RATE OPTIMIZATION FOR INCREMENTAL REDUNDANCY AND REPETITION CODING IN RELAY NETWORKS

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ABSTRACT

The deployment of relay nodes is a viable solution to fulfil the demands of high throughput and ubiquitous access for future mobile radio communications. A relay can cooperate by transmitting either a repetition of the previously transmitted signal (repetition coding) or by sending *new* code bits which have not been transmitted before (incremental redundancy). In this paper, a general optimization method for the achievable code rates in an adaptive relay network with repetition coding and incremental redundancy is presented.

1. INTRODUCTION

In relay networks different forwarding techniques at the relay such as amplify-and-forward (AF) and decode-and-forward (DF) have been investigated [1]. Adaptive relaying was introduced in [2]. This adaptivity takes the advantages of AF and DF and minimizes their disadvantages. Relay networks with such adaptivity are named adaptive relay networks.
There are two methods by which a relay can cooperate, namely repetition coding (RC) and incremental redundancy (IR) [3]. In repetition coding the relays (or source) retransmit repetitions of the source's first transmission. The destination can combine all the repetitions using maximum ratio combining (MRC).
In the case of incremental redundancy (IR), the relays (or the source) transmit additional parity bits which have not been transmitted before. The number of code bits transmitted from each relay may be different. The destination merges these bits (code combining) and tries to decode the resulting code.
For IR the relay must[1] successfully decode the received bits from the source, otherwise it cannot generate incremental redundancy. However, in the case of repetition coding this is not an issue. The relay can repeat the signal (e.g. amplify-and-forward) without successfully decoding it, moreover, repetition coding is simpler to implement in practice.
However, the problem with repetition coding (RC) is that for the sake of MRC the relay (or source) repeats the whole codeword. Contrarily, in the case of IR a relay can transmit theoretically even a single bit to the destination. The objective of this paper is to optimize and compare the performances of repetition coding and incremental redundancy in an adaptive relay network with specific setup.
In [4,5], a comparison of IR and RC for relay networks is performed. Reference [4] uses decode-and-forward with space time transmissions from the relays (the performance is identical to beamforming). In that contribution the authors activated all successfully decoding relays. As distributed beamforming is difficult to implement in practice, in this work we consider orthogonal multiple access. In orthogonal multiple access the activation of all relays is not optimal, hence, relay(s) are selected for contribution. Moreover, we consider adaptive relaying instead of pure decode-and-forward. In [5], only a single relay with decode-and-forward is considered. In [6] bits are assigned randomly to each transmission, in our simulation results we compare our optimization to the random assignment as done in [6] but with equal probability of assignment. Our new contributions are summarised as
- determining *optimal rates* for IR and RC.
- considering an *adaptive multi-relay network with orthogonal access.*
- In the case of IR, the lengths of sequences transmitted from the source and each of the relays *may not be the same.*
The rest of the paper is organized as follows. Sec. 2 presents the system model and Sec. 3 introduces resource allocation based on marginal returns. Sec. 4 optimizes the rates and number of transmissions for incremental redundancy and repetition coding. Finally, Sec. 5 discusses the simulation results, while, Sec. 6 concludes the paper.

2. RELAY NETWORK

First, the general channel model of the relay network is introduced. Second and third subsections explain relaying with incremental redundancy and with repetition coding.

2.1 System Model

The system model in Fig. 1 is restricted to a parallel dual-hop network. The source S encodes its K bits data sequence $\mathbf{u}$ using an ideal code. The modulation results in the sequence $\mathbf{x}_s$ of M_s symbols. For an **ideal coding scheme** [7], whenever, the accumulated mutual information $M_s \cdot C(\gamma)$ at a receiver (relay or destination) is greater than or equal to the info length K, then we have successful decoding at that receiver. Here, $C(\gamma)$ is the $\mathcal{M}$-ary AWGN channel capacity with γ being the SNR from the source to the receiver. We assume a general $\mathcal{M}$-ary modulation scheme which maps $m = log_2(\mathcal{M})$ bits to one of the possible $\mathcal{M}$ symbols. For the sake of the optimization procedure provided in this paper, we restrain only to the modulation schemes whose capacity is increasing concave function of the channel SNR. Moreover, it is assumed that each node uses the same $\mathcal{M}$-ary modulation and transmits with a constant power equal to 1. The source-relay, source-destination, and relay-destination links are assumed to be i.i.d. Rayleigh block-fading including path loss. The path loss exponent is 2 and complex AWGN has variance N_0. Next, the channel model with respect to IR and RC is provided.

