GREEDY PURSUITS FOR COMPRESSED SENSING OF JOINTLY SPARSE SIGNALS

Dennis Sundman, Saikat Chatterjee and Mikael Skoglund

School of Electrical Engineering and ACCESS Linnaeus Centre
KTH - Royal Institute of Technology, SE-10044 Stockholm, Sweden
denniss@kth.se, sach@kth.se, skoglund@kth.se

ABSTRACT

For compressed sensing with jointly sparse signals, we present a new signal model and two new joint iterative-greedy-pursuit recovery algorithms. The signal model is based on the assumption of a jointly shared support-set and the joint recovery algorithms have knowledge of the size of the shared support-set. Through experimental evaluation, we show that the new joint algorithms provide significant performance improvements compared to regular algorithms which do not exploit a joint sparsity.

1. INTRODUCTION

Compressed sensing (CS) [1, 2] is based on a sparse signal model and utilizes an under-determined system of linear equations for data acquisition and reconstruction. The CS process is in general computationally light, and straight forward, in the acquisition part and computationally heavy in the reconstruction part. For the reconstruction part, we note in the literature three classes of algorithms: convex relaxation based, non-convex and iterative-greedy-pursuits.

Due to the complexity in the reconstruction part of CS, iterative-greedy-pursuit algorithms have recently attracted much attention. Examples of such algorithms are Matching pursuit [3], Orthogonal matching pursuit (OMP) [4], Look ahead orthogonal matching pursuit [5] and Subspace pursuit (SP) [6]. From a measurement vector, the main principle of iterative-greedy-pursuit algorithms is to estimate the underlying support-set of a sparse vector followed by evaluating the associated signal values. The support-set is the set of indices corresponding to non-zero elements of a sparse vector. To estimate the support set and the associated signal values, the iterative-greedy-pursuit algorithms use linear algebraic tools, e.g. the matched filter and least squares solution.

Recently we have seen a rising interest in solving CS for multiple jointly sparse signals [7]. We refer to such a problem as the *joint CS problem* and application scenarios referenced in the literature are magnetoencephalography [8, 9, 10, 11], spectrum analysis [12] and wireless sensor networks [13]. A joint CS problem has two main aspects: (1) system set-up, (2) signal model. Depending on these two aspects, a reconstruction algorithm needs to be developed.

First, for the system set-up, we notice that [8] and [9] use a single sensor (one sensing matrix) to sample signals. Duarte et al. [7] are a bit more general and provide a system set-up where multiple sensors are present. Next, we observe that [8, 9] have a signal model defined in such a way that all sparse signals have one common support-set. We refer to the signal model of [8, 9] as the common support-set model. On the other hand, the signal model in [7] is defined in such a way that the signals have common and individual signal parts. We refer to this signal model as the mixed signal model.

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We now discuss existing reconstruction algorithms for the joint CS problem. Based on a single sensor system set-up and a common support-set model, several greedy-pursuit algorithms have been proposed [8, 9]. These algorithms can not be applied for the more general multiple sensor system set-up of [7], as well as the mixed signal model. For the multiple sensor system set-up along with the mixed signal model, convex relaxation as well as iterative greedy-pursuit algorithms were proposed in [7].

In this paper, for the joint CS problem, we present a new signal model and develop two new iterative-greedy-pursuit algorithms. Our system set-up is the same as [7] where multiple sensors are present, but the signal model is different. For the signal model, we use common and individual support-sets to characterize jointly sparse signals. Unlike [7], there is no restriction on the associated signal values. We refer to this new signal model as the *mixed support-set* model. The mixed support-set model is shown to be a generalization over all previously proposed models. To take advantage of the mixed support-set model, we develop two new iterative-greedy-pursuit algorithms based on regular OMP and regular SP. We refer to these new algorithms as *joint OMP* and *joint SP*. The only knowledge the proposed algorithms use is the cardinalities of the common and individual support-sets. The new algorithms utilize the strategy of jointly finding the common support-set iteratively. Through experimental evaluations, we show significant improvements compared to the regular OMP and SP.

Notations: Let a matrix be denoted as $\mathbf{A} \in \mathbb{R}^{M \times N}$ and a vector as $\mathbf{x} \in \mathbb{R}^{M \times N}$. $\mathcal{I}$ is the support-set of $\mathbf{x}$, which is defined in the next section. $\mathbf{A}_{\mathcal{I}}$ is the submatrix consisting of the columns in $\mathbf{A}$ corresponding to the elements in a set $\mathcal{I}$. Similarly $\mathbf{x}_{\mathcal{I}}$ is a vector formed by the components of $\mathbf{x}$ that are indexed by $\mathcal{I}$. The pseudo inverse of $\mathbf{A}$ is denoted as $\mathbf{A}^{\dagger}$ and the matrix transpose as $\mathbf{A}^T$.

