MULTISCALE BLOCK COMPRESSED SENSING WITH SMOOTHED PROJECTED LANDWEBER RECONSTRUCTION

James E. Fowler, Sungkwang Mun, and Eric W. Tramel

Department of Electrical & Computer Engineering
Geosystems Research Institute
Mississippi State University, MS USA
fowler@ece.msstate.edu, {sm655,ewt16}@msstate.edu

ABSTRACT

A multiscale variant of the block compressed sensing with smoothed projected Landweber reconstruction algorithm is proposed for the compressed sensing of images. In essence, block-based compressed-sensing sampling is deployed independently within each subband of each decomposition level of a wavelet transform of an image. The corresponding multiscale reconstruction interleaves Landweber steps on the individual blocks with a smoothing filter in the spatial domain of the image as well as thresholding within a sparsity transform. Experimental results reveal that the proposed multiscale reconstruction preserves the fast computation associated with block-based compressed sensing while rivaling the reconstruction quality of a popular total-variation algorithm known for both its high-quality reconstruction as well as its exceedingly large computational cost.

1. INTRODUCTION

There has been increasing interest in deploying the paradigm of compressed sensing (CS) for the sampling and reconstruction of image data. Since real-world images are almost always compressible in some transform domain, the mathematics of CS apply directly to image data, and any existing CS reconstruction technique can be used without modification to recover image data just like any other signal. That said, improved reconstruction quality can result from intelligent use of prior knowledge (statistical dependencies, structure, etc.) that one often has about the imagery at hand. As a consequence, CS recovery techniques are being increasingly tailored specifically to image data.

For example, a number of recent CS strategies (e.g., [1–4]) are deployed assuming that the image is both sampled and reconstructed in the domain of a discrete wavelet transform (DWT). Such wavelet-domain CS permits known statistical models (e.g., [2, 4]) for wavelet coefficients to be exploited in reconstruction. Additionally, the degree of CS subsampling can be adapted to the wavelet decomposition—often, the baseband is retained in full with no subsampling (e.g., [1, 3]), while the degree of subsampling is increased for successively higher-resolution decomposition levels (e.g., [1]).

One of the primary challenges for CS on image data is the large computational cost typically associated with CS reconstruction for multidimensional signals. One approach to mitigating such computational burdens is to limit CS sampling to relatively small blocks (e.g., [5, 6]). Reconstruction schemes based on this block-based CS paradigm, such as the block CS with smoothed projected Landweber reconstruction (BCS-SPL) described in [6], provide very quick image recovery in only a fraction of the time typically required for techniques that feature full-image CS sampling. The drawback of block-based CS sampling is typically a reduced quality of image reconstruction due to the fact that CS sampling generally works better the more global it is.

It this paper, we propose to ameliorate the reconstruction quality of block-based CS while retaining its light computational burden and extremely fast execution. Specifically, we propose a multiscale algorithm that deploys BCS-SPL [6] in the domain of a wavelet transform. The resulting technique, multiscale BCS-SPL (MS-BCS-SPL), is shown in experimental results to provide a significant gain in reconstruction quality over the original BCS-SPL while being only slightly slower. Additionally, MS-BCS-SPL is observed to usually outperform not only an alternative multiscale algorithm described in [1] but also the popular total-variation (TV) recovery [7] which is known for its high-quality reconstruction at the cost of an extremely long runtime.

2. BACKGROUND

In brief, CS is a mathematical paradigm which permits, under certain conditions, signals to be sampled at sub-Nyquist rates via linear projection into a dimension much lower than that of the original signal, yet which still allows exact recovery of the signal from the samples. More specifically, suppose that we want to recover real-valued signal $\mathbf{x}$ with length N from M samples such that $M \ll N$. In other words, we want to recover $\mathbf{x}$ from

$$\mathbf{y} = \mathbf{A}\mathbf{x}, \tag{1}$$

where $\mathbf{y}$ has length M, and $\mathbf{A}$ is an $M \times N$ measurement matrix with subsampling rate, or *subrate*, being $S = M/N$. Because the number of unknowns is much larger than the number of observations, recovering every $\mathbf{x} \in \Re^N$ from its corresponding $\mathbf{y} \in \Re^M$ is impossible in general; however, if $\mathbf{x}$ is sufficiently sparse in some domain, then exact recovery of $\mathbf{x}$ is possible—this is the fundamental tenet of CS theory.

