PERFORMANCE ANALYSIS OF COOPERATIVE MIMO SYSTEMS USING DECODE AND FORWARD RELAYING

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ABSTRACT
In this paper, considering a source, a relay and a destination with multiple antennas, we provide the exact Bit Error Probability (BEP) of dual-hop cooperative Multiple Input Multiple Output (MIMO) systems using Decode and Forward (DF) relaying for Binary Phase Shift Keying (BPSK) modulation assuming independent and identically distributed (i.i.d.) Rayleigh fading channels. When the source or the relay has multiple antennas, it employs an Orthogonal Space Time Block Code (OSTBC) to transmit. The receiver uses a Maximum Likelihood (ML) decoder. We derive the expressions of the Moment Generating Functions (MGF), the Probability Density Functions (PDF) of the Signal-to-Noise Ratio (SNR), the exact and asymptotic BEP at the destination then we compare theoretical results to simulation ones. The analysis is performed in the presence or absence of the direct link. We also derive the diversity order and we study the effect of the number of antennas equipped with the source, the relay and the destination on the end-to-end BEP.

1. INTRODUCTION
MIMO is an emerging technology which consists in using multiple transmit and receive antennas in order to create spatial diversity between transmitter and receiver. The success of MIMO technology has led to the concept of cooperative communications where multiple nodes equipped with a single antenna cooperate together in order to form a virtual MIMO array and offer cooperative diversity. Cooperative system is a promising solution for high data-rate coverage required in future wireless communications systems. Combining a cooperative system with a Multiple Input Multiple Output transmission is an interesting way to overcome the channel impairments and to improve the system performances. The Symbol Error Probability (SEP) of cooperative systems using Amplify and Forward (AF) or Decode and Forward (DF) relaying was derived in [1]-[2]-[3]. In the AF protocol, each relay amplifies the received signal using an adaptive gain. In the DF protocol, each relay decodes the received signal from the source and transmits only if it has correctly decoded. In this paper, we are interested only in DF relaying. Exact end-to-end BER analysis of dual-hop orthogonal space-time transmission with multiple antenna is found in [4] for AF and DF relaying. Performance analysis of Amplify and Forward based cooperative diversity in MIMO relay channels was studied in [5]. Power allocation strategies in cooperative MIMO networks were investigated in [6].

In this paper, we present new simple expressions of the exact and asymptotic BEP at the destination for different scenarios of cooperative MIMO systems. Then we derive the expression of the diversity order.

The paper is organized as follows. Sections 2, 3 and 4 presents the performance analysis of cooperative MIMO systems using DF relaying respectively in the absence and presence of a direct link. For the case where we don’t have a direct link, we study two scenarios. The first concerns systems where the relay has a single transmit antenna and the second is about systems where the relay has multiple transmit antennas. Section 5 gives some numerical and simulation results. Section 6 draws some conclusions.

2. PERFORMANCE ANALYSIS OF COOPERATIVE MIMO SYSTEMS WITH A RELAY EQUIPPED WITH A SINGLE TX ANTENNA IN THE ABSENCE OF A DIRECT LINK

2.1 System Model

As shown in Figure 1, we consider a dual-hop wireless communication system in which the source S has **n_s** transmit antennas and the destination D has **n_d** receive antennas. The relay R supports **n_r** receive antennas.

![Figure 1: Cooperative MIMO system with a relay equipped with a single TX antenna.](image)

As shown in Figure 1, we consider a dual-hop wireless communication system in which the source S has **n_s** transmit antennas. S is communicating with the destination D equipped with **n_d** receive antennas through a relay R which supports multiple receive antennas, **n_r**, and a single transmit antenna. The transmission mode is assumed to be done over orthogonal channels and is composed of three phases:

- **Phase 1**: the source transmits the signal to the relay through its **n_s** antennas by using an OSTBC. The **n_r** × **n_s**
channel matrix of the first hop is

\[ H_{S,R} = \begin{pmatrix} h_{S,R}^{(1,1)} & \cdots & h_{S,R}^{(1,n_R^S)} \\ \vdots & \ddots & \vdots \\ h_{S,R}^{(n_I^R,1)} & \cdots & h_{S,R}^{(n_I^R,n_R^S)} \end{pmatrix} \]

We assume that each element of \( H_{S,R} \) is an i.i.d. complex Gaussian random variable (r.v.) with zero mean and variance \( \rho_{S,R} \). The received signal at the relay from the source is given by

\[ Y_{S,R} = \sqrt{E_S / n_R^S} H_{S,R} S + N_{S,R} \quad (1) \]

where \( E_S \) is the transmitted energy per symbol by the source, \( H_{X,Y} \) is the channel matrix of the link between \( X \) and \( Y \) and \( N_{X,Y} \) is an additive white complex gaussian noise with a variance equal to \( N_0 \). \( S \) is the code matrix which depends on the number of transmit antennas of the source.