2. MIXED SUPPORT-SET MODEL

Using the general multiple sensor system set-up (as in [7]), we first describe the joint CS problem and then the new mixed support-set signal model. First, for the l'th sensor, we have the sparse signal $\mathbf{x}_l$ which is observed for the joint CS problem as

$$\mathbf{y}_l = \mathbf{A}_l \mathbf{x}_l + \mathbf{w}_l, \qquad \forall l \in \{1, 2, ..., L\}, \tag{1}$$

where $\mathbf{y}_l \in \mathbb{R}^{M \times 1}$ is a measurement vector, $\mathbf{A}_l \in \mathbb{R}^{M \times N}$ a measurement matrix, $\mathbf{w}_l \in \mathbb{R}^{N \times 1}$ is the measurement error, and $M < N$. $\mathbf{A}_l$ and $\mathbf{w}_l$ are independent across l. The signal vector $\mathbf{x}_l$ has K_l non-zero components with indices $\mathcal{I}_l = \{i : x_l(i) \neq 0\}$. $\mathcal{I}_l$ is referred to as the support-set of $\mathbf{x}_l$ and the cardinality is $|\mathcal{I}_l| = \|\mathbf{x}_l\|_0 = K_l$. Using the set of $\{\mathbf{y}_l\}_{l=1}^{L}$, the joint CS reconstruction problem endeavors for finding $\{\mathbf{x}_l\}_{l=1}^{L}$ by exploiting some shared structure among the l sensors defined by the underlying signal model.

Now, we describe the new mixed support-set signal model with a shared structure where the signal vector $\mathbf{x}_l$ consists of two parts

$$\mathbf{x}_l = \mathbf{z}_l^{(c)} + \mathbf{z}_l^{(p)}, \qquad \forall l \in \{1, 2, ..., L\}. \tag{2}$$

In the new model (2) both $\mathbf{z}_l^{(c)}$ and $\mathbf{z}_l^{(p)}$ have independent non-zero components. There are $K_l^{(p)}$ non-zero values associated with $\mathbf{z}_l^{(p)}$. For simplicity we assume that the non-zero values are located uniformly at random over the support-set $\mathcal{I}_l^{(p)} \in \{1, 2, \dots, N\}$, and $\forall l \in \{1, 2, \dots, L\}$. For $\mathbf{z}_l^{(c)}$ there are similarly $K^{(c)}$ non-zero components with the constraint that the support-set is shared, $\mathcal{I}_l^{(c)} = \mathcal{I}^{(c)}, \forall l \in \{1, 2, \dots, L\}$. The elements of $\mathcal{I}^{(c)}$ are the same (common) to all signals, but unknown to the reconstructor[1]. This gives a support-set $\mathcal{I}_l$ for each set of signals as

$$\mathcal{I}_l = \mathcal{I}^{(c)} \cup \mathcal{I}_l^{(p)}, \qquad \forall l \in \{1, 2, ..., L\}. \tag{3}$$

We define $K_{l,\max} = |\mathcal{I}^{(c)}| + |\mathcal{I}_l^{(p)}| = K^{(c)} + K_l^{(p)}$. Note that the support-sets can intersect, so $K_{l,\max} \geq K_l$.

From the set of measurement vectors $\{\mathbf{y}_l\}_{l=1}^L$, the task is to find a good estimate $\{\hat{\mathbf{x}}_l\}_{l=1}^L$ using the a-priori knowledge of the cardinality of the support-sets $\{K_l^{(p)}\}_{l=1}^L$ and $K^{(c)}$.

2.1 Generalization over existing signal models

The purpose of this section is to clarify what is new in the mixed support-set model. Let us compare our model with the mixed signal model of [7], where $\mathbf{x}_l$ is composed of common and individual parts

$$\mathbf{x}_l = \mathbf{z}^{(c)} + \mathbf{z}_l^{(p)}, \qquad \forall l \in \{1, 2, ..., L\}. \tag{4}$$

Here $\mathbf{z}^{(c)}$ represents a common sparse signal part and $\mathbf{z}_l^{(p)}$ represents the individual (private) signal part for the l'th sensor. Note that $\mathbf{z}^{(c)}$ is fixed for all the data set. Comparing (2) and (4) we can say that the new mixed support-set model is a generalization over the mixed signal model in the sense that the former allows for different values in the common signal part $\mathbf{z}_l^{(c)}$.

