AN ALGEBRAIC DERIVATIVE-BASED METHOD FOR R WAVE DETECTION

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ABSTRACT

In this paper, a new robust method for R wave detection in ECG signal is proposed by using algebraic derivative estimation based technique. In fact, this new and efficient method relies on differential algebra, non-commutative algebra together with operational calculus. This technique allows noisy signal to be filtered via iterated time integrals and R wave slope to be emphasized. The ECG signal is then Hilbert transformed to be enhanced and threshold compared. The performance of the algorithm was tested by using the annotated records of MIT-BIH Arrhythmia Database. The robustness of the proposed R wave detector in presence of noise was also tested according to records from MIT-BIH Noise Stress Test Database. The overall performance is quite good even SNR as low as 6 dB.

1. INTRODUCTION

The Electrocardiogram ECG represents the electrical activity of the heart. This is characterized by a number of waves P, QRS, T related to the heart activity. The QRS complex is the most perceptible waveform within ECG beat. Its high amplitude makes QRS detection easier than other waves. Detection of R-peaks and consequently QRS complex have many applications including R-R interval analysis, ST segment examination, ECG data compression and ECG waveform classification. Different approaches have been proposed to QRS detection. An extensive review of the approaches proposed in the last decade can be found in [15]. Recently, new approaches are mainly based on artificial neural networks [22], genetic algorithms, wavelet transforms [16], hidden Markov models as well filter banks techniques [1]. In noisy-free case, all these algorithms have satisfactory performances but performances decrease when the noise corrupts measured signal. Consequently, these techniques require a pre-treatment step using the classical numeric filters which introduce signal distortions in frequency or temporal components. Indeed, we need methods preserving the signal information contents and particularly R wave positions, in the presence of noise. Our aim in this paper is to present a new method which permits essentially to overcome the difficulties that arise when considering the pre-treatment of the ECG signal.

The algorithm based on algebraic-derivative allows to emphasize large slope of R wave. To enhance QRS complex, the differentiated signal is then Hilbert transformed. The output is finally examined using decision rules to detect the occurrence of QRS complexes.

Noise treatment is the mean challenge of QRS detectors since noise is often present in all ECG signals. It comes from various sources: muscular activity, movements artifacts, power line interference and baseline wandering due to the respiration. In addition, P and T waves with high amplitudes may be considered challenging noise as they winder R wave detection.

Algebraic derivative, initially developed in [18], has already proved to be robust with respect to noise in image processing [14], signal processing [8], automatic control [9] as well as in biomedical engineering. This feature is due to the iterated time integrals which play the role of low pass filter attenuating the noise power. Indeed, disturbing noise is viewed here as highly fluctuating phenomena [7]. This investigation is essentially stimulated by the theory of M. Fliess. The underlying theoretical framework using, in particular, module theory and Mikusiński operational calculus can be found in [19]. Recently, problem in biomedical engineering can be tackled with this new method which is of algebraic flavour. For instance, the works of Mboup [21], which aim at the neural information processing by the detection of neuronal spikes, show promising results in neuroscience field.

The Hilbert transformation is of widespread interest because it is applied in the theoretical description of many systems and directly implemented in the form of Hilbert analog or digital filters (transformers). Hilbert transformer (or filter) finds numerous applications, especially in modern digital signal processing. This terminology is applied here as a part of R wave detection algorithm.

The remaining of the paper is organized as follows: section 2 outlines the algebraic derivative-based method as well as the properties of the Hilbert transform. Section 3 gives an overview of the R wave detection steps of our algorithm. Section 4 applies the methodology on reals ECG data excerpted from the MIT-BIH arrhythmia database [17] and gives the simulation results compared to other algorithms and section 5 draws the conclusion.