2.2 Relaying with Incremental Redundancy

In the case of relaying with incremental redundancy (IR), the source first encodes the signal $\mathbf{u}$ using an ideal coding scheme, resulting in the code word $\mathbf{x}_M$ as shown in Fig. 2. Afterwards, the application of puncturing and modulation results in the symbol sequence $\mathbf{x}_s$ with length M_s and code rate $R_s = K/(M_s)$ bits/symbol. The block $\mathrm{P_S}$ performs puncturing at the source. In this scheme, the lth relay tries to decode the received signal $\mathbf{y}_{r_l}$. In the case of success, the

[1] Soft reencoding methods exist but are not considered here.

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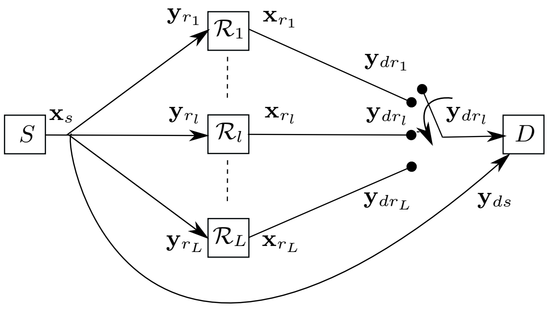

Figure 1: System model

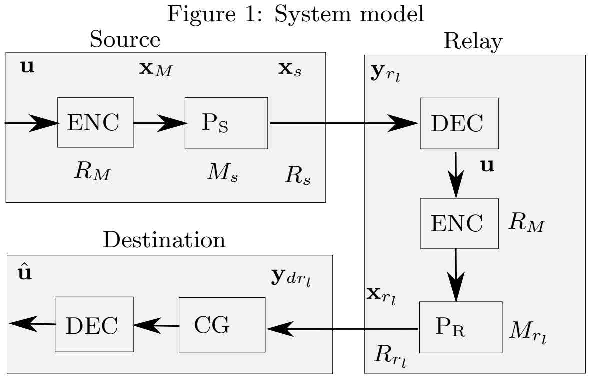

Figure 2: Block diagram depicting the procedure of incremental redundancy for a single relay.

relay also uses an ideal code with puncturing (unit $\mathrm{P_R}$) and generates the $\mathcal{M}$-ary symbol sequence $\mathbf{x}_{r_l}$. Otherwise, the relay remains silent, i.e.,

$$\mathbf{x}_{r_l} = \begin{cases} \bar{\mathbf{x}}_M & \text{for } M_s \cdot C(\gamma_{r_l s}) \geq K \\ 0 & \text{for } M_s \cdot C(\gamma_{r_l s}) < K \end{cases} . \tag{1}$$

In (1), $\bar{\mathbf{x}}_M$ represents symbols sequence obtained by modulating the *new* additional parity bits from the mother code word $\mathbf{x}_M$ with length M_{r_l} and code rate $R_{r_l} = K/(M_{r_l})$. Every relay transmits different bits of the word $\mathbf{x}_M$. Here, $\gamma_{r_l s}$ is the SNR on the link between source and lth relay.
The mutual information (per M_s symbols) delivered by the source at the lth relay and at the destination is given by $M_s \cdot C(\gamma_{r_l s})$ and $M_s \cdot C(\gamma_{ds})$ respectively. Let $\mathcal{R} = \{1, 2, 3..., L\}$ be the set of indices of all relays. The mutual information delivered at the destination by the lth relay is given by $M_{r_l} \cdot C(\gamma_{dr_l})$, where, $l \in \mathcal{D}$ and $\mathcal{D} \subset \mathcal{R}$ is the set of indices of relays with successful decoding. γ_{dr_l} and γ_{ds} represent the SNRs from the source and from the lth relay to the destination respectively. Consequently, the mutual information of all received channel symbols after code combining (CG) at the destination is given by

$$I_{tot} = M_s \cdot C(\gamma_{ds}) + \sum_{l \in \mathcal{D}} M_{r_l} \cdot C(\gamma_{dr_l}). \tag{2}$$

2.3 Relaying with Repetition Coding

In this scheme, forward error correction in conjunction with repetition coding is applied as shown in Fig. 3. The source applies an ideal coding scheme (as forward error correction code) with code rate R_s to the sequence $\mathbf{u}$. This encoding and $\mathcal{M}$-ary modulation results in the code word $\mathbf{x}_s$ with length M_s. Afterwards, the unit REP stores the signal and repeats it on request. The number of total transmissions