The common support-set model [8, 9] used in magnetoencephalography has no individual signal parts at all. Their model is

$$\mathbf{x}_l = \mathbf{z}_l^{(c)}, \qquad \forall l \in \{1, 2, ..., L\}. \tag{5}$$

Therefore our model (2) also generalizes the model (5).

The regular CS algorithms OMP and SP were not constructed to exploit the knowledge of a shared structure. Furthermore, none of the algorithms presented in [8, 9, 7] can solve the joint CS problem based on the new mixed support-set model (2). Therefore, in the following two sections, we develop two new iterative-greedy-pursuit algorithms.

3. JOINT ORTHOGONAL MATCHING PURSUIT

In this section, based on the regular OMP algorithm, we present a new joint OMP algorithm for solving the joint CS problem (1). This algorithm works for multiple sensors and is based on the new general mixed support-set model (2). For clarity in the algorithmic notation, we define three functions as follows

$$\text{resid}(\mathbf{y}, \mathbf{A}) \triangleq \mathbf{y} - \mathbf{A}\mathbf{A}^{\dagger}\mathbf{y}, \tag{6}$$

$$\text{max\_indices}(\mathbf{x}, k) \triangleq \{\text{the set of indicies corresponding to the } k \text{ largest amplitude components of } \mathbf{x}\}, \tag{7}$$

and

$$\text{add}_1(\mathbf{s}, \mathcal{I}) \triangleq \{\text{In the } N \times 1 \text{ vector } \mathbf{s}, \text{ add 1 to every index corresponding to the elements in the support-set } \mathcal{I}\}. \tag{8}$$

3.1 Joint Orthogonal Matching Pursuit

We now describe the new joint OMP algorithm that uses the new mixed support-set model. For developing the joint OMP algorithm, we modify the regular OMP algorithm so that it can use an estimate of the common support-set as an initial support-set. Our assumption is that over the iterations, the estimate of the common support-set will improve. The necessary modifications to the regular OMP is presented in subsection 3.2.

We now show the steps of the joint OMP in algorithm 1. In the k'th iteration stage, the algorithm finds a temporary

Algorithm 1 : joint OMP

Input: $\{\mathbf{A}_l\}_{l=1}^L$, $\{K_l^{(p)}\}_{l=1}^L$, $K^{(c)}$,$\{\mathbf{y}_l\}_{l=1}^L$
Initialization:
1: $k \leftarrow 0$ ('Iteration counter')
2: $\mathcal{I}^{(c)} \leftarrow \emptyset$ ('Initial common support-set')
Iterations:
1: **repeat**
2: $k \leftarrow k + 1$
3: $\mathbf{s} \leftarrow \mathbf{0}_{N \times 1}$
4: **for** $\forall l \in \{1, 2, ..., L\}$ **do**
5: $K_{l,\max} \leftarrow K^{(c)} + K_l^{(p)} - \text{size}(\mathcal{I}^{(c)})$
6: $(\mathcal{I}_l, \hat{\mathbf{x}}_l, n_l) \leftarrow \text{OMP}(\mathbf{A}_l, K_{l,\max}, \mathbf{y}_l, \mathcal{I}^{(c)})$
7: $\mathbf{s} \leftarrow \text{add}_1(\mathbf{s}, \mathcal{I}_l)$
8: **end for**
9: $\mathcal{I}^{(c)} \leftarrow \text{max\_indices}(\mathbf{s}, k)$
10: **until** $(k = K^{(c)})$
Output: $\{\hat{\mathbf{x}}_l\}_{l=1}^L$, $\{\mathcal{I}_l\}_{l=1}^L$

$K_{l,\max}$ (step 5), which is passed on to the underlying modified OMP together with the estimated common support-set $\mathcal{I}^{(c)}$ from the previous iteration (step 6). An N-sized zero vector $\mathbf{s}$ adds for each estimated support-set $\mathcal{I}_l$ an one at the location of the predicted non-zero element (step 7). If the modified OMP algorithm finds one support-set element that is common for all data, the value of $\mathbf{s}$ at this index will have the maximum size of L. The indices corresponding to the k largest elements in $\mathbf{s}$ are then chosen as the common support-set $\mathcal{I}^{(c)}$ (step 9). Once an element is chosen, it will be fed to the modified OMP algorithm in the next iteration and always remain in $\mathcal{I}^{(c)}$. Because a chosen index will always remain in the common support-set, it is important to carefully select each index to add to the common support-set. We find that, in practice, the use of $\text{add}_1(\cdot, \cdot)$ and $\text{max\_indices}(\cdot, \cdot)$ allows us for a good estimate of $\mathcal{I}^{(c)}$.

