UNIFORM BLIND EQUALIZATION OF TWO-PATH CHANNELS WITH ZEROS ON THE UNIT CIRCLE

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ABSTRACT

Blind equalization of channels with z-transform having zeros on the unit circle or even just close-by is a very difficult task as the equalizer length tends to infinity. We propose a new blind equalizer for constant-modulus (CM) signals that uniformly equalizes inter-symbol interference caused by two-path propagations with arbitrary position of the zeros of the channel transfer function in the z-domain. The equalizer achieves perfect signal recovery in the noise-free case. It avoids the divergence of the inverse of the channel by truncating the zero-forcing equalizer filter at a certain order and uses the observation that the remainder is proportional to a single unknown transmitted symbol. A CM cost function is then optimized with respect to the unknown parameters of a two-path channel and that one symbol. We investigate our new equalizer theoretically and in numerical Monte-Carlo simulations and present results for the variance of the path parameters and the equalized symbols. Furthermore, we compare our algorithm with the classical parametric CMA.

1. INTRODUCTION

Equalization is an essential task in wireless communications, because the received signals are often subject to inter-symbol interference (ISI) caused by multi-path propagation. A widespread approach is to include training sequences in the transmitted signal which are used by the equalizer to estimate the channel and to recover the original signal. In order to avoid wasting channel capacity training sequences can be omitted and a blind equalizer can be used which exploits certain a priori known properties of signals and channels. The constant-modulus-algorithm (CMA) is by far the most known and studied method for blind channel equalization. It was first introduced for blind equalization of quadrature amplitude modulation (QAM) signals in [1] and of PAM and FM signals in [2]. A review including a large list of publications on the constant modulus criterion can be found in [3]. Like many equalizers, the CMA suffers from the fact that a channel transfer function that exhibits zeros on the unit circle cannot be inverted by a finite-impulse-response (FIR) filter in the case of baud-spaced processing. When a zero moves towards the unit circle, the equalization requires exceedingly long FIR filters [4]. If the received signal is oversampled, the channel can be modeled as a single-input-multiple-output (SIMO) system. Then, a finite zero-forcing filter exists (even if the channel is non-minimum phase), if and only if the sub-channels do not share common zeros, see [5] or [6]. However, Tugnait proved that multi-path channels with delays equaling integer multiples of the symbol rate do not fulfill this condition and are therefore not identifiable from the fractionally sampled received data [7]. Our algorithm makes use of the constant-modulus property of the transmitted symbols and therefore can, as it is known for the classical CMA [8], equalize this kind of channels as well.

In [4] a method is proposed which can equalize channels with zeros located close to, but not exactly on the unit circle.

We focus on a two-path channel model and propose a parametric, batch-processing variant of the CMA that circumvents the problem of a diverging filter length as the zeros approach the unit circle or even lie exactly on it. In our approach we use a finite-length equalizer filter which is partitioned into two parts: The first part consists of the exact zero-forcing filter truncated at a specific order and the second part is proportional to the transmitted symbol at the position of truncation. For a given batch of received data, we estimate the channel parameters and the unknown symbol by minimizing a CM cost-function. Thus, we include the estimation of this symbol in the optimization process and use it to complete the filter. Equalization is then performed based on the estimated parameters using the truncated FIR filter. By this means, the algorithm can invert two-path channels with zeros located exactly on the unit circle.

In principal, the method can be extended to channel models comprising more than two paths, in doing so the dimension and complexity of the problem increase. Here, we focus on a two-path channel model which seems to be very restrictive at first glance, but there exist many situations where it is commonly applied: It serves as a simple model for a line-of-sight microwave channel known as Rummlers model [9]. Applying this model in an urban environment, microcellular path-loss can be predicted from ultra-high-frequency to microwave bands [10]. Furthermore, it is generally accepted as a model for the HF-communication channel [11]. In that context, it is known as the Watterson model [12] which underlies the ITU recommendation [13] for testing HF modems. Almost all HF-channel models proposed by the ITU are slowly time-varying and can be considered as nearly stationary within short batches of data which can be processed with our new blind equalizer.