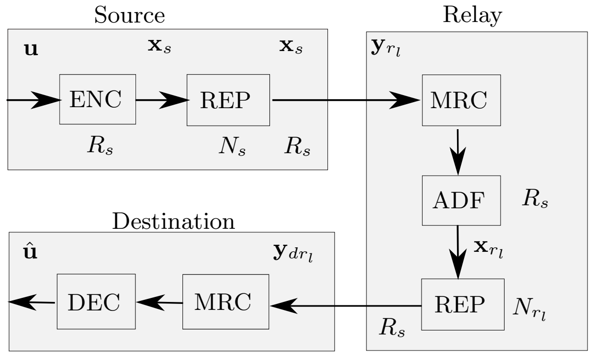

Figure 3: Block diagram depicting the procedure of repetition coding for a single relay.

from the source are denoted by N_s.
Each relay receives N_s transmissions from the source and combines the received signals using maximum ratio combining. The SNR of the source-relay link is denoted by $\gamma_{r_l s}$. Thus, the SNR of the signal after maximum ratio combining at the lth relay is $N_s \cdot \gamma_{r_l s}$. Subsequently, the relaying technique ADF [8] is applied. In this technique the relay tries to decode the signal obtained after maximum ratio combining. In the case of decoding success, the signal $\mathbf{x_s}$ is generated and transmitted from the relay. Otherwise, amplify-and-forward is used. Thus, the transmitted signal is given by

$$\mathbf{x}_{r_l} = \begin{cases} \mathbf{x}_s & \text{for } M_s \cdot C(N_s \cdot \gamma_{r_l s}) \geq K \\ \alpha \cdot \mathbf{y}_{r_l} & \text{for } M_s \cdot C(N_s \cdot \gamma_{r_l s}) < K \end{cases} . \tag{3}$$

Here, the inequality $M_s \cdot C(N_s \cdot \gamma_{r_l s}) \geq K$ represents the condition for successful decoding for an ideal code and α is the amplification factor at the relay. The number of retransmissions of the signal $\mathbf{x}_{r_l}$ from the lth relay is denoted by N_{r_l}.
The destination receives N_s repetitions from the source and N_{r_l} (each of length M_s symbols) repetitions from the lth relay. These signals are combined using maximum ratio combining. The SNR from all the nodes at the destination after maximum ratio combining becomes

$$\gamma^{ADF} = N_s \cdot \gamma_{ds} + \sum_{l \in \mathcal{D}} \gamma^{ADF}_{l,N_{r_l}} + \sum_{l \in \mathcal{R} \setminus \mathcal{D}} \gamma^{ADF}_{l,N_{r_l}}. \tag{4}$$

Here, $\gamma^{ADF}_{l,N_{r_l}} = N_{r_l} \cdot \gamma_{dr_l}$ for $l \in \mathcal{D}$ is the SNR via lth relay using decode-and-forward and $\gamma^{ADF}_{l,N_{r_l}} = \frac{N_s \cdot \gamma_{r_l s} \cdot N_{r_l} \cdot \gamma_{dr_l}}{N_s \cdot \gamma_{r_l s} + N_{r_l} \cdot \gamma_{dr_l} + 1}$ for $l \in \mathcal{R} \setminus \mathcal{D}$ is the SNR via lth relay using amplify-and-forward as derived in [9].
The next section explains the marginal returns based resource allocation technique. Later on, this technique will be used to find the optimum rates for different nodes.

3. RESOURCE ALLOCATION WITH MARGINAL RETURNS

In this section a resource allocation technique based on diminishing Marginal Returns (MR) [9] is described. Let, v_l represents the number of resources assigned to the lth relay. Moreover, $f(v_1, ...v_l, ...v_L) : \mathbb{Z}^L_+ \to \mathbb{R}_+$ denotes a monotonically increasing objective function with diminishing marginal returns w.r.t v_l. The Marginal return (MR) Δ_{l,v_l} w.r.t. v_l is the gain of the objective function per unit increase of v_l i.e., $\Delta_{l,v_l} := f(v_1, ...v_l + 1, ...v_L) - f(v_1, ...v_l, ...v_L)$. Diminishing means that Δ_{l,v_l} is decreasing (or remains constant) w.r.t. v_l i.e., $\Delta_{l,v_l} \geq \Delta_{l,v_l+1}$.

To clarify the concept of MR, a small toy example with the corresponding solution is presented. Imagine an objective function shall achieve the value $f(v_1,..v_l,..v_L) = f_{th}$ at the minimum cost $\sum_{l=1}^{L} v_l$.