**Phase 2**: if the relay correctly decodes, it forwards the decoded symbol with symbol energy \( E_R \) to the destination; otherwise, it remains idle. The received signal at the destination from the relay, \( Y_{R,D} \), can be written as

\[ Y_{R,D} = \sqrt{E_R} H_{R,D} \tilde{S} + N_{R,D} \quad (2) \]

where \( \tilde{S} = s_j \) if symbol \( s_j \) has been correctly decoded, \( \tilde{S} = 0 \) otherwise and \( H_{R,D} \) is an \( n_D^R \times 1 \) channel vector.

**Phase 3**: the destination decodes the received signals from the relay using a ML decoder.

### 2.2 Statistical description of the SNR

The Signal to Noise Ratio (SNR) at the relay is given by

\[ \Gamma_{S,R} = \sum_{i=1}^{n_I^R} \sum_{j=1}^{n_R^S} \Gamma_{S,R}^{(i,j)} \quad (3) \]

where

\[ \Gamma_{S,R}^{(i,j)} = \frac{E_S}{n_I^R n_R^S} | h_{S,R}^{(i,j)} |^2 \quad (4) \]

\( \Gamma_{S,R}^{(i,j)} \) is the SNR between the \( i \)-th relay’s receive antenna and the \( j \)-th source’s transmit antenna, and \( h_{S,R}^{(i,j)} \) represents the channel coefficient between the \( i \)-th Rx antenna of \( Y \) and the \( j \)-th Tx antenna of \( X \). The SNR at the relay follows a chi-square distribution with \( 2n_I^R n_R^S \) degrees of freedom. Therefore, the PDF of \( \Gamma_{S,R} \) is given by

\[ p_{\Gamma_{S,R}}(\gamma) = f(\gamma, \Gamma_{S,R}, n_I^R n_R^S) \quad (5) \]

where \( f(\gamma, a, l) \) is the chi-square distribution with \( 2l \) degrees of freedom and average mean \( a \times 1 \):

\[ f(\gamma, a, l) = \frac{\gamma^{l-1} e^{-\gamma/2}}{a^l (l-1)!}, \gamma \geq 0 \quad (6) \]

and

\[ \Gamma_{S,R}^{(i,j)} = E(\Gamma_{S,R}^{(i,j)}) = \frac{E_S}{n_I^R n_R^S} E(|h_{S,R}^{(i,j)}|^2) \quad (7) \]

where \( E(\cdot) \) is the expectation operator. \( \Gamma_{S,R}^{(i,j)} \) is independent of \( i \) and \( j \).

The BEP at the relay can be written as

\[ P_{e,SR} = A \int Q(\sqrt{\gamma} B \Gamma_{S,R}(\gamma)) d\gamma \quad (9) \]

where \( A \) and \( B \) depends on the considered modulation (\( A = 1 \) and \( B = 2 \) for BPSK modulation).

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{+\infty} e^{-u^2/2} du \quad (10) \]

and

\[ \Psi(l, X, Y) = A \left[ 1 - \mu_{X,Y} \right] \sum_{k=0}^{l-1} C_k^l \left( 1 + \mu_{X,Y} \right)^k \quad (11) \]

where

\[ C_k^l = \frac{n_1^R}{n_1^R (n_l^R - k)!} \quad (12) \]

\[ \mu_{X,Y} = \sqrt{\frac{2n_I^R n_R^S E_R \Gamma_{X,Y}^{(i,j)}}{B \Gamma_{X,Y}^{(i,j)}} + 2} \quad (13) \]

and

\[ \Gamma_{X,Y} = n_I^R n_R^S \Gamma_{X,Y}^{(i,j)} \quad (14) \]

The BEP at the destination can be written as

\[ P_{e,D} = P_{e,SR} + (1 - P_{e,SR}) P_{e,RD} \quad (15) \]

where \( P_{e,RD} \) represents the BEP of the \((R-D)\) link between the relay and the destination.