The paper is organized as follows: In Section 2 we introduce the parametric channel model and derive our equalizer algorithm. Section 3 contains an analytical calculation of the first order variance of the estimated parameters. Then, in Section 4 we present a comparison of analytical and numerical results for our new uniform equalizer and a classical parametric CMA based on a FIR filter.

2. DATA MODEL AND ALGORITHM DEVELOPMENT

The blind equalizer outlined below is a batch-processing algorithm relying on a parametric channel model and exploiting the CM property of the transmitted symbols. The substantial new modification consists in employing a truncated parametric equalizer filter which is therefore finite. We omit noise in the derivation of the algorithm, but investigate statistical efficiency in the presence of noise in a theoretical analysis and with help of simulations. The considered channel model has the form

$$h(t) = \delta(t) + \lambda\delta(t-\tau) \tag{1}$$

with complex-valued path attenuation $\lambda = \alpha e^{j\phi}$ $(\alpha, \phi \in \mathbb{R})$ and path delay $\tau \in \mathbb{R}$. (As the channel can only be identified up to an unknown overall factor, we set the amplitude of the first path equal to one.)

The received lowpass signal $x(t)$ is the convolution of the channel response $h(t)$ with the transmitted signal

$$y(t) = \sum_{i=-\infty}^{\infty} s_i g(t-iT), \tag{2}$$

where $g(t)$ is the combined transmitter and receiver filter and $\{s_n\}$ the discrete information-bearing sequence of symbols. In the fol-

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lowing, we assume that the received data $x_n = x(nT/M)$ are oversampled by a factor of M times the symbol rate T and the path delay is $\tau = LT/M$. The delay estimation can be carried out beforehand, see e.g. [14], independently of the estimation of λ. For a stationary delay, the estimation is rather robust for sufficiently large observation periods.

As we assume a two-path channel, the equation for the distorted signal in the noise-free case

$$x_n = y_n + \lambda y_{n-L} \tag{3}$$

can be rewritten as

$$y_n = x_n - \lambda y_{n-L}\,. \tag{4}$$

A K-fold recursive expansion of the above equation leads to the following representation for the transmitted sample y_n

$$y_n = \sum_{k=0}^{K} x_{n-kL}(-\lambda)^k + y_{n-(K+1)L}(-\lambda)^{K+1} \tag{5}$$

which can be verified by inserting the formula in (3). We observe that the transmitted signal can be recovered with help of a truncated zero-forcing filter and a residual term which is proportional to the transmitted signal at the position of truncation.

Based on (5) we define the parametric filter for calculating the symbol sequence $\{z_n\}$:

$$z_n = \sum_{k=0}^{K} x_{nM-kL}(-\lambda)^k + z_{nM-(K+1)L}(-\lambda)^{K+1} \tag{6}$$

The difference to $\{y_n\}$ in (5) is, that $\{z_n\}$ is baud spaced and synchronized, i.e. the signal is sampled at positions where the pulse-shape filter fulfills the first Nyquist condition. Consequently, there is no ISI due to the pulse-shape filter and $\{z_n\}$ shows the CM property in the case of no noise and perfect equalization. In the cost-function we will consider equalizer outputs for certain $n \in [1, \tilde{N}]$ (where $\tilde{N}$ is the number of samples in a block) which can be related to the same starting symbol z_0. First of all, the condition $nM - (K+1)L = 0$ yields $K = Mn/L - 1$. Next, we have to select those $n = \bar{n}\kappa$ for which $\bar{n}$ runs consistently from $1, \ldots, N$ (where N is the number of samples involved in the equalization process) and $\kappa = \triangle L/M$. $\triangle$ has to be chosen as the smallest possible integer for which $\triangle \cdot L$ is an integer multiple of M. Then, the equalizer outputs for $\bar{n} = 1, \ldots, N$ are given by

$$z_{\bar{n}} = \sum_{k=0}^{\bar{n}\triangle-1} x_{(\bar{n}\triangle-k)L}(-\lambda)^k + z_0(-\lambda)^{\bar{n}\triangle} \tag{7}$$

Due to the CM property we can set $z_0 = e^{j\theta_0}$. Furthermore, the received signals have to be properly scaled in order to be able to yield CM equalized signals. Therefore, we multiply them by an unknown real constant γ which is determined within the optimization of the cost-function.