According to [9], the solution can be obtained by initializing v_l $\forall l$ with the minimum possible values (0 in this case). Afterwards, v_1 or v_2 or ... v_L are incremented (repeatedly) depending on which v_l provides the maximum MR. The optimal solution is reached, when $f(v_1,..v_l,..v_L) \geq f_{th}$ is achieved. At this point, the optimization stops with $\{v_1^*,...v_l^*,...v_L^*\} = \{v_1,...v_l,...v_L\}$ being the solution.

As an example, let, $f(v_1, v_2) = \log(1+v_1) + 0.8 \cdot \log(1+v_2)$ and $f_{th} = 1.5$ holds. v_1 and v_2 are initialized with the minimum possible values ($v_1 = v_2 = 0$). At $[v_1, v_2] = [0, 0]$, v_1 and v_2 give marginal returns 0.693 and 0.554 respectively. As $0.693 > 0.554$, v_1 is incremented and we get $[v_1, v_2] = [1, 0]$. At $[v_1, v_2] = [1, 0]$, v_1 and v_2 give marginal returns 0.4055 and 0.554 respectively. As $0.554 > 0.405$, v_2 is incremented and we get $[v_1, v_2] = [1, 1]$. Afterwards, we get marginal returns of 0.4055 and 0.3244 for $v_1 = 1$ and $v_2 = 1$ respectively. Here, $0.4055 > 0.3244$, which leads to the increment of v_1 and $[v_1, v_2] = [2, 1]$ is obtained. These values deliver $f(2, 1) = 1.65$, which is greater than $f_{th} = 1.5$. Therefore, the optimization stops and $[v_1^*, v_2^*] = [2, 1]$ is the solution.

4. RATE OPTIMIZATION FOR IR AND RC

In this section the selection of the optimal code rates R_s and R_{r_l} $\forall l$ for relaying with IR are presented. Furthermore, the optimization of the code rate R_s and the number of retransmissions N_{r_l} $\forall l$ with RC is also provided. This optimization requires global channel knowledge at a central node.

4.1 Rate Optimization for Incremental Redundancy

Optimizing the code rate means that the minimum number of possible symbols ($M_s + \sum_{l\in\mathcal{D}} M_{r_l}$) should be selected to ensure successful decoding at the destination. Mathematically, this can be stated as

$$[\mathbf{M}^*\ M_s^*] = \underset{[\mathbf{M}, M_s]}{\text{argmin}} \left[\left(M_s + \sum_{l\in\mathcal{D}} M_{r_l} \right) \right] \quad (5)$$

such that,

$$M_s \cdot C(\gamma_{ds}) + \sum_{l\in\mathcal{D}} M_{r_l} \cdot C(\gamma_{dr_l}) \geq K, \quad (6)$$

where, $\mathbf{M} = [M_{r_1}, ..., M_{r_L}]$ holds. Moreover, we have to find the optimum value of M_s. However, whenever M_s changes then a relay may switch from being active to passive and vice-versa. This process introduces a further non-linearity in the system making the optimization difficult. Therefore, we choose M_s such that all possible number of relays have successfully decoded as given below.

for $\hat{l} = \{0, 1, 2, 3, ..., L\}$

$$[\bar{\mathbf{M}}_{\hat{l}}\ \bar{M}_{s_{\hat{l}}}] = \underset{[\mathbf{M}, M_s]}{\text{argmin}} \left[\left(M_s + \sum_{l\in\mathcal{D}} M_{r_l} \right) \right] \quad (7)$$

such that,

$$M_s \cdot C(\gamma_{ds}) + \sum_{l\in\mathcal{D}} M_{r_l} \cdot C(\gamma_{dr_l}) \geq K, \quad (8)$$

$$M_s \geq \frac{K}{C(\gamma_{r_{\hat{l}}s})}. \quad (9)$$

end loop

$$[\mathbf{M}^*\ M_s^*] = \underset{[\bar{\mathbf{M}}_{\hat{l}}, \bar{M}_{s_{\hat{l}}}]}{\text{argmin}} \left[\bar{M}_{s_{\hat{l}}} + \left(\bar{\mathbf{M}}_{\hat{l}} \cdot [\mathbf{1}]_{L\times 1} \right) \right]$$

In this problem, $\hat{l}$ corresponds to the number of relays with successful decoding and thus using decode-and-forward. We assume that the relays are ordered in descending order w.r.t. $\gamma_{r_l s}$. Thus, $\hat{l} = ||\mathcal{D}||$ also represents the index of a relay with the weakest link to the source among the relays with successful decoding i.e, $\hat{l} = \text{argmin}(\gamma_{r_l s}), l \in \text{D}$ holds. Therefore, inequality (9) ensures that all $\hat{l}$ relays with indices in the set $\mathcal{D}$ decode successfully. Moreover, $\hat{l} = 0$ corresponds to the case when no relay successfully decodes and let $K/C(\gamma_{r_0 s}) = K/m$ holds.