There are several issues that arise when the signal $\mathbf{x}$ is an image. From the perspective of practical implementation, the dimensionality of the sampling process in (1) grows very quickly as the size of image $\mathbf{x}$ increases due to the multidimensional nature of image data. This leads to a huge memory required to store the sampling operator when $\mathbf{A}$ is implemented as a matrix within the CS sensing process. Additionally, a large $\mathbf{A}$ yields a huge memory and computational burden within the CS reconstruction process. However, the use of structurally random matrices (SRMs) (e.g.,

This material is based upon work supported by the National Science Foundation under Grant No. CCF-0915307.

[8, 9]) can significantly mitigate these issues. In essence, an SRM provides a sampling process (operator $\mathbf{A}$) consisting of a random permutation, a simple and computationally efficient transform (such as a block cosine or Hadamard transform), and a random subsampling process, all of which can be performed with little computation or memory.

As an alternative to SRMs for alleviating the huge computation and memory burdens associated with the measurement matrix $\mathbf{A}$ within both the sensing and reconstruction processes, one can adopt a philosophy long used in image-processing fields when an image is too large to be feasibly processed in its entirety—namely, break the image into smaller blocks and process the blocks independently. An approach for such block-based CS (BCS) for 2D images was proposed in [5].

In BCS, an image is divided into $B \times B$ blocks and sampled using an appropriately-sized measurement matrix. That is, suppose that $\mathbf{x}_j$ is a vector representing, in raster-scan fashion, block j of input image $\mathbf{x}$. The corresponding $\mathbf{y}_j$ is then

$$\mathbf{y}_j = \mathbf{\Phi}\mathbf{x}_j, \tag{2}$$

where $\mathbf{\Phi}$ is an $M_B \times B^2$ measurement matrix such that the subrate for the image as a whole is $S = M_B/B^2$. It is straightforward to see that (2) applied block-by-block to an image is equivalent to a whole-image measurement matrix $\mathbf{A}$ in (1) with a constrained structure; specifically, $\mathbf{A}$ is constrained to have a block-diagonal structure,

$$\mathbf{A} = \begin{bmatrix} \mathbf{\Phi} & 0 & \cdots & 0 \\ 0 & \mathbf{\Phi} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & \mathbf{\Phi} \end{bmatrix}. \tag{3}$$

In [5], BCS was proposed wherein the sampling of an image is driven by random matrices applied on a block-by-block basis, while the reconstruction is a variant of the projected Landweber (PL) reconstruction[1] that incorporates a smoothing operation intended to reduce blocking artifacts. Since it combines BCS with a smoothed PL (SPL) reconstruction, in [6], the overall technique was called BCS-SPL.

Ideally, the CS sampling operator should be "global" in the sense that the entire signal $\mathbf{x}$ should contribute to each and every measurement taken in producing $\mathbf{y}$ in (1). However, a block-diagonal structure as in (3) defeats such maximally holistic sampling. As a consequence, BCS-based techniques such as BCS-SPL, while capable of exceedingly fast reconstruction, can be at a disadvantage in terms of reconstruction quality due to their reliance on a block-based sampling operator. In the next section, we propose a modification to the BCS-SPL algorithm designed to improve its reconstruction-quality performance while maintaining its block-based sampling and corresponding fast reconstruction. Specifically, we deploy BCS-SPL within the wavelet domain of the image $\mathbf{x}$ to provide multiscale sampling and reconstruction.

[1] PL reconstruction, when incorporating hard thresholding for enforcement of sparsity, is often known as *iterative hard thresholding* (IHT) within the CS community (e.g., [10]).