2. FUNDAMENTAL RELATIONSHIPS

2.1 Algebraic derivative

We address the problem of estimating the first order derivative. More precisely, let $x(t)$ denotes the signal we want to estimate. We assume that $x(t)$ is smooth enough to permit a polynomial approximation of a chosen order p locally over an interval $I_{t-}^{T} = [t-T, t]$ where p is an integer. This integer corresponds to the derivation order, here $p = 1$. This assumption is motivated by the Weierstrass's Theorem which states that function, continuous in a finite closed interval can be approximated to a desired accuracy by a polynomial function. Because polynomials are the simplest functions, and computers can directly evaluate polynomials, this theorem has both practical and theoretical relevance. With this motivation, the signal can be approximated around $t = 0$ for example by the first degree polynomial time function: $p_1(t) = a_0 + a_1 t$, $t \geq 0$, $a_1, a_2 \in \mathbf{R}$. Accurate identification of the coefficients is immediate: a_0 and a_1 are respectively the estimators of the signal and its derivative around the time constant $t = 0$, i.e, $a_0 = x(0)$ and $a_1 = \frac{dx}{dt}|_{t=0}$. Rewrite thanks to classic operational calculus (cf [19]) p_1 as $P_1 = \frac{a_0}{s} + \frac{a_1}{s^2}$. We look for annihilate a_0 because we want to estimate a_1. Thus, multiply both sides by s:

$$sP_1 = a_0 + \frac{a_1}{s}$$

Take the derivative of both sides with respect to s, in order to omit a_0, which correspond in time domain to the multiplication by $-t$ (cf [19]);

$$P_1(s) + s\frac{dP_1(s)}{ds} = -\frac{a_1}{s^2} \quad (1)$$

The time derivative; i.e, $s\frac{dP_1(s)}{ds}$, is removed by multiplying both sides of equations (1) by s^{-n}, $n > 1$, for instance $n = 2$. This operation allows to obtain only iterated time integrals. Since $x(t)$ is noisy, there is an error in computing polynomial coefficients and henceforwards we can only obtain an estimate of a_1 denoted by $\hat{a}_1$. Finally, the above equation leads in the time domain to the linear estimator

$$\begin{aligned}\hat{a}_1(t) &= 6\frac{\int_{t-T}^{t}\tau x(\tau)d\tau - \int_{t-T}^{t}\int_{t-T}^{\tau}x(\kappa)d\kappa d\tau}{T^3} && \text{(2a)}\\ &= 6\frac{\int_{t-T}^{t}\tau x(\tau)d\tau - \int_{t-T}^{t}(T-\tau)x(\tau)d\tau}{T^3} && \text{(2b)}\\ &= 6\frac{\int_{t-T}^{t}(2\tau - T)x(\tau)d\tau}{T^3} && \text{(2c)}\end{aligned}$$

There are no derivatives with respect to time involved, but only integrations. This is particularly valuable in the presence of high frequency perturbations or measurements corrupted by noise. Besides, we may mention that almost all local polynomial modeling problems use the principle of sliding estimation window. An optimal choice of the size of this window remains an issue. In this approach, it is established that a quit short time window is sufficient for obtaining accurate value of a_1. We note that the role of this window is to localise the model fitting around the instant t. We use an T-point window for simplicity, although other window functions can be used. More information can be found in the original paper [18].

2.2 Hilbert transform

Let's $x(t)$ a real signal. Generally speaking, the Hilbert transform of a real signal $x(t)$ is defined by [11]:

$$\begin{aligned}x_H(t) &= \mathscr{H}\{x(t)\} && \text{(3a)}\\ &= \frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{x(\lambda)d\lambda}{t-\lambda} && \text{(3b)}\\ &= \frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{x(t-\lambda)d\lambda}{\lambda} && \text{(3c)}\\ &= x(t) * \frac{1}{\pi t} && \text{(3d)}\end{aligned}$$