The marginal return per symbol w.r.t. M_s and M_{r_l} is given by $\Delta_{M_s} = C(\gamma_{ds})$ and $\Delta_{M_{r_l}} = C(\gamma_{dr_l})$ respectively. It is clear that Δ_{M_s} and $\Delta_{M_{r_l}}$ $\forall l$ are constants and do not change w.r.t. M_s and M_{r_l} respectively, therefore, we can apply the marginal return based solution to solve problem (7) for a specific value of $\hat{l}$ as shown in Algorithm 1. As the marginal

Algorithm 1

for $\hat{l} = \{0, 1, 2, ..., L\}$

1: Initialization: $M_s = \frac{K}{C(\gamma_{r_{\hat{l}}s})}$, $M_{r_l} = 0\ \forall l$

2: Calculate $\Delta_{M_s} = C(\gamma_{ds})$ and $\Delta_{M_{r_l}} = C(\gamma_{dr_l})$

3: $l^* = \underset{l\in\mathcal{D}}{\text{argmax}}(\Delta_{M_{r_l}})$

4: **if** $\Delta_{M_s} < \Delta_{M_{r_{l^*}}}$ **then**

5: $\quad M_{r_{l^*}} = \frac{K - M_s \cdot C(\gamma_{ds})}{C(\gamma_{dr_{l^*}})}$

6: **else**

7: $\quad M_s = \frac{K}{C(\gamma_{ds})}$

8: **end if**

9: $\bar{\mathbf{M}}_{\hat{l}} = [M_{r_1}, M_{r_2}, ..., M_{r_L}]$, $\bar{M}_{s_{\hat{l}}} = M_s$

end for

$[\mathbf{M}^*\ M_s^*] = \underset{[\bar{\mathbf{M}}_{\hat{l}}, \bar{M}_{s_{\hat{l}}}]}{\text{argmin}} \left[\bar{M}_{s_{\hat{l}}} + \left(\bar{\mathbf{M}}_{\hat{l}} \cdot [\mathbf{1}]_{L\times 1} \right) \right]$

return $\Delta_{M_{r_l}} \forall l \in \mathcal{D}$ is constant, the selection of a single relay R_{l^*} among the relays with successful decoding is optimal. In this algorithm, if $\Delta_{M_s} < \Delta_{M_{r_{l^*}}}$ holds then we continuously increment $M_{r_{l^*}}$, which leads directly to the equality given at line 5. Otherwise, M_s is progressively incremented which results in $M_s = \frac{K}{C(\gamma_{ds})}$. Thus, the optimal rates are $R_s^* = K/M_s^*$ and $R_{r_l}^* = K/M_{r_l}^* \forall l$ bits/symbol.

4.2 Rate Optimization for Repetition Coding

The goal is to minimize the total number of code bits transmitted under the condition of successful decoding at the destination. Let, the vector $\mathbf{N} = [N_{r_1}, N_{r_2}, ..., N_{r_L}]$ represents the number of transmissions from all the relays and for the sake of simplicity $N_s = 1$ is assumed. Similar to the arguments given in Sec. 4.1, we explicitly consider all possible numbers of relays with successful decoding as given below.

for $\hat{l} = \{0, 1, 2, .., L\}$

$$[\bar{\mathbf{N}}_{\hat{l}}\ \bar{M}_{s_{\hat{l}}}] = \underset{[\mathbf{N}, M_s]}{\text{argmin}} \left[M_s \cdot \left(N_s + \sum_{l=1}^{L} N_{r_l} \right) \right] \quad (10)$$

such that,

$$M_s \cdot \left(C\left(\gamma^{ADF}\right) \right) = K, \quad (11)$$

$$N_s = 1, N_{r_l} \in \{1, 2, 3, 4, ...\infty\}. \quad (12)$$

$$M_s \geq \frac{K}{C(N_s \cdot \gamma_{r_{\hat{l}}s})}. \quad (13)$$

end loop

$$[\mathbf{N}^* \; M_s^*] = \underset{[\bar{\mathbf{N}}_{\hat{l}}, \bar{M}_{s_{\hat{l}}}]}{\operatorname{argmin}} \left[\bar{M}_{s_{\hat{l}}} \cdot \left(N_s + \bar{\mathbf{N}}_{\hat{l}} \cdot [\mathbf{1}]_{L\times 1}\right)\right] \tag{14}$$