3. MS-BCS-SPL

3.1 Multiscale BCS

The sampling operator $\mathbf{A}$ for MS-BCS-SPL is split into two components—a multiscale transform $\mathbf{\Omega}$ (e.g., a DWT) and a multiscale block-based measurement process $\mathbf{\Phi}'$ such that $\mathbf{A} = \mathbf{\Phi}'\mathbf{\Omega}$, and (1) becomes

$$\mathbf{y} = \mathbf{\Phi}'\mathbf{\Omega}\mathbf{x}. \tag{4}$$

Assume that $\mathbf{\Omega}$ produces L levels of wavelet decomposition; thus, $\mathbf{\Phi}'$ consists of L different block-based sampling operators, one for each level. That is, let the DWT of image $\mathbf{x}$ be

$$\tilde{\mathbf{x}} = \mathbf{\Omega}\mathbf{x}. \tag{5}$$

Subband s at level l of $\tilde{\mathbf{x}}$ is then divided into $B_l \times B_l$ blocks and sampled using an appropriately-sized $\mathbf{\Phi}_l$ (note that $l = L$ is the highest-resolution level). That is, suppose $\tilde{\mathbf{x}}_{l,s,j}$ is a vector representing, in raster-scan fashion, block j of subband s at level l, with $s \in \{H, V, D\}$, and $1 \leq l \leq L$. Then,

$$\mathbf{y}_{l,s,j} = \mathbf{\Phi}_l \tilde{\mathbf{x}}_{l,s,j}. \tag{6}$$

Since the different levels of wavelet decomposition have different importance to the final image reconstruction quality, we adjust the sampling process so as to yield a different subrate, S_l, at each level l. In all cases, we set the subrate of the DWT baseband to full sampling, i.e., $S_0 = 1$. Then, we let the subrate for level l be

$$S_l = W_l S', \tag{7}$$

such that the overall subrate becomes

$$S = \frac{1}{4^L} S_0 + \sum_{l=1}^{L} \frac{3}{4^{L-l+1}} W_l S'. \tag{8}$$

Given a target subrate S and a set of level weights W_l, one can easily solve (8) for S', yielding a set of level subrates S_l via (7). However, this process will typically produce one or more $S_l > 1$. Thus, we modify the solution to enforce $S_l \leq 1$ for all l. Specifically, after finding S' and S_1 via (8) and (7), we check if $S_1 > 1$. If so, we set $S_1 = 1$, remove its corresponding term from the sum in (8), and then solve

$$S = \frac{1}{4^L} S_0 + \frac{3}{4^L} S_1 + \sum_{l=2}^{L} \frac{3}{4^{L-l+1}} W_l S' \tag{9}$$

for S', again using (7) to redetermine S_l for $l = 2, \ldots, L$. We repeat this process as needed to ensure that all $S_l \leq 1$.

For the experimental results to follow later, we use level weights,

$$W_l = 16^{L-l+1}, \tag{10}$$

which we have found to perform well in practice. The resulting level subrates S_l for various target subrates S for a DWT with $L = 3$ levels are shown in Table 1.

3.2 Multiscale Reconstruction

The BCS-SPL reconstruction algorithm couples a full-image Wiener-filter smoothing process with a sparsity-enhancing thresholding process in the domain of some full-image sparsity transform $\mathbf{\Psi}$. Interleaved between the smoothing and

Table 1: Subrates S_l at level l for target overall subrate S for a DWT with $L = 3$ levels. In all cases, the baseband is given full sampling ($S_0 = 1.0$).