Here $\mathscr{H}\{.\}$ means the Hilbert transform and $*$ denotes the usual convolution product. The integrals in definition (3b) and (3c) are improper because the integrand goes to infinity for $\lambda = t$. Therefore, the integral is defined as the Cauchy principal value (Sign P). Equation (3d) shows that the Hilbert transform of $x(t)$ is obtained by filtering the signal through a linear filter with impulse response $\frac{1}{\pi t}$.
A useful way of looking at the Hilbert transform, and perhaps a more intuitive definition, is in the frequency domain

$$\begin{aligned}X_H(\omega) &= X(\omega).[-j.sgn(\omega)] && \text{(4a)}\\ &= X(\omega).H(\omega) && \text{(4b)}\end{aligned}$$

where the transfer function of the Hilbert transform $H(\omega)$ is given by:

$$H(\omega) = \begin{cases} -j, & \text{for } \omega > 0 \\ j, & \text{for } \omega < 0 \\ 0, & \text{for } \omega = 0 \end{cases} \tag{5}$$

In fact, a Hilbert transform simply shifts all positive frequency components by -90^o and all negative-frequency components by $+90^o$. The amplitude always remains constant throughout this transformation. As outlined in figure 2 Hilbert Transform requires a Fast Fourier Transform (FFT) and an Inverse FFT (IFFT). The implementation of the Hilbert Transform is fairly simple in the frequency domain. The so-colled *analytic* signal is then defined as:

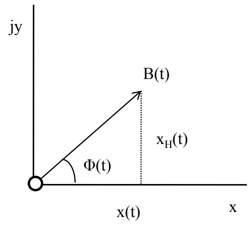

Figure 1: The cartesian plane representing the analytic signal $z(t) = x(t) + jx_H(t)$=$B(t)e^{j\phi(t)}$

$$z(t) = x(t) + jx_H(t) \tag{6}$$

It is witnessed that $x(t) = Re\{z(t)\}$ and $x_H(t) = Im\{z(t)\}$ where Re is the real part of $z(t)$ though Im is the imaginary part of $z(t)$. The name of *analytic* denotes numerous types of complex functions which satisfy the Cauchy Riemann conditions for differentiability and is traditionally called *analytic* functions. By taking the Fourier Transform of expression (6), we get

$$Z(\omega) = X(\omega) + jX_H(\omega) \tag{7}$$

by using equation (5), we obtain that the analytic signal has the following spectrum:

$$Z(\omega) = \begin{cases} 2X(\omega), & \text{for } \omega > 0 \\ X(0), & \text{for } \omega = 0 \\ 0, & \text{for } \omega < 0 \end{cases} \tag{8}$$

which is inverse transformed to obtain $z(t)$. Therefore, the Hilbert transform can be easily computed by taking the imaginary part of the analytic signal $z(t)$.
Since the *analytic* signal is complex it can always be put into polar form

$$z(t) = B(t)e^{j\phi(t)} \tag{9}$$

We can then unambiguously define the envelope $B(t)$(the amplitude) and the phase $\phi(t)$ of $z(t)$ by

$$B(t) = \sqrt{x^2(t) + x_H^2(t)}, \qquad \phi(t) = tan^{-1}\left(\frac{x_H(t)}{x(t)}\right) \tag{10}$$

which gives

$$\omega(t) = \phi'(t) = \frac{x_H(t)'x(t) - x(t)'x_H(t)}{B^2(t)} \tag{11}$$

for the instantaneous frequency (IF) in the complex plane. The signal (equation 9) is represented in the cartesian plane as shown in figure 1. The IF obtained by this means is often meaningless. A recent method pioneered by Huang and al. called Empirical Mode Decomposition (EMD) [13] was introduced to address this problem. An innovative approach based on iterated application of the Hilbert Transform (IHT) that has asymptotical convergence property faster than that of EMD, is described in [10].
It's clear that the envelope $B(t)$ coincides with $x(t)$ when $x_H(t) = 0$. Therefore, the maximum contribution of the envelope $B(t)$ when $x(t) = 0$ is given by its Hilbert transform. These remarks can be exploited in our case by the following way: We can turn the first differential of the ECG into analytic signal using the Hilbert transform. The maximum contribution of the first derivative envelope(B($\frac{d}{dt}(ECG)$)) is given by its Hilbert transform.