\[ P_{e,RD} = \Psi(n_D^R, R, D) \quad (16) \]

where

\[ (1 - \mu_{R,D}) \frac{n_D^R}{2} \sum_{k=0}^{n_D^R - 1} C_k^{n_D^R - 1} \left( 1 + \mu_{R,D} \right)^k \quad (17) \]

where

\[ \mu_{R,D} = \sqrt{\frac{B \Gamma_{R,D}^{(i,j)}}{B \Gamma_{R,D}^{(i,j)} + 2}} \quad (18) \]

and

\[ \Gamma_{R,D}^{(i,j)} = E_R (|h_{R,D}^{(i,j)}|^2) = \frac{E_R \rho_{R,D}}{N_0} \quad (19) \]
2.3 Asymptotic BEP

At a high SNR, the relay always correctly decodes the emitted symbols. Therefore, \( P_{e,SR} \approx 0 \) and peD becomes [7]

\[
P_{e,D}^{\text{asym}} \approx \frac{C_{n_r}^{d_R} n_{r}^{S} - 1}{(4^{\Gamma_{l}(1)} n_r)^{d_R}} + \frac{C_{n_r}^{d_R} n_{r}^{S} - 1}{(4^{\Gamma_{l}(1)} n_r)^{d_R}}
\]

(19)

If \( n_r^D < n_r^D n_r^S \), the second term of pedasymotique can be neglected at high SNR and the diversity order is \( n_r^D \). Similarly, when \( n_r^D n_r^S < n_r^D \), the diversity order is \( n_r^D n_r^S \). Therefore, the diversity order is

\[
d = \min(n_r^D, n_r^D n_r^S)
\]

(20)

3. PERFORMANCE ANALYSIS OF COOPERATIVE MIMO SYSTEMS WITH A RELAY EQUIPPED WITH MULTIPLE TX ANTENNAS IN THE ABSENCE OF A DIRECT LINK

3.1 System Model

As shown in Figure 2, the relay has now \( n_r^R \) transmit antennas. All transmissions are made over orthogonal channels. The cooperation protocol has 3 phases:

- **Phase 1**: The source transmits \( N = \text{lcm}(n_r^S, n_r^R) \) symbols to the relay where \( \text{lcm} \) is the abbreviation of least common multiple.

- **Phase 2**:
  - If the relay R correctly decodes \( i \) symbols with \( 0 \leq i < n_r^R \) then it remains idle because it can’t form any code matrix.
  - If the relay R correctly decodes \( j \) symbols with \( kn_r^R \leq j < (k+1)n_r^R \) then it transmits \( k \) matrices of code, thus it transmits \( kn_r^R \) symbols and the rest of the symbols, \( N - kn_r^R \), are not emitted.
  - If the relay R correctly decodes the totality of the \( N \) symbols then it transmits \( \frac{N}{n_r^R} \) matrices of code.

- **Phase 3**: The destination decodes the signals received from the relay using a ML decoder.

3.2 Exact BEP

The probability that the relay will transmit \( k \) matrices of code is:

\[
p_k = \sum_{j=kn_r^R}^{(k+1)n_r^R - 1} C_{n_r}^{d_R} (1 - P_{e,SR})^{j} P_{e,SR}^{(N-j)}
\]

(21)

where \( j \) is the number of symbols correctly received by the relay and \( P_{e,SR} \) is given by (8).

The BEP at the destination is given by

\[
P_{e,D} = p_0 + \sum_{k=1}^{N} p_k \left( \frac{kn_r^R}{N} P_{e,RD} + \frac{(N - kn_r^R)}{N} \right)
\]

(22)

where \( P_{e,RD} = \Psi(n_r^R, n_r^R, R, D) \), \( \Psi(l, X, Y) \) is given by psi, and

\[
\Gamma_{(m,n)}^{(m,n)} = \frac{E_{r} (|h_{m,n}^{R,R}|^2)}{N_0 n_r^R} = \frac{E_{r} P_{R,D}}{N_0 n_r^R}
\]

(23)

3.3 Asymptotic BEP

The asymptotic BEP at the destination is given by [7]

\[
P_{e,D}^{\text{asym}} \approx \frac{C_{n_r}^{d_R} n_{r}^{S} - 1}{(4^{\Gamma_{l}(1)} n_r)^{d_R}} + \frac{C_{n_r}^{d_R} n_{r}^{S} - 1}{(4^{\Gamma_{l}(1)} n_r)^{d_R}}
\]

(24)

Therefore, the diversity order is

\[
d = \min(n_r^D, n_r^D n_r^S)
\]

(25)

For \( n_r^R = 1 \), we obtain the same result as pedasymotique which is valid when the relay has a single transmit antenna.