3.2 Modified OMP

As mentioned in subsection 3.1, we now describe the modified OMP in algorithm 2. Instead of initializing with an empty support-set and begin iterating from the first component as in the regular OMP, we here allow the algorithm to take an initial support-set and continue building the final support-set from this. This modification boils down to the regular OMP as shown by Tropp and Gilbert [4] when the

[1] For easy practical implementation, we assume that the elements are uniformly distributed over $\mathcal{I}^{(c)}$ and $\mathcal{I}_l^{(p)}$.

Algorithm 2 : OMP (including modifications)

Input: $\mathbf{A}$, $K_{\max}$, $\mathbf{y}$, $\mathcal{I}_{\text{ini}}$.
Initialization:
1: $\mathcal{I}_0 \leftarrow \mathcal{I}_{\text{ini}}$
2: $\mathbf{r}_0 \leftarrow \text{resid}(\mathbf{y}, \mathbf{A}_{\mathcal{I}_0})$
3: $k \leftarrow |\mathcal{I}_0|$
Iteration:
1: **repeat**
2: $k \leftarrow k + 1$
3: $i_{\max} \leftarrow \text{max\_indices}\left(\mathbf{A}^T \mathbf{r}_{k-1}, 1\right)$
4: $\mathcal{I}_k \leftarrow \mathcal{I}_{k-1} \cup i_{\max}$
5: $\hat{\mathbf{x}}_{\mathcal{I}_k} \leftarrow \mathbf{A}^{\dagger}_{\mathcal{I}_k} \mathbf{y}$
6: $\mathbf{r}_k \leftarrow \text{resid}(\mathbf{y}, \mathbf{A}_{\mathcal{I}_k})$
7: **until** $(k = K_{\max})$
Output:
1: $\hat{\mathcal{I}} \leftarrow \mathcal{I}_k$
2: $\hat{\mathbf{x}}$ such that $\hat{\mathbf{x}}_{\mathcal{I}_k} = \mathbf{A}^{\dagger}_{\mathcal{I}_k}\mathbf{y}$ and $\hat{\mathbf{x}}_{\bar{\mathcal{I}}_k} = \mathbf{0}$
3: $n_r \leftarrow \|\mathbf{r}_k\|_2$

initial support-set $\mathcal{I}_{\text{ini}} = \emptyset$. The modified OMP algorithm starts with finding a residual (step 2 of Initialization), where $\mathcal{I}_0 = \mathcal{I}_{\text{ini}}$. If the initial support-set $\mathcal{I}_{\text{ini}} = \emptyset$, the matrix $\mathbf{A}_{\mathcal{I}_0}$ is empty and the residual becomes $\mathbf{y}$. At the k'th iteration stage the modified OMP algorithm forms the matched filter, identifies the index corresponding to the largest amplitude (step 3) and adds this to the support-set (step 4). It proceeds with solving a least squares problem with the selected indices (step 5), subtracts the least squares fit and produces a new residual (step 6). This process is updated until $K_{\max}$ components have been picked in the support-set. Normally the regular OMP also has a stopping criterion when the residual norm is non-decreasing. We have removed this stopping criterion to force the modified OMP such that it picks $K_{\max}$ elements.

In addition to the sparse signal estimate $\hat{\mathbf{x}}$, we also output the estimated support-set and the final residual norm. The residual norm is here given for completion and could be used as a stopping criterion in the joint algorithm, although we do not take this approach.

4. JOINT SUBSPACE PURSUIT

For solving the joint CS problem (1), we develop the joint SP algorithm based on the mixed support-set model and a modified SP algorithm. The joint SP algorithm uses the functions (6), (7) and (8), defined in section 3.

4.1 Joint Subspace Pursuit

We here describe the new joint SP algorithm that uses the new mixed support-set model. For developing the joint SP algorithm, we modify the regular SP algorithm so that it can use an estimate of the common support-set as an initial support-set. The necessary modifications to the regular SP is presented in subsection 4.2.