S	S_1	S_2	S_3
0.1	1.0000	0.1600	0.0100
0.2	1.0000	0.5867	0.0367
0.3	1.0000	1.0000	0.0667
0.4	1.0000	1.0000	0.2000
0.5	1.0000	1.0000	0.3333

thresholding operations lie Landweber steps in the form of $\mathbf{x} \leftarrow \mathbf{x} + \mathbf{\Phi}^T (\mathbf{y} - \mathbf{\Phi}\mathbf{x})$, where $\mathbf{\Phi}$ is some measurement matrix. Fig. 1 illustrates how the BCS-SPL reconstruction is modified to accommodate the situation in which CS sampling takes place within a multiscale transform $\mathbf{\Omega}$ as in (4). In essence, the resulting MS-BCS-SPL reconstruction applies a Landweber step on each block of each subband in each decomposition level separately using the appropriate block-based $\mathbf{\Phi}_l$ for the current level l. As in the original BCS-SPL, Wiener filtering takes place in the spatial domain of the image, while some thresholding operator is applied in the domain of full-frame sparsity transform $\mathbf{\Psi}$ to promote sparsity.

```
function x̃ = MS-BCS-SPL(y, {Φ_{l, 1≤l≤L}}, Ψ, Ω)
  for each level l
    for each subband s ∈ {H,V,D}
      for each block j
        x̃^(0)_{l,s,j} = Φ_l^T y_{l,s,j}
  i = 0
  do
    x^(i) = Ω^{-1} x̃^(i)
    x̂^(i) = Wiener(x^(i))
    x̂̃^(i) = Ω x̂^(i)
    for each level l
      for each subband s ∈ {H,V,D}
        for each block j
          x̂̂^(i)_{l,s,j} = x̂̃^(i)_{l,s,j} + Φ_l^T (y_{l,s,j} − Φ_l x̂̃^(i)_{l,s,j})
    x̌̌^(i) = Ψ Ω^{-1} x̂̂^(i)
    x̌^(i) = Threshold(x̌̌^(i))
    x̄̃^(i) = Ω Ψ^{-1} x̌^(i)
    for each level l
      for each subband s ∈ {H,V,D}
        for each block j
          x̃^(i+1)_{l,s,j} = x̄̃^(i)_{l,s,j} + Φ_l^T (y_{l,s,j} − Φ_l x̄̃^(i)_{l,s,j})
    D^(i+1) = ||x̃^(i+1) − x̂̂^(i)||_2
    i = i + 1
  until |D^(i) − D^(i−1)| < 10^{-2}
  x̃ = x̃^(i)
```

Figure 1: MS-BCS-SPL reconstruction of a 2D image; Wiener$(\cdot)$ is pixel-wise adaptive Wiener filtering using a neighborhood of 3×3, while Threshold$(\cdot)$ is a thresholding process.

4. RESULTS

We now evaluate the performance of the MS-BCS-SPL algorithm described above on several grayscale images of size 512×512. We compare to the original BCS-SPL algorithm [6] as well as to the TV reconstruction described in [7] and a multiscale variant of GPSR as described in [1]. Both MS-BCS-SPL and BCS-SPL use a dual-tree DWT (DDWT) [11] as the sparsity transform $\mathbf{\Psi}$ with bivariate shrinkage [12] applied within the DDWT domain to enforce sparsity as described in [6]. MS-BCS-SPL uses a 3-level DWT with the popular 9/7 biorthogonal wavelets as the sampling-domain transform $\mathbf{\Omega}$. At decomposition level l of $\mathbf{\Omega}$, blocks of size $B_l \times B_l$ are individually sampled in the DWT domain using the scrambled block-DCT SRM sampling operator of [8]; we use blocks of sizes $B_l = 16$, 32, and 64 for decomposition levels $l = 1$, 2, and 3, respectively ($l = 3$ is the highest-resolution level). On the other hand, BCS-SPL uses $B \times B$ block-based sampling applied directly on the image in its ambient domain; here, $B = 32$. TV uses the scrambled block-Hadamard SRM of [9] to provide a fast whole-image CS sampling. Finally, the multiscale GPSR (MS-GPSR) is implemented similarly to MS-BCS-SPL—GPSR reconstruction is applied independently to each DWT level using the same $\mathbf{\Omega}$ as MS-BCS-SPL; subrates in the individual levels follow Table 1 with sampling using a scrambled block-DCT SRM applied to the entire DWT level. We use our implementation[2] of BCS-SPL and MS-BCS-SPL, ℓ_1-MAGIC[3] for TV, and the GPSR implementation[4] from its authors.