Let, $K/C(\gamma_{r_0 s}) = K/m$ holds. Similar to Sec. 4.1, we try to solve problem (10) for a specific $\hat{l}$ by calculating the marginal returns. Here, the marginal return (MR) per symbol w.r.t. M_s for the function $M_s \cdot \left(C\left(\gamma^{ADF}\right)\right)$ with given $[N_{r_1}, .., N_{r_l}, .., N_{r_L}]$ can be written as

$$\Delta_{M_s} = \frac{(M_s+1)C\left(\gamma^{ADF}\right) - M_s \cdot C\left(\gamma^{ADF}\right)}{N_s + \sum_{l=1}^{L} N_{r_l}} \tag{15}$$

$$= \frac{C\left(\gamma^{ADF}\right)}{1 + \sum_{l=1}^{L} N_{r_l}}. \tag{16}$$

This is the gain in function $M_s \cdot \left(C\left(\gamma^{ADF}\right)\right)$ with per unit increase of M_s. Increase of M_s by 1 unit (as shown in Fig. 4 with $t=1$ and $N_s = 1$) costs extra $1 + \sum_{l=1}^{L} N_{r_l}$ symbols due to the repetition coding of the relays. Therefore, we have in the denominator $1 + \sum_{l=1}^{L} N_{r_l}$ to get the gain per cost of 1 symbol.

Similarly, MR for the same function per symbol w.r.t. N_{r_l} for given M_s and $[N_{r_1}, .., N_{r_l}, .., N_{r_L}]$ can be expressed as

$$\Delta_{l,N_{r_l}} = \frac{M_s}{M_s}\Big(C(N_s\gamma_{ds} + \gamma^{ADF}_{1,N_{r_1}} + ..\underline{\gamma^{ADF}_{l,N_{r_l}+1}} + ...\gamma^{ADF}_{L,N_{r_L}}) - C(N_s\gamma_{ds} + \gamma^{ADF}_{1,N_{r_1}} + ..\underline{\gamma^{ADF}_{l,N_{r_l}}} + ..\gamma^{ADF}_{L,N_{r_L}})\Big). \tag{17}$$

As shown in Fig. 4, a single unit increase of N_{r_l} costs M_s symbols (with $t = 0$), therefore, in (17) we have division by M_s to obtain the gain per cost of 1 symbol. After finding the marginal returns we look if they are diminishing.

We consider a modulation scheme having a capacity $\left(C\left(\gamma^{ADF}\right)\right)$ being a monotonically increasing concave function of γ^{ADF}. It implies that the marginal return $\Delta_{l,N_{r_l}}$ is diminishing w.r.t. N_{r_l}, due to space limitations the proof for this statement is skipped here. Moreover, it is clear that marginal return Δ_{M_s} is constant w.r.t. M_s. Thus, problem (10) can be solved by marginal returns based resource allocation (Sec. 3). We start to solve problem (10) for a specific value of $\hat{l}$ of the loop as shown in Algorithm 2. First, M_s and N_{r_l} $\forall l$ are initialized with the minimum possible values i.e, $\frac{K}{C(N_s \cdot \gamma_{r_{\hat{l}}s})}$ and 0 respectively. M_s or $N_{r_{l^*}}$ are incremented (line 7) continuously, depending on which provides the maximum MR per symbol. This continues until successful decoding is achieved at line 11.

MR Δ_{M_s} is independent of M_s and remains constant as long as N_s and N_{r_l} are unchanged. On the other hand, $\Delta_{l^*,N_{r_{l^*}}}$ is diminishing ($\Delta_{l^*,N_{r_{l^*}}} \geq \Delta_{l,N_{r_l}+1}$ $\forall l$), therefore, once the condition $\Delta_{M_s} \geq \Delta_{l^*,N_{r_{l^*}}}$ is satisfied, it will hold true for all the forthcoming iterations until successful decoding at line 11. Thus, from the condition of successful decoding, we obtain directly the equation $M_s = \frac{K}{C(\gamma^{ADF})}$ instead of incrementing M_s repeatedly to reach the same equation.