We now show the steps of the joint SP in algorithm 3. In contrast to the joint OMP, the joint SP algorithm does not need any iterator because it uses the sum-residual-norm as stopping criteria. Another difference to the joint OMP is that the joint SP algorithm has to keep track of its old data (step 4). The maximum $K_{l,\max}$ (which is found in the initialization) is then fed into the modified SP algorithm (step 5). Similarly to the joint OMP, the N-sized zero vector $\mathbf{s}$ is used to find the common support-set $\mathcal{I}^{(c)}$ (step 6 and 9). The sum-residual-norm is formed in (step 8). The difference to the joint OMP is that, in each iteration, the joint SP picks the indices corresponding to the $K^{(c)}$ largest components of $\mathbf{s}$ (instead of the k largest, as in joint OMP). This common support-set is always of the same size $K^{(c)}$, but refined through iterations. We stop when the value of the sum-residual-norm no longer decreases.

Algorithm 3 : joint SP

Input: $\{\mathbf{A}_l\}_{l=1}^L$, $\{K_l^{(p)}\}_{l=1}^L$, $K^{(c)}$,$\{\mathbf{y}_l\}_{l=1}^L$
Initialization:
1: $r_n \leftarrow \infty$; $\hat{\mathbf{x}}_l \leftarrow \mathbf{0}_{N\times 1}, \forall l \in \{1, 2, \ldots L\}$
2: $\mathcal{I}^{(c)} \leftarrow \emptyset$; $\mathcal{I}_l \leftarrow \emptyset, \forall l \in \{1, 2, \ldots L\}$
3: $K_{l,\max} = K^{(c)} + K_l^{(p)}, \forall l \in \{1, 2, \ldots L\}$
Iteration:
1: **repeat**
2: $\mathbf{s} \leftarrow \mathbf{0}_{N\times 1}$
3: **for** $\forall l \in \{1, 2, ..., L\}$ **do**
4: $(r_n^{\text{old}}, \mathcal{I}_l^{\text{old}}, \hat{\mathbf{x}}_l^{\text{old}}) \leftarrow (r_n, \mathcal{I}_l, \hat{\mathbf{x}}_l)$
5: $(\mathcal{I}_l, \hat{\mathbf{x}}_l, n_l) \leftarrow \text{SP}(\mathbf{A}_l, K_{l,\max}, \mathbf{y}_l, \mathcal{I}^{(c)})$
6: $\mathbf{s} \leftarrow \text{add}_1(\mathbf{s}, \mathcal{I}_l)$
7: **end for**
8: $r_n \leftarrow \sum_{l=1}^L n_l$
9: $\mathcal{I}^{(c)} \leftarrow \text{max\_indices}(\mathbf{s}, K^{(c)})$
10: **until** $(r_n \geq r_n^{\text{old}})$
Output: $\{\hat{\mathbf{x}}_l^{\text{old}}\}_{l=1}^L$, $\{\mathcal{I}_l^{\text{old}}\}_{l=1}^L$

Since the joint SP algorithm stops on the criterion of sum-residual-norm it turns out to converge using less iterations, why it is also less computationally intensive than the joint OMP.

4.2 Modified SP

As mentioned in subsection 4.1, we here describe the modified SP in algorithm 4. The initialization phase has, compared to the regular SP, been modified in a similar way as the OMP algorithm so that it can use an initial support-set. This algorithm boils down to the regular SP as defined by Dai and Milenkovic [6] when $\mathcal{I}_{\text{ini}} = \emptyset$. At k'th iteration stage, the modified SP algorithm forms the matched filter $\mathbf{A}^T\mathbf{r}_{k-1}$, identifies the indices corresponding to the $K_{\max}$ largest amplitudes followed by joining with the old support-set (step 3 of Iteration). This support-set $\mathcal{I}'$ is likely to be bigger than $K_{\max}$. The algorithm solves a least squares problem with the selected indices of $\mathcal{I}'$ and identifies the new indices cor-

Algorithm 4 : SP (including modifications)