The reconstruction performance of the various algorithms under consideration is presented in Table 2. In most cases, the wavelet-domain sampling and multiscale reconstruction of MS-BCS-SPL provides a substantial gain in reconstruction quality over the image-domain sampling of BCS-SPL, generally on the order of a 1- to 3-dB increase in peak signal to noise ratio (PSNR). Additionally, MS-BCS-SPL outperforms TV reconstruction in most instances despite the fact that TV has the advantage of full-image sampling; the gains of MS-BCS-SPL over TV are particularly significant at the lowest subrates. MS-BCS-SPL also generally outperforms MS-GPSR even though the latter globally samples each resolution level. The primary exception is the Barbara image—although MS-BCS-SPL outperforms TV at the lowest subrates, MS-GPSR is slightly better. However, TV dominates the performance comparison for Barbara at the higher subrates. Fig. 2 depicts typical reconstruction results for Lenna at a subrate of $S = 0.1$.

As can be seen in Table 3, in terms of execution times, reconstruction with MS-BCS-SPL is only slightly slower than BCS-SPL, each running for about half a minute on a dual-core 2.8-GHz machine. On the other hand, the execution times of both MS-GPSR and TV are some two orders of magnitude longer, with TV requiring nearly two hours to reconstruct a single image even with fast SRM implementation of the sampling operator.

5. CONCLUSION

MS-BCS-SPL provides a multiscale variant of the original BCS-SPL reconstruction by deploying block-based CS sam-

[2] http://www.ece.msstate.edu/~fowler/BCSSPL/

[3] http://www.acm.caltech.edu/l1magic/

[4] http://www.lx.it.pt/~mtf/GPSR/

pling within the domain of a wavelet transform. The corresponding multiscale reconstruction applies the Landweber step at the core of BCS-SPL to each block in each subband at each decomposition level independently. The resulting MS-BCS-SPL algorithm achieves a 1- to 3-dB gain in reconstruction PSNR over the original BCS-SPL that features sampling and reconstruction in the original spatial domain of the image. As a result, MS-BCS-SPL effectively retains the fast execution speed associated with block-based CS while rivaling the quality of CS reconstructions such as TV that employ full-image sampling.

The general advantages of block-based CS are a reduced computational complexity in reconstruction as well as a greatly simplified sampling-operator implementation in both the reconstruction as well as in the sampling process. A multiscale BCS in the wavelet domain like we have proposed here retains these advantages for reconstruction; however, the decomposition of the measurement process as $\mathbf{A} = \mathbf{\Phi}'\mathbf{\Omega}$ entails that the transform $\mathbf{\Omega}$ wrecks the block-diagonal structure of $\mathbf{\Phi}'$, producing a dense $\mathbf{A}$ that can be challenging to implement within a CS sensing device. Thus, the improved performance of MS-BCS-SPL reconstruction can be viewed as arriving at the expense of a more complicated sampling process within the sensing device.