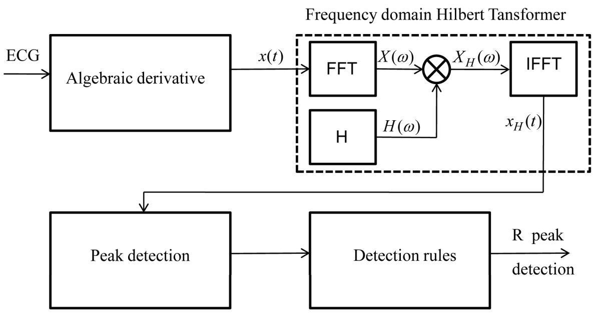

Figure 2: Block diagram of the R wave detector.

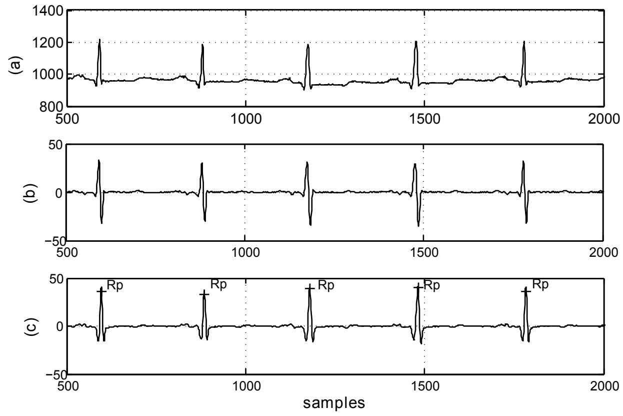

Figure 3: R wave position definition: (a) ECG signal (record(100)); (b) The output of the algebraic derivative; (c) Hilbert transform.

Recall that if the real valued signal $x(t)$ is even then $X(\omega)$ is purely real-valued while if $x(t)$ is odd then $X(\omega)$ is purely imaginary-valued. Now if $X(\omega)$ is purely real valued then certainly $X_H(\omega)$ is purely imaginary valued(and vice-versa). Therefore, the Hilbert transformation changes any even term to an odd term and any odd term to an even term.

Let's take the even part of the signal. The Hilbert transform will be an odd function. That is to mean it will cross zero on the x-axis every time there is an inflexion point in the original waveform. Similarly, a crossing of the zero between consecutive positive and negative inflexion points in the original waveform will be represented as a peak in its Hilbert transformed.

This interesting property can be exploited to develop an elegant and much easier way to find the peak of the QRS complex in the ECG waveform corresponding to a zero crossing in its first derivative waveform $\frac{d(ECG)}{dt}$.

3. ALGORITHM FOR DETECTING R WAVE

The fact of applying the first derivative followed by the Hilbert transform doesn't detect QRS complex, but it only aims to enhance the peaks corresponding to QRS complex and then find region for high probability to locate R wave. A second stage to detect R wave is required. It consists to detect peaks in the signal. When a peak is activated, peak detection function returns all samples reaching the current peak height. Thus, each time a peak is detected, it is classified as either QRS complex or noise. The algorithm uses the peaks height, peaks locations (relative to the last QRS peak) and maximum Hilbert transformation in order to classify peaks.

The following is an outline of the basic detection rules for the algorithm detailed in [12] and [20]:

1. Ignore all peaks that precede or follow larger peaks by less than a waiting time equal to *Dist* (*refractory period*).
2. If the peak is larger than the detection threshold we call it QRS complex, otherwise we call it noise.
3. If no QRS is detected within 1.5 R-to-R intervals so there is a peak larger than half the detection threshold, and the peak followed the preceding detection by at least 360*ms*, we classify that peak as a QRS complex (*back search*).

Figure 2 shows the blocks diagram of the basic operations of our algorithm for beats detection.

A true QRS peak is expected to appear after a delay equal to *Dist* following the latest detected QRS peak. *Dist* is related to the expected QT interval of the current RR interval. Initially, $Dist = 200ms$. It is updated further using a version of an empirical formula of the QT interval:

$$Dist = 0.4 * mRR \tag{12}$$

Here *mRR* represents a combination of averaged latest four interval RR_{mean} with the shortest one of them RR_{min}:

$$\frac{7RR_{mean} + RR_{min}}{8}$$

When a beat is located nearer than *Dist* to the latest QRS complex, this candidate is ignored if its amplitude is smaller than the latest detected complex.