Input: $\mathbf{A}$, $K_{\max}$, $\mathbf{y}$, $\mathcal{I}_{\text{ini}}$
Initialization:
1: $\mathcal{I}_0 \leftarrow \text{max\_indices}\left(\mathbf{A}^T\mathbf{y}, K_{\max}\right) \cup \mathcal{I}_{\text{ini}}$
2: $\mathbf{r}_0 \leftarrow \text{resid}(\mathbf{y}, \mathbf{A}_{\mathcal{I}_0})$
3: $k \leftarrow 0$
Iteration:
1: **repeat**
2: $k \leftarrow k + 1$
3: $\mathcal{I}' \leftarrow \mathcal{I}_{k-1} \cup \text{max\_indices}\left(\mathbf{A}^T\mathbf{r}_{k-1}, K_{\max}\right)$
4: $\hat{\mathbf{x}}$ such that $\hat{\mathbf{x}}_{\mathcal{I}'} = \mathbf{A}^{\dagger}_{\mathcal{I}'}\mathbf{y}$ and $\hat{\mathbf{x}}_{\bar{\mathcal{I}}'} = \mathbf{0}$
5: $\mathcal{I}_k \leftarrow \text{max\_indices}(\hat{\mathbf{x}}, K_{\max})$
6: $\mathbf{r}_k \leftarrow \text{resid}(\mathbf{y}, \mathbf{A}_{\mathcal{I}_k})$
7: **until** $(\|\mathbf{r}_k\|_2 \geq \|\mathbf{r}_{k-1}\|_2)$
8: $k \leftarrow k - 1$ ('Previous iteration count')
Output:
1: $\hat{\mathcal{I}} \leftarrow \mathcal{I}_k$
2: $\hat{\mathbf{x}}$ such that $\hat{\mathbf{x}}_{\mathcal{I}_k} = \mathbf{A}^{\dagger}_{\mathcal{I}_k}\mathbf{y}$ and $\hat{\mathbf{x}}_{\bar{\mathcal{I}}_k} = \mathbf{0}$
3: $n_r \leftarrow \|\mathbf{r}_k\|_2$

responding to the $K_{\max}$ largest amplitudes (step 4 and 5 of Iteration) followed by finding the residual (step 6). This process is repeated until the residual norm no longer decreases.

In addition with the sparse signal estimate $\hat{\mathbf{x}}$, we also output the estimated support-set $\hat{\mathcal{I}}$ and the final residual norm.

5. SIMULATION RESULTS

In the simulations we are interested in finding how much performance can be gained by exploiting the mixed support-set model with the new algorithms (joint OMP and joint SP) over the regular algorithms (regular OMP and regular SP). We report the results for clean and noisy measurement cases. In the noisy case we have chosen signal-to-measurement-noise-ratio (SMNR) 20 dB, i.e., $10\log_{10}\frac{\mathbb{E}\{\|\mathbf{x}\|_2^2\}}{\mathbb{E}\{\|\mathbf{w}\|_2^2\}} = 20$. Note that we drop the subscript l because we are averaging over all sensors l.

To compare the algorithms, we use two performance measurements. The first performance measure is the signal-to-reconstruction-noise-ratio (SRNR) which is defined as

$$\mathrm{SRNR} = 10\log\mathbb{E}\left\{\frac{\|\mathbf{x}\|_2^2}{\|\mathbf{x}-\hat{\mathbf{x}}\|_2^2}\right\}. \quad (9)$$

The second performance measure is the the distortion $d(\mathcal{I},\hat{\mathcal{I}}) = 1 - (|\mathcal{I}\cap\hat{\mathcal{I}}|/|\mathcal{I}|)$ which measures the performance of the support-set estimation [14]. The distortion of the support-set is a valuable performance measurement since the algorithms we compare endeavor to estimate the underlying support-set. Considering a large number of realizations (data vectors), we can compute the average of $d(\mathcal{I},\hat{\mathcal{I}})$. We define the average support-cardinality error (ASCE) as follows

$$\mathrm{ASCE} = \mathbb{E}\left\{d(\mathcal{I},\hat{\mathcal{I}})\right\} = 1 - \mathbb{E}\left\{\frac{|\mathcal{I}\cap\hat{\mathcal{I}}|}{|\mathcal{I}|}\right\}. \quad (10)$$

Next we describe the simulation setup. In any CS setup, all sparse signals are expected to be exactly reconstructed if the number of measurements are more than a certain threshold value. The computational complexity to test this uniform reconstruction ability is exponentially high. Instead, we can rely on empirical testing, where SRNR and ASCE is computed for random measurement matrix ensemble. We define the fraction of measurements

$$\alpha = \frac{M}{N}. \quad (11)$$

Using α, the testing is performed as follows:

1. Given the signal parameter N, choose an α (such that M is an integer).
2. Randomly generate a set of $M \times N$ sensing matrices $\{\mathbf{A}_l\}_{l=1}^{L}$ where the components are drawn independently from an i.i.d. Gaussian source (i.e. $a_{m,n} \sim \mathcal{N}\left(0, \frac{1}{M}\right)$) and then scale the columns of $\mathbf{A}_l$ to unit-norm.
3. Generate support-sets $\mathcal{I}^{(c)}$ and $\{\mathcal{I}_l^{(p)}\}_{l=1}^{L}$ of cardinality $K^{(c)}$ and $\{K_l^{(p)}\}_{l=1}^{L}$, respectively. The support-sets are uniformly chosen from $\{1,2,...,N\}$.
4. Randomly generate a set of signal vectors $\{\mathbf{x}_l\}_{l=1}^{L}$ following (2), where $\{\mathbf{z}_l^{(c)}\}_{l=1}^{L}$ and $\{\mathbf{z}_l^{(p)}\}_{l=1}^{L}$ corresponding to the non-zero components (support-sets determined in step 3). The non-zero components in the vectors are chosen independently from a Gaussian source.
5. Compute the measurements $\mathbf{y}_l = \mathbf{A}_l\mathbf{x}_l + \mathbf{w}_l, \forall l \in \{1,2,...,L\}$. Here $\mathbf{w}_l \sim \mathcal{N}(\mathbf{0},\sigma_l^2\mathbf{I}_M)$.
6. Apply the CS algorithms on the data $\{\mathbf{y}_l\}_{l=1}^{L}$.

In the simulation procedure above, for each $l \in \{1,2,\ldots,L\}$, Q sets of sensing matrices are created. For each data set and each sensing matrix, P sets of data vectors are created. In total, we will average over $L\cdot Q\cdot P$ data to evaluate the performance.

5.1 Parameters and simulation set-up

For the plots presented in this paper, we have chosen: $N = 500$, $K^{(c)} = 10$, and $\forall l, K_l^{(p)} = K^{(p)} = 10$. We have chosen $L = 10$ for which we have chosen number of $\mathbf{A}_l$'s to 50 (i.e. $Q = 50$) and the number of data-sets $\mathbf{x}$ to 50 (i.e. $P = 50$), giving a total number of $10\cdot50\cdot50 = 25000$ data for statistics.

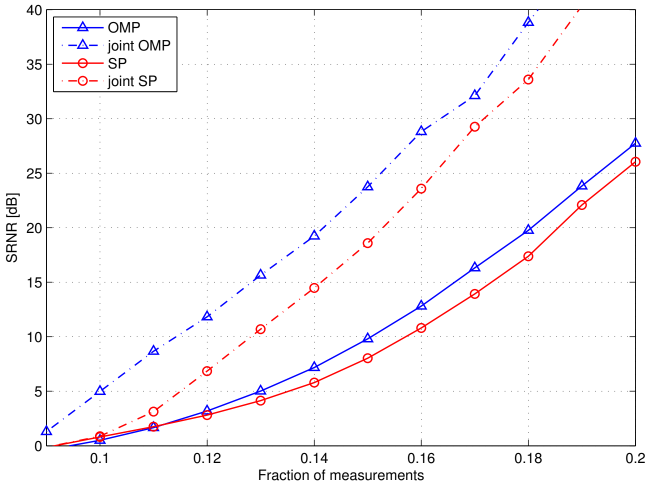

Figure 1: SRNR for clean measurements with $K^{(c)} = 10$, $K^{(p)} = 10$.

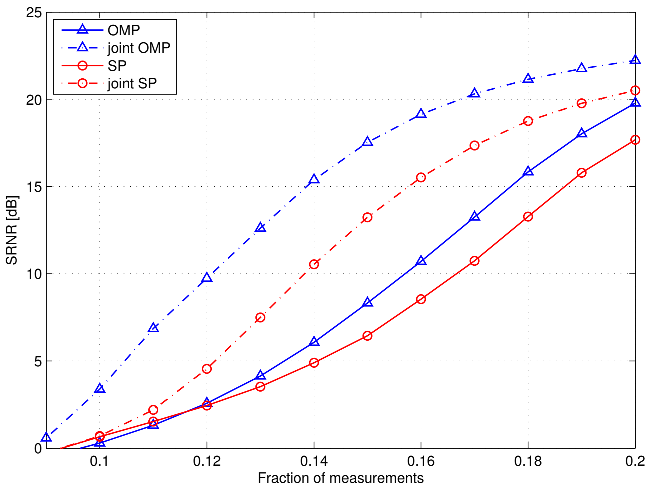

Figure 2: SRNR for noisy measurements with SMNR = 20 dB, $K^{(c)} = 10$, $K^{(p)} = 10$.