The cost-function in which N equalized symbols are considered depends on the received data, the path parameter, the path delay, the initial symbol, the overall scaling factor γ, and the particular transmitted symbol sequence $\mathbf{s}$:

$$c(\mathbf{x};\lambda,\tau,\gamma,z_0|\mathbf{s}) = \sum_{\bar{n}=1}^{N} (|\tilde{z}_{\bar{n}}|^2 - 1)^2 \tag{8}$$

with

$$\tilde{z}_{\bar{n}} = \gamma \sum_{k=0}^{\bar{n}\triangle-1} x_{(\bar{n}\triangle-k)L}(-\lambda)^k + e^{j\theta_0}(-\lambda)^{\bar{n}\triangle}\quad. \tag{9}$$

Because all equalized symbols within one block are related to the same z_0, the length of the filter depends on $\bar{n}$. We note, that the cost-function has a trivial local minimum for $\gamma = 0$ and $\alpha = 1$ which has to be excluded in the optimization procedure.

For fixed τ the cost-function is a polynomial in the parameters $\alpha, \phi, z_0, \gamma$. In principle, even in the noise-free case there could exist further local minima besides the one at the values of the true parameters, i.e. for which $|\tilde{z}_{\bar{n}}|^2 = 1$ holds for all $\bar{n}$. But, it is very unlikely that at these further local minima the cost-function has zeros, as well.

3. STATISTICAL EFFICIENCY FOR LARGE N

In our theoretical performance analysis we will compute the first-order variance of the estimated path parameter and compare it with the results for a parametric CMA using an infinite-length equalizer. The corresponding cost-function for this parametric CMA is defined as follows

$$c(\mathbf{x};\lambda,\gamma|\mathbf{s}) = \sum_{n=1}^{N} (|z_n|^2 - \gamma)^2 \tag{10}$$

with

$$z_n = \sum_{k=0}^{\infty} x_{nM-kL}(-\lambda)^k \tag{11}$$

and depends only on the parameters α, ϕ, γ. In the following analysis it is assumed that the beforehand estimated delay τ equals the true one.

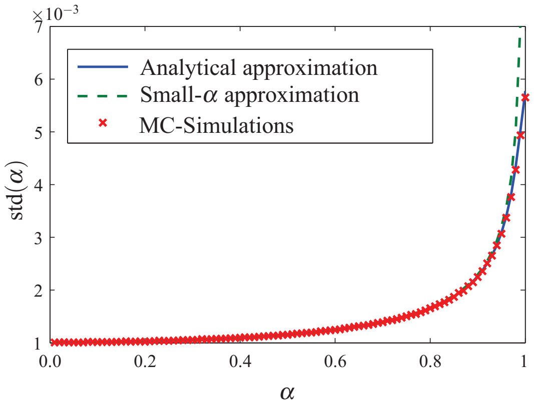

Figure 1: Standard deviation for the path parameter α: Comparison of analytical approximations and simulations for varying α. Block length is $N = 100$ and additive noise has $\sigma_n = 0.01$.

Let ρ denote the vector of parameters to be estimated from the cost-function. In the absence of noise in the data $\mathbf{x}_0$, the minimum of the cost-function is located at the position of the true parameters ρ_0. In the presence of additional noise $\delta\mathbf{x}$ on the signal, if the signal changes to $\mathbf{x} = \mathbf{x}_0 + \delta\mathbf{x}$, the position of the minimum will change correspondingly from ρ_0 to $\hat{\rho} = \rho_0 + \delta\rho$. (In case of complex-valued receive data, we define the real-valued receive vector $\mathbf{x} = (x^{(r)}_{1-WL}, x^{(i)}_{1-WL}, \ldots, x^{(r)}_{NM}, x^{(i)}_{NM})^T$, where W is the filter length and $\cdot^{(r)}, \cdot^{(i)}$ indicate real and imaginary parts of the samples, respectively. We consider the real-valued cost-function as a function of real variables only.) It is assumed, that $\delta\mathbf{x}$ is Gaussian distributed and zero-mean with $\mathrm{E}\left[\delta\mathbf{x}\delta\mathbf{x}^T\right] = \sigma_n^2\mathbf{I}$.