4. PERFORMANCE ANALYSIS OF COOPERATIVE MIMO SYSTEMS IN THE PRESENCE OF A DIRECT LINK

4.1 System Model

In this section and as shown in Figure 3, there are a relaying link (\( S-R-D \)) and a direct link (\( S-D \)). The destination coherently combines the received signals from the source and the relay using a Maximum Ratio Combiner (MRC).
4.2 Conditional BER at the destination
If the relay has correctly decoded, the SNR at the destination can be written as:

\[ \Gamma_{D|R} = \frac{\sum_{i=1}^{n_D} \sum_{j=1}^{n_D} \Gamma_{S,D}^{(i,j)}(\cdot) + \sum_{k=1}^{n_D} \Gamma_{R,D}^{(k,1)}}{1 + \frac{\Gamma_{S,D}}{n_D} + \frac{\Gamma_{R,D}}{n_D}} \]  

(26)

where

\[ \Gamma_{S,D}^{(i,j)} = \left( \frac{E_S}{n_i^S n_j^S} \right)^2 \]  

(28)

\[ \Gamma_{R,D}^{(k,1)} = \left( \frac{E_R}{n_k^R} \right)^2 \]  

(29)

The BEP at the destination is given by

\[ P_e^{D|R} = A \int Q(\sqrt{B} \Gamma_{D|R}(\cdot)) d\Gamma \]  

(30)

where \( \Gamma_{D|R}(\cdot) \) is the PDF of the received SNR at the destination.

\[ M_{D|R}(s) = LT \left( \Gamma_{D|R}(\cdot) \right) \]  

(31)

where LT is the Laplace transform. The MGF of the SNR at the destination can be written as

\[ M_{D|R}(s) = E(e^{-s \Gamma_{D|R}}) \]  

(32)

where

\[ \Gamma_{S,D}^{(i,j)} = \frac{\Gamma_{S,D}}{n_i^S n_j^S} \]  

(33)

and

\[ \Gamma_{R,D}^{(k,1)} = \frac{\Gamma_{R,D}}{n_k^R} \]  

(34)

Thus

\[ M_{D|R}(s) = \frac{1}{1 + \frac{\Gamma_{S,D}}{n_D}} \times \frac{1}{1 + \frac{\Gamma_{R,D}}{n_D}} \]  

(35)

Using a fraction decomposition, we have

\[ M_{D|R}(s) = \sum_{i=1}^{n_D} \frac{a_i}{1 + \frac{\Gamma_{S,D}}{n_i^S}} + \sum_{k=1}^{n_D} \frac{b_k}{1 + \frac{\Gamma_{R,D}}{n_k^R}} \]  

(36)

where \( a_i \) and \( b_k \) are the residues which were computed using the software Mathematica. The PDF of the SNR can be calculated using the inverse Laplace Transform of the MGF as follows

\[ p(\Gamma_{D|R}(\cdot)) = LT^{-1}(M_{D|R}(\cdot)) \]  

(37)

Therefore, when the relay has correctly decoded, the BEP at D can be written as

\[ P_e^{D|R} = \sum_{i=1}^{n_D} a_i \Psi(\frac{\Gamma_{S,D}}{n_i^S}, S, D) + \sum_{k=1}^{n_D} b_k \Psi(\frac{\Gamma_{R,D}}{n_k^R}, D, R) \]  

(38)

where \( \Psi \) and \( \Psi \) were previously defined in \( f \) and \( psi \).