The adaptive detection threshold used in 2 and 3 is calculated by using estimates of QRS peaks and noise peak heights. At each time a peak is classified as a QRS complex, it is added to a buffer containing the last eight QRS peaks. Otherwise, the peak is added to a another buffer containing the recent eight non-QRS peaks (noise peaks). The detection threshold is set according to:

$$\begin{aligned}\text{Detection\_Threshold= Mean\_Noise\_Peak} \\ \text{+TH*(Mean\_Peak-Mean\_Noise\_Peak)}\end{aligned} \tag{13}$$

Where TH is the threshold coefficient. We estimate the mean of QRS height (respectively noise height and R-R intervals) using the mean of the last eight peak values. We note that some initial threshold estimate in the beginning of the algorithm is needed. We initialize noise buffer to 0 and the QRS buffer to the eight maximum consecutive peaks in 1-second interval. The initial threshold is initialized according to formula (13). Figure 3 illustrates an example of R wave detection by the proposed algorithm. We can see in figure 3(b) the original signal after the derivative estimation. The resulting signal contains only R wave positions (refer to figure 3(c)). The '+' marker corresponds to the R-position Rp.

4. RESULTS AND DISCUSSIONS

To analyze the performance of the proposed technique, on real world signals, we choose the ECG samples excerpted from the MIT-BIH Arrhythmia database [17]. It contains 48 half-hour recording of annotated ECG with a sampling rate of $360Hz$. Furthermore, beat annotations include R wave marks.

Two benchmark parameters are used to compare the performance of the proposed detection algorithm: the sensitivity and the positive predictivity of the beat detection are defined as following [15]

$$Se(\%) = 1 - \frac{FN}{TP+FN} = \frac{TP}{TP+FN}\%, \tag{14}$$

$$+P(\%) = 1 - \frac{FP}{TP+FP} = \frac{TP}{TP+FP}\%, \tag{15}$$

where (TP) is the number of true positive, (FN) is the number of false negative and (FP) is the number of false positives. The (*Se*) reports the percentage of true beats that were correctly detected by the algorithm. The positive predictivity $(+P)$ reports the percentage of beat detection which were in reality true beats.

Annot	TP	FN	FP	Se(%)	+P(%)	DER
17658	17610	48	20	99.76	89.89	0.34

Table 1: Recapitulate results for the proposed R wave detector of a 5*mn* record (not including MIT 107 and 108)

Method	DER	Se(%)	+P(%)
Proposed detector	1.05	99.38	99.45
Combined adaptive threshold [5]	1.44	99.92	98.64
Wavelet transform [16]	1.09	99.50	99.42
Linear adaptive filtering [22]	2.41	99.15	98.47
Bandpass filtering [12] and back search	2.91	99.15	97.98
Bandpass filtering [20]	3.46	99.15	97.58
Filter banks [1]	3.22	99.26	97.58

Table 2: DER, Se(%), +P(%) scores for the proposed the R wave detector compared to literature for the record 105

The Detection Error Rate (DER) and the Average time error are calculated as follow:

$$\text{DER}(\%) = \frac{FP+FN}{\text{Total number of QRS complex}}\% \tag{16}$$

$$\text{Average time error(ms)} = \frac{\sum_{i=1}^{TP} |\text{Detected\_QRS\_Time-Actual\_QRS\_Time}|}{TP} \tag{17}$$

The robustness of the estimation method in the presence of noise is quantified by the noise stress test using recordings from the MIT-BIH Noise Stress Database [17]. The ECG recordings were created using two clean records (118 and 119) from the MIT-BIH ECG Database to which calibrated amounts of noise containing baseline wander, muscle artifact, and electrode motion artifact. The SNR of each record is ranged from 6 to 24 dB.