We use the general results of [15],[16] to calculate analytical expressions for the performance of the uniform equalizer: The variance of the parameters which are estimated batchwise from the global minimum of a cost-function c reads:

$$\begin{aligned}\mathrm{var}(\rho) &= \mathrm{E}\left[\delta\rho\delta\rho^T\right] \\ &= \sigma_n^2(\mathbf{D}^{(\rho\rho)})^{-1}\mathbf{D}^{(\rho x)}\mathbf{D}^{(x\rho)}(\mathbf{D}^{(\rho\rho)})^{-1}\end{aligned} \tag{12}$$

with

$$\mathbf{D}_{ik}^{(x\rho)} = \left.\frac{\partial^2 c}{\partial x_i \partial \rho_k}\right|_{\xi_0}, \ \mathbf{D}_{kl}^{(\rho\rho)} = \left.\frac{\partial^2 c}{\partial \rho_k \partial \rho_l}\right|_{\xi_0} \quad (13)$$

and $\xi_0 = (\mathbf{x}_0^T, \rho_0^T)^T$. The variance can be computed for a particular symbol sequence $\mathbf{s}$. On the other hand we are interested in the average over all possible symbol sequences with a given probability distribution $w(\mathbf{s})$.

In the following, we assume that the information bearing symbols are uncorrelated, i.e.

$$\mathrm{E}[s_n s_m^*] = \delta_{n,m} \ ; \ \mathrm{E}[s_n s_m] = 0 \ . \quad (14)$$

The elements of the Hessian matrix of the cost-function (8) at ξ_0 where $\tilde{z}_{\bar{n}} = s_n$ read

$$\left.\frac{\partial^2 c}{\partial \eta_m \partial \eta_n}\right|_{\xi_0} = 2\sum_{\bar{n}=1}^{N} \left(\frac{\partial \tilde{z}_{\bar{n}}}{\partial \eta_n}\tilde{z}_{\bar{n}}^* + \tilde{z}_{\bar{n}}\frac{\partial \tilde{z}_{\bar{n}}^*}{\partial \eta_n}\right)\left(\frac{\partial \tilde{z}_{\bar{n}}}{\partial \eta_m}\tilde{z}_{\bar{n}}^* + \tilde{z}_{\bar{n}}\frac{\partial \tilde{z}_{\bar{n}}^*}{\partial \eta_m}\right). \quad (15)$$

The calculation of the first-order partial derivatives yields

$$\frac{\partial \tilde{z}_{\bar{n}}}{\partial \alpha} = -\sum_{k=1}^{\bar{n}\triangle} y_{(\bar{n}\triangle - k)L}(-\alpha)^{k-1} e^{jk\phi} \quad (16)$$

$$\frac{\partial \tilde{z}_{\bar{n}}}{\partial \phi} = j\sum_{k=1}^{\bar{n}\triangle} y_{(\bar{n}\triangle - k)L}(-\alpha)^{k} e^{jk\phi} = j\alpha\frac{\partial \tilde{z}_{\bar{n}}}{\partial \alpha} \quad (17)$$

$$\frac{\partial \tilde{z}_{\bar{n}}}{\partial \theta_0} = je^{j\theta_0}(-\lambda)^{\bar{n}\triangle}, \ \frac{\partial \tilde{z}_{\bar{n}}}{\partial \gamma} = s_n - e^{j\theta_0}(-\lambda)^{\bar{n}\triangle}. \quad (18)$$

