat{\boldsymbol{\Gamma}}}_{d,L_1}$ (red star). One sensor is defective for the three cases.

These results confirm the interest of the proposed L_1-rectification approach.

5. CONCLUSION

In the array processing framework, we have introduced a multiple regression scheme in the L_1-norm which exploits the structural properties of the covariance resulting from the array symmetries. The proposed method is a rectification technique which enhances the estimated array covariance matrix. In the case of a uniform circular array with a defective sensor, this method is able to fully restore the covariance matrix considered without default: it outperforms conventional L_2 rectification methods. Moreover, when applied to the sample covariance matrix without defective sensor, it yields the same performance as L_2 rectification. As a conclusion, the proposed L_1 rectification method brings robustness against sensor failure and there is only benefit to use it instead of L_2 rectification.

For future work, the theoretical statistical properties of the proposed method will be investigated.

REFERENCES

[1] I. Barrodale and F. Roberts. An improved algorithm for discrete l_1 linear approximation. *SIAM Journal of Numerical Analysis*, 10:839–848, 1973.

[2] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambride University Press, 2002.

[3] E. Candes and T. Tao. Decoding by linear programming. *IEEE Trans. on Information Theory*, 51(12):4203–4215, December 2005.

[4] P. Forster. Generalized cross spectral matrices for array of arbitrary geometry. *IEEE Trans. Sig. Proc.*, 49(5), May 2001.

[5] R. Maronna, R. Martin, and V. Yohai. *Robust Statistics*. John Wiley, 2006.

[6] S. Portnoy and R. Koenker. The gaussian hare and the laplacian tortoise: Computability of squared-error versus absolute-error estimators. *Statistical Science*, 12:279–300, 1997.

[7] M. Powell. *Approximation theory and methods*, chapter 14 & 15. Cambridge University Press, 1988.