Table 2: Reconstruction PSNR in dB

Algorithm	*Subrate* 0.1	0.2	0.3	0.4	0.5
Lenna					
MS-BCS-SPL	**31.55**	**34.67**	**36.67**	**37.90**	**39.01**
BCS-SPL	28.01	31.55	33.69	35.37	36.88
TV	29.87	32.90	35.04	36.81	38.41
MS-GPSR	30.30	33.61	35.21	36.32	37.75
Barbara					
MS-BCS-SPL	23.82	25.08	26.05	27.36	28.84
BCS-SPL	22.40	23.77	25.38	27.01	28.66
TV	22.96	24.48	**26.26**	**28.40**	**30.79**
MS-GPSR	**24.04**	**25.28**	26.09	27.47	29.62
Peppers					
MS-BCS-SPL	**31.05**	**34.20**	**35.69**	**36.75**	**37.68**
BCS-SPL	28.98	32.08	33.84	35.21	36.44
TV	30.37	33.13	34.71	35.90	37.01
MS-GPSR	29.30	31.86	33.05	34.22	35.79
Mandrill					
MS-BCS-SPL	21.39	**23.00**	**24.64**	**25.53**	26.45
BCS-SPL	20.52	21.81	22.88	23.94	25.08
TV	20.51	22.02	23.44	24.91	**26.49**
MS-GPSR	**21.52**	22.90	24.31	25.06	26.25
Goldhill					
MS-BCS-SPL	**29.00**	**31.06**	**32.78**	**33.73**	34.66
BCS-SPL	27.08	29.10	30.48	31.79	33.10
TV	27.53	29.86	31.64	33.20	**34.84**
MS-GPSR	28.45	30.44	32.16	32.95	34.10

Table 3: Reconstruction time for Lenna at subrate of 0.3

Algorithm	*Time (sec.)*
BCS-SPL	30
MS-BCS-SPL	46
MS-GPSR	1,173
TV	6,584

REFERENCES

[1] P. Schniter, L. C. Potter, and J. Ziniel, "Fast Bayesian matching pursuit: Model uncertainty and parameter estimation for sparse linear models," *IEEE Transactions on Signal Processing*, 2008, submitted.

[2] L. He and L. Carin, "Exploiting structure in wavelet-based bayesian compressive sensing," *IEEE Transactions on Signal Processing*, vol. 57, no. 9, pp. 3488–3497, September 2009.

[3] B. Han, F. Wu, and D. Wu, "Image representation by compressive sensing for visual sensor networks," *Journal of Visual Communication and Image Representation*, vol. 21, no. 4, pp. 325–333, May 2010.

[4] Y. Kim, M. S. Nadar, and A. Bilgin, "Compressed sensing using a Gaussian scale mixtures model in wavelet domain," in *Proceedings of the International Conference on Image Processing*, Hong Kong, September 2010, pp. 3365–3368.

[5] L. Gan, "Block compressed sensing of natural images," in *Proceedings of the International Conference on Digital Signal Processing*, Cardiff, UK, July 2007, pp. 403–406.

[6] S. Mun and J. E. Fowler, "Block compressed sensing of images using directional transforms," in *Proceedings of the International Conference on Image Processing*, Cairo, Egypt, November 2009, pp. 3021–3024.

[7] E. Candès, J. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Communications on Pure and Applied Mathematics*, vol. 59, no. 8, pp. 1207–1223, August 2006.

[8] T. T. Do, T. D. Tran, and L. Gan, "Fast compressive sampling with structurally random matrices," in *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing*, Las Vegas, NV, March 2008, pp. 3369–3372.

[9] L. Gan, T. T. Do, and T. D. Tran, "Fast compressive imaging using scrambled block Hadamard ensemble," in *Proceedings of the European Signal Processing Conference*, Lausanne, Switzerland, August 2008.

[10] T. Blumensath and M. E. Davies, "Iterative hard thresholding for compressed sensing," *Applied and Computational Harmonic Analysis*, vol. 27, no. 3, pp. 265–274, November 2009.

[11] N. G. Kingsbury, "Complex wavelets for shift invariant analysis and filtering of signals," *Journal of Applied Computational Harmonic Analysis*, vol. 10, pp. 234–253, May 2001.

[12] L. Şendur and I. W. Selesnick, "Bivariate shrinkage functions for wavelet-based denoising exploiting interscale dependency," *IEEE Transactions on Signal Processing*, vol. 50, no. 11, pp. 2744–2756, November 2002.

(a) MS-BCS-SPL, 31.55 dB

(b) BCS-SPL, 28.01 dB

(c) TV, 29.87 dB

(d) MS-GPSR, 30.30 dB

Figure 2: Reconstructions of the 512×512 Lenna image (shown in detail) for a subrate of $S = 0.1$.