In case of the infinite-length filter (11) we have

$$\frac{\partial z_n}{\partial \alpha} = -\sum_{k=1}^{\infty} y_{nM-kL}(-\alpha)^{k-1} e^{jk\phi} \quad (19)$$

$$\frac{\partial z_n}{\partial \phi} = j\sum_{k=1}^{\infty} y_{nM-kL}(-\lambda)^{k} = -j\alpha\frac{\partial z_n}{\partial \alpha}. \quad (20)$$

Under the assumptions of statistically independent information bearing symbols we see that the partial derivatives $\frac{\partial \tilde{z}_{\bar{n}}}{\partial \alpha}$ and $\frac{\partial \tilde{z}_{\bar{n}}}{\partial \phi}$ as well as $\frac{\partial z_n}{\partial \alpha}$ and $\frac{\partial z_n}{\partial \phi}$ are uncorrelated with $\tilde{z}_{\bar{n}}^*$ and z_n^* respectively. Furthermore, $\frac{\partial \tilde{z}_{\bar{n}}}{\partial \theta_0}$ is only correlated with s_0^*, and $\frac{\partial \tilde{z}_{\bar{n}}}{\partial \gamma}$ correlates with s_n^* and z_0^*.

Then, it is straightforward to compute the elements of the Hessian matrix for large block length N, i.e. if the sum over n can be replaced by the expectation operator:

$$\mathbf{H}|_{\xi_0} = 4\begin{bmatrix} \frac{N-t}{1-\alpha^2} & 0 & 0 & -\frac{t}{\alpha} \\ 0 & \frac{\alpha^2(N-t)}{1-\alpha^2} & t & 0 \\ 0 & t & t & 0 \\ -\frac{t}{\alpha} & 0 & 0 & 2N+t \end{bmatrix} \quad (21)$$

with

$$t = \alpha^{2\triangle}\frac{1-\alpha^{2\triangle N}}{1-\alpha^{2\triangle}} \ . \quad (22)$$

For the CM cost-function (10) we find

$$\mathbf{H}|_{\xi_0} = 2N\begin{bmatrix} \frac{2}{1-\alpha^2} & 0 & 0 \\ 0 & \frac{2\alpha^2}{1-\alpha^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \ . \quad (23)$$

In the derivation we assumed a rectangular pulse shaping filter and a path delay larger than the symbol period. For different kinds of filters we would effectively have a different multi-path model than the two-path model (1).

For asymptotic block length $N \to \infty$ the variance of the path parameter obtained by optimizing the uniform CM cost-function (8) is

$$\mathrm{var}(\delta\alpha^2) = \sigma_n^2 \frac{(1-\alpha^4)(1+2\alpha^{2N+2}) + \frac{\alpha^2}{N}(\alpha^{2N}-1)(\alpha^{2N+4}+2\alpha^2+3)}{N(1+\alpha^2)(\alpha^2-1+\frac{\alpha^2}{N}(1-\alpha^{2N}))^2}. \quad (24)$$

For small α we have the approximation

$$\mathrm{var}(\delta\alpha^2) = \frac{\sigma_n^2}{N(1-\alpha^2)}. \quad (25)$$

For $\alpha = 1$ we obtain using the rule of L'Hôpital

$$\mathrm{var}(\delta\alpha^2) = \frac{\sigma_n^2}{3}\frac{N+2}{N}. \quad (26)$$

It turns out that the variance of the estimated path parameter does not diverge even if $\alpha = 1$.

To compare our uniform equalizer with the classical CMA, we cite some theoretical performance results for the CMA from [15]: For asymptotic block length $N \to \infty$ the variance of the path parameter obtained by optimizing the CM cost-function (11) is, see [15]

$$\mathrm{var}(\delta\alpha^2) = \frac{\sigma_n^2}{N(1-\alpha^2)}\left(1 + \frac{1}{N}\frac{\alpha^{2N+2}-\alpha^2}{1-\alpha^2}\right). \quad (27)$$

Applying twice the rule of L'Hôpital yields for $\alpha = 1$

$$\mathrm{var}(\delta\alpha^2) = \frac{\sigma_n^2}{2}\frac{N+1}{N}. \quad (28)$$

Also in case of the classical CMA the estimation of the path parameter does not diverge. But the MSE of the symbols estimated by the classical CM cost-function (11) diverges, if α approaches one (see [15]):

$$\mathrm{E}_s[\mathrm{var}(z)] = \frac{\sigma_n^2}{1-\alpha^2} + \mathscr{O}(N^{-1}). \quad (29)$$

This is obvious, because an infinite number of non-decreasing filter taps would be required, if $\alpha = 1$.