The BEP at D is written as

\[ P_e^D = P_e^{SR} \times P_e^{SD} + (1 - P_e^{SR}) \times P_e^{D|R} \]  

(39)

4.3 Asymptotic BEP
At high SNR, \( P_e^{SR} \ll 1 \) and \( Pe^D \) can be approximated by

\[ P_e^{as} \approx \frac{C e^{\alpha D}}{2^n_0 n_0^R - 1} \times \frac{C e^{\alpha D}}{2^n_0 n_0^R - 1} \]  

(42)

\[ \frac{4 \Gamma_{S,D}^{(i,j)}(\cdot) n_i^S \Gamma_{R,D}^{(k,1)}(\cdot) n_k^R}{(\Gamma_{S,D}^{(i,j)}(\cdot)) n_i^S + (\Gamma_{R,D}^{(k,1)}(\cdot) n_k^R)} \times \frac{1}{(\Gamma_{S,D}^{(i,j)}(\cdot)) n_i^S + (\Gamma_{R,D}^{(k,1)}(\cdot) n_k^R)} \]  

The diversity order is

\[ d = \min(n_i^S(n_k^R + n_0^D), n_k^R(1 + n_i^S)) \]  

(43)

5. SIMULATION AND THEORETICAL RESULTS
In this section, we compare the derived theoretical results to simulation ones. We plot the BEP at the destination versus \( E_b / N_0(dB) \) where \( E_b \) is the transmitted energy per bit for a dual-hop DF MIMO systems using a BPSK modulation. We have allocated the same power to the source and relay: \( E_R = E_S = E_b / 2 \). In order to take into account the path loss, the average power of the channel coefficient between the i-th Rx antenna of Y and the j-th Tx antenna of X is given by

\[ E(\lvert h_{i,j}^{(i,j)} \rvert^2) = \frac{\beta}{d_{XY}^\alpha} \]  

(44)

where \( d_{XY} \) is the normalized distance between X and Y and \( \alpha \) is the path loss exponent. Note that \( d_{XY} = \frac{d_{XY}}{d_{SR}} \), \( d_{XY} \) is the effective distance in meters between X and Y, \( d_{SR} \) is an arbitrary reference distance and \( \beta \) is the path loss at the reference distance. We have used the following parameters \( \beta = 1, \alpha = 3, d_{SR} = 0.4 \) and \( d_{BD} = 0.6 \).

Figure 4 shows the exact, asymptotic BEP and simulation results for a cooperative MIMO systems using DF relaying for the case where the relay has a single transmit antenna in the absence of a direct link. For this scenario the diversity order is: \( \min(n_i^S(n_k^R + n_0^D), n_k^R(1 + n_i^S)) \).

(31) 13 (n_i^S = 2, n_k^R = 1, n_0^D = 3) offers better performance than the 2112 (n_i^S = 2, n_k^R = 1, n_0^D = 2) and 2111 (n_i^S = 2, n_k^R = 1, n_0^D = 1). In fact, the diversity order is equal to three for the first system however the two others have respectively a diversity order equal to two and one. Thus, if the diversity increases, the performances of the system improve.
It is clear that the diversity order is: \( n_D \) where the relay has multiple transmit antenna in the absence and \( n_R \) where all nodes have a single antenna. We see that the BEP for the scenario \( 2222 \) \((n^s_1 = 2, n^R_1 = 2, n^D_1 = 2)\) where we have multiple transmit antennas at the relay is lower than the same scenario but with a single transmit antenna at the relay, \( 2212 \) \((n^s_1 = 2, n^R_1 = 2, n^D_1 = 1, n^D_2 = 2)\), and the cooperative SISO system where all nodes have a single antenna.

Figure 5 shows the BEP plots, computed from the analytical results and simulation ones for a MIMO relay system where the relay has multiple transmit antenna in the absence of a direct link. For this scenario, the diversity order is: \( \min(n^s_1 n^R_1, n^s_1 n^D_1) \). We see that the BEP for the scenario \( 2222 \) \((n^s_1 = 2, n^R_1 = 2, n^D_1 = 2)\) where we have multiple transmit antennas at the relay is lower than the same scenario but with a single transmit antenna at the relay, \( 2212 \) \((n^s_1 = 2, n^R_1 = 2, n^D_1 = 1, n^D_2 = 2)\), and the cooperative SISO system where all nodes have a single antenna.

Figure 6 compares the BEP of cooperative MIMO systems in the presence and absence of a direct link. It is clear that the obtained performances are better in the presence of a direct link.

6. CONCLUSION

In this paper, considering a source, a relay and a destination with multiple antennas, we study the end-to-end performance of dual-hop cooperative MIMO systems employing DF relaying. We have derived the PDF and MGF of the SNR at the destination. Numerical results have shown that the BEP analysis provided in the paper is in accordance with the simulation results for various scenarios.

REFERENCES