Now, we assume a situation in which $\Delta_{M_s} < \Delta_{l^*,N_{r_{l^*}}}$ always holds true until successful decoding at the destination in line 11 occurs. Everything works fine until before the last increment of $N_{r_{l^*}}$. We know that N_{r_l} can only be incremented by 1 (repetition of one frame) and that a unit increment of $N_{r_{l^*}}$ costs M_s additional symbols as shown in Fig. 4. It may happen that for the last retransmission we require $b < M_s$ symbols to successfully decode at the destination, however, we can transmit at least M_s symbols. Thus, $M_s - b$ symbols are transmitted though they were not needed. On the other hand, a unit increment of M_s costs

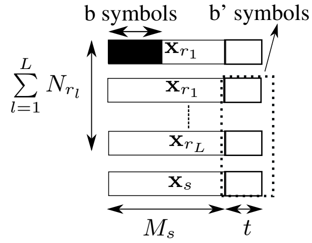

Figure 4: Repetitions from all the nodes, t denotes the possible increment in M_s. As an example: The source transmits one time frame $\mathbf{x}_s$, relay $\mathcal{R}_1$ two times frame $\mathbf{x}_{r_1}$, while, relay $\mathcal{R}_L$ transmits $\mathbf{x}_{r_L}$.

$1 + \sum_{l=1}^{L} N_{r_l}$ symbols. Instead of incrementing $N_{r_{l^*}}$ for the last time (which allocates $M_s - b$ symbols in vain), we continuously increment M_s and may need at most b' symbols, where $b \leq b' < M_s$ holds. In this case, though the marginal return per symbol $\Delta_{l^*,N_{r_{l^*}}}$ may be greater than Δ_{M_s}, the last increment of $N_{r_{l^*}}$ does not provide the optimum solution. This situation is dealt in line 6 and 12 of the algorithm. In line 6, the optimal frame length (M_s^-) required to successfully decode at the destination without incrementing $N_{r_{l^*}}$ is calculated. In line 12, it is checked if the total number of bits required for successful decoding without the increment of $N_{r_{l^*}}$ is less than that with the increment of $N_{r_{l^*}}$. If this is true, then instead of incrementing $N_{r_{l^*}}$, we continuously increment M_s, which automatically leads to the equation given in line 14.

Finally, in line 16 the so far incremented values of M_s and N_{r_l} $\forall l$ are considered as the optimum for a specific $\hat{l}$. At the end, the optimum rate R_s^* is equal to $K/(M_s^*)$ and $\mathbf{N}^*$ being the optimum number of transmissions required from all the relays.

Algorithm 2

for $\hat{l} = \{0, 1, 2, ..., L\}$
1: Initialization: $M_s = \frac{K}{C(N_s \cdot \gamma_{r_{\hat{l}}s})}$, $M_s^- = \infty$, $N_{r_l} = 0$ $\forall l$
2: **repeat**
3: Calculate Δ_{M_s} and $\Delta_{l,N_{r_l}}$ using (15) and (17)
4: $l^* = \underset{l}{\operatorname{argmax}}(\Delta_{l,N_{r_l}})$
5: **if** $\Delta_{M_s} < \Delta_{l^*,N_{r_{l^*}}}$ **then**
6: $M_s^- = \frac{K}{C(\gamma^{ADF})}$
7: $N_{r_{l^*}} = N_{r_{l^*}} + 1$
8: **else**
9: $M_s = \frac{K}{C(\gamma^{ADF})}$
10: **end if**
11: **until** $M_s \cdot C(\gamma^{ADF}) \geq K$
12: **if** $M_s^- \cdot (N_s + \sum_{l=1}^{L} N_{r_l} - 1) < M_s \cdot (N_s + \sum_{l=1}^{L} N_{r_l})$ **then**
13: $N_{r_{l^*}} = N_{r_{l^*}} - 1$
14: $M_s = \frac{K}{C(\gamma^{ADF})}$
15: **end if**
16: $\bar{\mathbf{N}}_{\hat{l}} = [N_{r_1}, N_{r_2}, ..., N_{r_L}]$, $\bar{M}_{s_{\hat{l}}} = M_s$
end for
$[\mathbf{N}^* \; M_s^*] = \underset{[\bar{\mathbf{N}}_{\hat{l}}, \bar{M}_{s_{\hat{l}}}]}{\operatorname{argmin}} \left[\bar{M}_{s_{\hat{l}}} \cdot \left(N_s + \bar{\mathbf{N}}_{\hat{l}} \cdot [\mathbf{1}]_{L\times 1}\right)\right]$