5.2 Analysis of the simulation results

We provide four figures showing the results of the numerical simulations.

Figure 1 and Figure 2 shows the SRNR of the reconstructed signal for a clean and a noisy (SMNR = 20 dB) measurement case, respectively. In the clean case, we notice that the joint OMP performs almost 15 dB better than the regular OMP at $\alpha = 0.15$. In the noisy case this number is slightly lower with about 10 dB improved performance (still $\alpha = 0.15$). Corresponding numbers for the SP-based

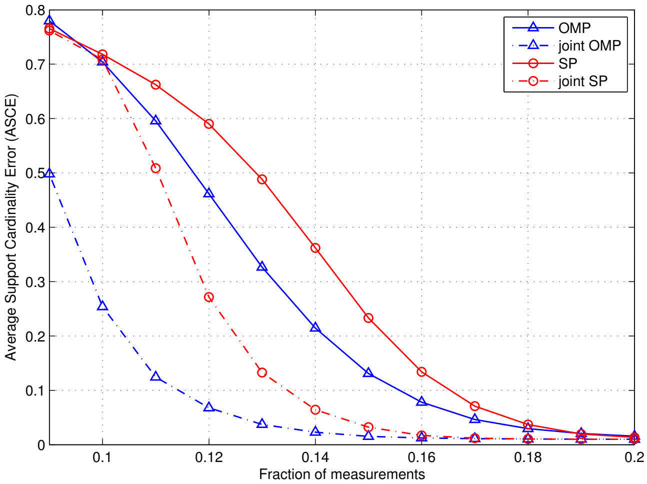

Figure 3: ASCE for clean measurements with $K^{(c)} = 10$, $K^{(p)} = 10$.

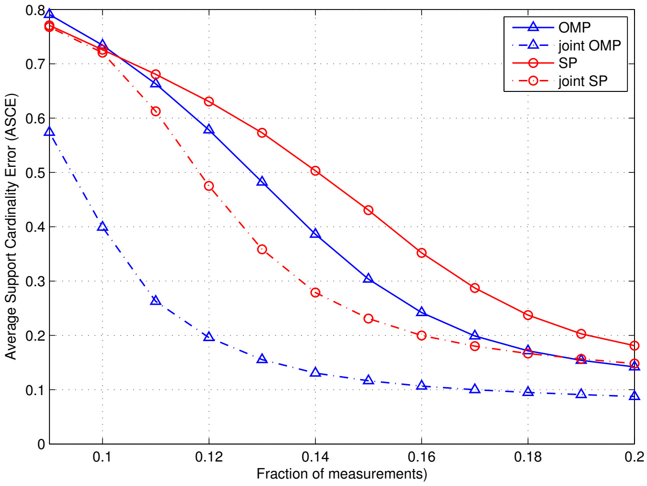

Figure 4: ASCE for noisy measurements with SMNR = 20 dB, $K^{(c)} = 10$, $K^{(p)} = 10$.

algorithm is about 10 dB improvement for the clean measurements and 8 dB improvement for the noisy case, both at $\alpha = 0.15$.

In Figure 3 and Figure 4 we similarly show the ASCE of the reconstructed signal for a clean and a noisy (SMNR = 20 dB) case. In the noisy case, we notice that none of the algorithms can perfectly find the support-set, as expected. We also notice that the joint OMP very quickly converges to its best performance. Also, the joint OMP is significantly better than all the other algorithms at low α's.

It is interesting to notice that the performance gain of joint OMP over regular OMP is higher than the gain of joint SP over regular SP. The reason for this is understood by how the algorithm picks the common support-set. The joint OMP picks one common support-set component at each iteration and hence iterates 10 times. In each iteration the regular OMP is called once for each sensor. The joint SP on the other hand, starts with an estimate of the full common support-set and iteratively refines it. Thus, the joint OMP is more careful when choosing the support-set and progresses at a slower pace compared to the joint SP. It was observed during the experiments that because of this difference in implementation, the joint OMP has a significantly higher computational complexity.

6. CONCLUSION

In this paper, a new mixed support-set model and two new greedy pursuit algorithms was proposed. We find that the model is a generalization of existing signal models presented in the literature earlier. To solve the joint CS problem based on this model, we developed two new algorithms, joint OMP and joint SP, based on regular OMP and SP, respectively. By experimental evaluations, we conclude that greedy pursuit algorithms can exploit joint sparsity information embedded in data.

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