In contrast to the classical CMA the MSE of the symbols estimated by our uniform CM cost-function (8) does not diverge even if $\alpha = 1$. This enables our algorithm to uniformly equalize 2-path channels with stronger first or second path or even cases were both paths have exactly equal amplitude. This is important for example, if the channel is time-varying and the trajectories of both path amplitudes are crossing.

4. SIMULATIONS

We carried out Monte-Carlo (MC) simulations with several hundreds of thousands MC runs for estimating the performance of the new uniform algorithm as a function of the path weight α. The case $\alpha = 1$ corresponds to the situation where the channel transfer function has zeros on the unit circle. We used PSK-4 signals in all experiments.

Figure 1 depicts analytical and numerical results for the standard deviation of the path parameter α estimated with our uniform blind equalizer. The batch size $\tilde{N}$ is chosen such that a maximum of $N = 100$ samples are involved in the filtering process, so the maximum value of k in (9) is 100. We observe good agreement of both curves and establish that there is no divergence for α approaching

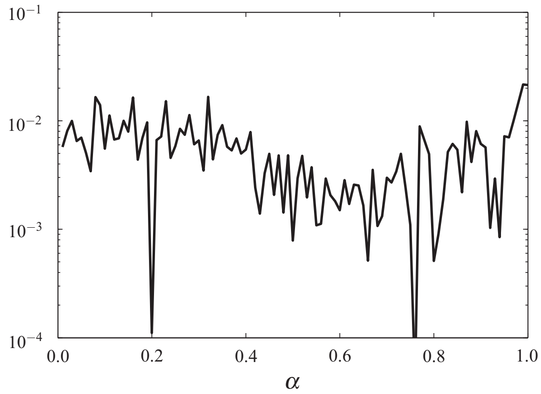

Figure 2: Relative error of the standard deviation for the path parameter α between MC-simulations and analytical approximation. Block length is $N = 100$ and additive noise has $\sigma_n = 0.01$.

one. The curve "Small-α approximation" corresponds to the term (25). It shows good agreement with the theoretical results as well, except for α-values close to one.

Figure 2 affirms the closeness of the MC-results and the analytic approximation (24): The relative error is nearly constant in the whole range of α and rarely larger than 10^{-2}.

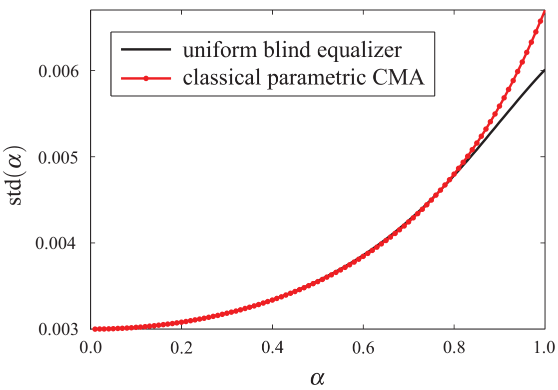

Figure 3: Analytic results for the standard deviations of the path parameter α. Block length is $N = 10$ and additive noise has $\sigma_n = 0.01$.

In order to compare the classical parametric CMA with our uniform blind equalizer, we show the analytic standard deviations of the path weight for both equalizers in Figure 3 and the corresponding results of MC-simulations in 4. In this case, the batch size is chosen such that $N = 10$. The results are very similar for small values of α. For α close to one, the classical CMA shows a larger standard deviation, however, both estimators do not diverge.