5. RESULTS

In this section the framework presented in Sec. 4 is applied to a specific relay setup. Four relays ($L = 4$) were placed in between a source and a destination. The distance between source and destination is 1. Moreover, 16-QAM was used as modulation scheme. Here, Rand-IR and Rand-RC represent the strategies with random allocation of the bits for Incremental Redundancy (IR) and Repetition Coding (RC) respectively. This is similar to the allocation of bits in [6], however, each bit or transmission is allocated with equal probability to a node. In Rand-IR the source continuously transmits symbols until one or more relays decode successfully. Afterwards, a symbol is randomly (with equal probability) allocated to the source or to each of the relay with successful decoding. In the case of Rand-RC, first the source chooses randomly M_s in the range[2] $(K/m, K/C(\gamma_{ds}))$, where, $m = 4$ represents the number of code bits mapped to a channel symbol by the modulator. Afterwards, N_s or N_{r_l} $\forall l$ are randomly incremented with equal probability. These scenarios do not require channel knowledge for the resource allocation.

In the scenario No-Relay, the source chooses the optimum rate and transmits to the destination without relay.

Opt-IR and Opt-RC corresponds to the optimization methods provided in Sec. 4 for IR and RC respectively and require global channel knowledge. Fig. 5 shows the average throughput for the mentioned strategies. Throughput η is defined as the info length K divided by the sum of the symbols transmitted from the source and all of the relays.

This figure shows that Opt-RC performs 2 dB worse than Opt-IR. It means that higher complexity for implementing IR (as compared to that of RC) can be avoided by using RC with the loss of 2 dB. At high SNR, both of the strategies reach the maximum of the capacity of 16-QAM.

It is evident that Opt-IR and Opt-RC outperform No-Relay by 5 dB and 3 dB respectively. Moreover, it is also clear that random resource allocation performs significantly worse than No-Relay. It means that if the source can adjust its rate according to the source-destination capacity then there is no need to use relay network with random resource allocation. The figure also shows that strategy Opt-IR outperforms strategy Rand-IR by 10 dB at low SNR and by 15 dB at high SNR.

The optimization of repetition coding (Opt-RC) provides gain of 10 dB over random symbols allocation (Rand-RC) at low SNR. At high SNR, Opt-RC saturates to 4, while Rand-RC saturates to the maximum possible throughput 2. Rand-RC saturates to 2 because first the source transmits $K/m < M_s < K/C(\gamma_{ds})$ symbols, which can not ensure successful decoding at the destination. Therefore, the source or relay transmits another M_s symbols which results in the loss of half of the throughput. This shows that the optimization of the rates using global channel knowledge for Opt-IR and Opt-RC can deliver significant gains.

This figure also reveals that the maximum number of transmissions (N_s^{max}) the source is allowed to transmit has no influence on the throughput, as long as the optimum code rate is chosen. As the optimization algorithm is only described for $N_s^{max} = 1$. For $N_s^{max} > 1$, the algorithm was run repeatedly for each value of N_s and then the optimum one was chosen.

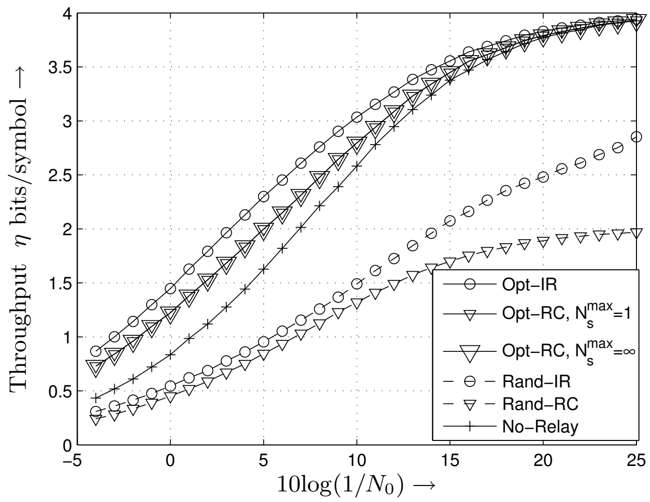

Figure 5: Throughput versus pseudo SNR $1/N_0$, with the relays are placed in the middle, N_s^{max} is the maximum number of transmissions the source is allowed to transmit.

6. CONCLUSION

In this paper, throughput analysis of incremental redundancy and repetition coding in parallel multi-relay network has been provided. A general optimization method for rate and number of transmissions allocation for both the strategies is provided. It is shown that for the given parallel relay network setup, incremental redundancy always outperforms repetition coding. Moreover, if the source is allowed to choose an optimum code rate, then only one transmission ($N_s = 1$) from the source is optimal.

[2]This range is natural, if we choose $M_s > K/C(\gamma_{ds})$ then it results in successful decoding and no cooperation is required. It will be very inefficient at low SNR.

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