The performance of our algorithm is influenced by a given set of parameters. There are two principle parameters within our algorithm(T,TH). The length T of the sliding window should be small enough to improve estimation. We test our algorithm for each record nine times with threshold coefficient TH ranging from 0.1 to 0.9. We choose the value which minimizes the Detection Error Rate. In this paper, we selected TH as the mean of all optimal value corresponding to lower DER for each record. The mean value is roughly 0.3.

In order to make objective comparison with other published works, we classify the reported results with respect to their score. On the first scenario, we analyze the first five minutes signals in 46 database records. The table 1 reports the detection performances of our algorithm. The results are $Se = 99.76\%$, $+P = 99.89\%$ and DER= 0.34%. Consequently, based on this score, the proposed technique outperforms Dinh's method [6] based on Cubic Spline method (DER= 0.75%), although it performs similarly to Ben Massaoud's method [3] based on Equalize of the maxima (DER= 0.34%).

On the second scenario, the algorithm is tested against the whole noisy MIT-BIH record 105. This later is used through the literature to test QRS detectors and therefore comparisons are possible. The predominant feature of this record is a high grade of noise and artifacts. The comparison made between the performance of the proposed algorithm and literature are listed in table 2. The reliability of the proposed detector compares favorably with that of other published results especially concerning the most noisy record 105 (see [1, 5, 12, 16, 20, 22]). Here, we should mention that the underling result is very encouraging since it would open promising investigation perspectives related to this new estimation method.

Next, a quantitative study of the noise tolerance test is performed. The performance of our detector is quit good for SNR as low as 6dB with high sensitivity values (about 94%) as well as high positive prediction (above 88%). The results are depicted table 3. At high SNR as well as low SNR, DER for our detector is superior to the other techniques examined specially in [4].

Record	SNR	DER	Se(%)	+P(%)
118 e24	24	0	100	100
118 e18	18	0	100	100
118 e12	12	1.27	99.73	98.99
118 e06	06	13.26	94.23	92.60
119 e24	24	0	100	100
119 e18	18	0	100	100
119 e12	12	1.16	99.90	88.84
119 e06	06	14.64	97.58	88.84

Table 3: DER, Se(%), +P(%) scores for the proposed QRS detector at 6 dB, 12dB, 18 dB and 24 dB for the record 118 and 119

Method	Av.time error(ms)
Proposed detector	5.58
Hilbert transform with 2^{nd} threshold [2]	7.08
Squaring function with automatic threshold [2]	7.90
2^{nd} derivative with automatic threshold [2]	6.5
Filter banks[1]	266

Table 4: Comparison results of QRS detection delay time and literature

Now let us discuss the delay time in QRS detection. Delay time is expressed by the average time error parameter. Simulation results show that our algorithm has minimal delay in detecting beat compared to other algorithms in literature [1, 2]. Comparative results are shown in table 4. In fact, applying algebraic derivative and the back search strategy result on a beat detection latency time of more than heartbeat interval. The detection delay will be roughly the sum of derivative delay plus the average of R-to-R interval if a back search detection is needed. The algebraic derivative for the used approximation introduces a delay in the estimation equals to $\frac{T}{2}$ samples length. See [18] for more explanation. This delay is off-line corrected after derivative step. The back search strategy is incorporated to correct FN. A back search is activated when no beat detection has occurred in a time interval superior 1.5 R-to-R interval. This strategy may result in a significant delay before a beat is detected.

5. CONCLUSION

We proposed an algebraic derivative-based algorithm for R wave detection of noisy ECG signals. We use an empirical technique based on local polynomial model to estimate the derivative. This model is popular because it is flexible with respect to derivative and computational easy to use. The algorithm is tested on several records from MIT-BIH arrhythmia database. These records are corrupted by different noises and artifacts. The results is assessed in term of benchmark parameters. Beat detection accuracy is comparable to other algorithm reported in literature. It can also be categorized by a real-time algorithm since it has a minimal beat detection delay. Furthermore, our algorithm could be implemented more easily comparatively to other algorithm based for instance on wavelet transform and neural networks. Further improvements to the algorithm may be easily achieved by using more features of the frequency components of the ECG or by using a more suitable model to fit ECG signal variation.

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