A different behavior can be observed in Figure 5 where the results of MC-simulations for the MSE of the equalized symbols are depicted. The symbol MSE for the classical parametric CMA diverges as α approaches one, because in this case an infinite number of filter coefficients would be necessary. For the uniform blind equalizer this divergence is removed, as it is expected due to the use of the truncated filter. The MSE curve shows only a slight increase when α increases. Similar results can be observed in Figure 6 which depicts the symbol MSE for a block length of $N = 100$. However, it turns out that the uniform equalizer shows better performance using shorter block lengths, at least if α approaches one.

The shown curves are based on simulations with a noise standard deviation of $\sigma_n = 0.01$. For larger values of σ_n we observed

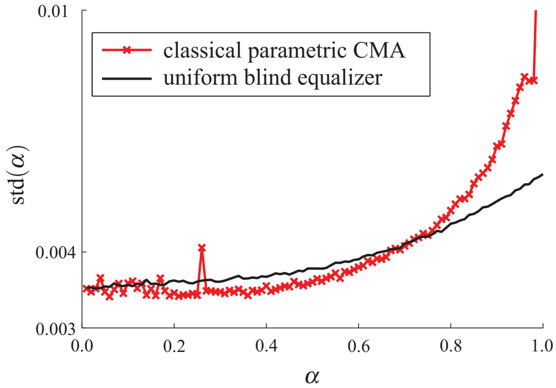

Figure 4: MC-simulations for the standard deviation of the path parameter α. Block length is $N = 10$ and additive noise has $\sigma_n = 0.01$.

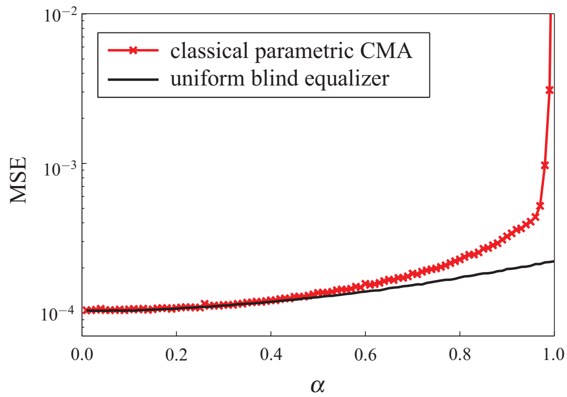

Figure 5: MC-simulations for the average MSE of the equalized symbols. Block length is $N = 10$ and additive noise has $\sigma_n = 0.01$.

a similar behavior regarding classical CMA and our uniform algorithm.

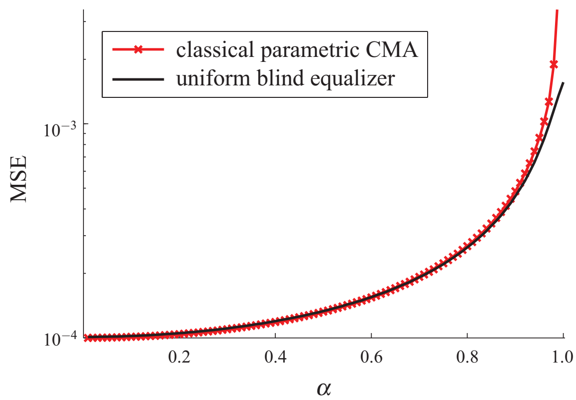

Figure 6: MC-simulations for the average MSE of the equalized symbols. Block length is $N = 100$ and additive noise has $\sigma_n = 0.01$.

5. CONCLUSION

We proposed a zero-forcing blind equalizer which can cope with two-path channels that have zeros on the unit circle in the z-domain.

As the proposed method exploits the constant-modulus property of the transmitted symbols it can handle as well cases in which subchannels have common zeros, a situation in which blind equalizers using second order statistics fail. In theoretical calculations and MC simulations we studied the performance of the new blind equalizer and found that it is able to equalize uniformly channels, unlike a parametric CMA that diverges for path weights of equal power.

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