A SELECTIVE NORMALIZED SUBBAND ADAPTIVE FILTER EXPLOITING AN EFFICIENT SUBSET OF SUBBANDS

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ABSTRACT

In this paper, we present a subband adaptive filter which selects a subset of subbands and utilizes them in updating the adaptive filter weight. The NSAF algorithm has a trade-off between the number of subbands and convergence speed. The proposed algorithm, thus, increases the number of subbands to acquire improved convergence speed. However, employing an increased number of subband filters raises computational complexity. We use only a subset of extended subbands so as not to have redundant computational complexity, while we maintain performance. To minimize performance degradation from the extended subbands, we show that the larger ratio of the corresponding squared error to an input power should be selected through a geometric interpretation. Throughout the experiments, we show that the proposed NSAF algorithm has good convergence performance compared with the conventional NSAF algorithm.

1. INTRODUCTION

The normalized least mean-square (NLMS) is one of the most popular and widely used adaptive filtering algorithms because of its simplicity and robustness. However, its defect is that correlated input signals significantly deteriorate its convergence behavior [1, 3]. To solve this problem, various approaches have been presented such as recursive least squares (RLS) [1, 3], affine projection algorithm (APA) [4], and subband adaptive filtering (SAF) [5, 8]. Among these, the SAF algorithm allocates the input signals and desired response into almost mutually exclusive subbands. This "re-whitening" delayless structure of SAF allows each subband to converge almost separately so the subband algorithms obtain faster convergence behavior. Lee and Gan presented the normalized SAF (NSAF) algorithm in [9] to improve convergence speed with almost the same computational complexity as the NLMS algorithm.

The NSAF algorithm, however, has enormous computational complexity when the number of taps is long such as when using network and acoustic echo cancelation applications. To reduce computation and to acquire faster convergence performance, several selective partial update algorithms have been proposed [10, 12]. Courville's approach incorporates the fullband weight model in each subband, coping with the structural problems [13]. The selective partial update algorithms update a selected subset of the filter coefficients. Abadi has applied these methods to the SAF to create simplified selective partial-update subband adaptive filtering (SSPU-SAF) algorithms [14]. Also, the NSAF shows that the number of subbands is closely related to the filter behavior and computational burden [9]. Therefore, Kim's algorithm adaptively changes the number of subbands at every iteration using this characteristic [15].

In this paper, we propose a selective subband scheme that improves the performance of the NSAF. The proposed algorithm enlarges the number of subbands and selects the most effective subset of subbands. The optimal subband selection criterion can be derived by using the principle of minimum disturbance between successive filter coefficients. We demonstrate that the proposed algorithm is much faster than the conventional NSAF algorithm while having a little bit lower computational complexity. In addition, we show that the proposed algorithm with doubled subbands has a better convergence rate than conventional NSAF when they have almost the same computational complexity.

This paper is organized as follows: In Section 2, we review NSAF and formulate the proposed algorithm. Section 3 contains the experimental results, which illustrate the convergence performance of the proposed algorithm, and Section 4 presents the conclusion.

2. SELECTIVE SUBBANDS NSAF (SS-NSAF)

Consider a desired signal $d(k)$ that originates from an unknown linear system

$$d(k) = \mathbf{u}(k)\mathbf{w}^{o} + v(k), \tag{1}$$

where $\mathbf{w}^{o}$ is an unknown column vector to be identified with an adaptive filter, $v(i)$ corresponds to measurement noise with zero mean and variance σ_v^2, and $\mathbf{u}(n)$ denotes a row input (regressor) vector with length M as follows:

$$\mathbf{u}(k) = [u(k) \;\; u(k-1) \;\; \cdots \;\; u(k-N+1)]. \tag{2}$$

2.1 Conventional NSAF

Fig. 1 shows a subband structure with the desired response $d(k)$. The filter output $y(k)$ is partitioned into M subbands by means of L-tap analysis filters $H_1(z), \cdots, H_M(z)$. These subband signals, $d_i(k)$ and $y_i(k)$ for $i = 1, \cdots, M$ are critically sub-sampled to a lower rate commensurate with their bandwidth. Let $\mathbf{w}(k)$ be an estimate for $\mathbf{w}^{o}$ at time index k. The decimated subband error signal is then defined as $e_{i,\mathrm{D}}(n) = d_{i,\mathrm{D}}(n) - \mathbf{u}_i(n)\mathbf{w}(n)$ where

$$\mathbf{u}_i(n) = [u_i(nM), u_i(nM-1), \ldots, u_i(nM-N+1)]. \tag{3}$$

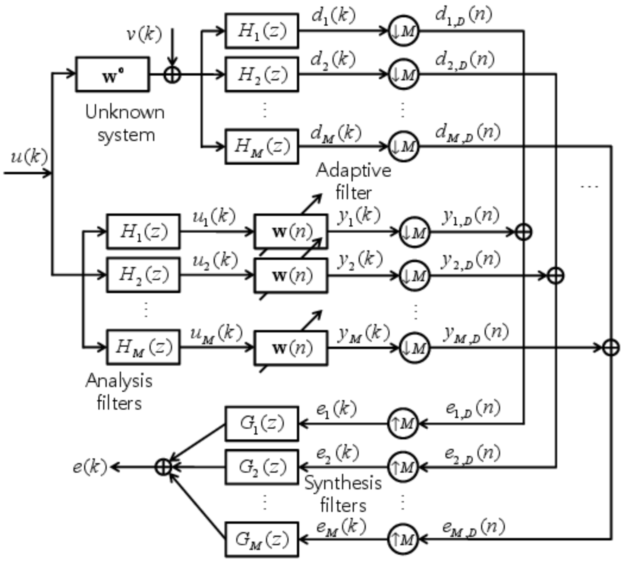

Figure 1: Structure of the NSAF

Note that the variable is indexing the original sequences, and n is used to index the decimated sequences. NSAF minimizes the squared Euclidean norm of the change in the tap-weight vector

$$\min_{\mathbf{w}(n+1)} \|\mathbf{w}(n+1)-\mathbf{w}(n)\|^2 \tag{4}$$

subject to the M constraints

$$d_{i,\mathrm{D}}(n)=\mathbf{u}_i(n)\mathbf{w}(n+1). \tag{5}$$

Using the Lagrange multipliers method, the cost function to be minimized is given by

$$J(n)=\|\mathbf{w}(n+1)-\mathbf{w}(n)\|^2+\sum_{i=1}^{M}\lambda_i(d_{i,D}(n)-\mathbf{u}_i(n)\mathbf{w}(n+1)) \tag{6}$$

To minimize the cost function, the differentiation $J(n)$ should be zero. Thus, the differentiation of $J(n)$ is

$$\frac{\partial J(n)}{\partial \mathbf{w}(n+1)}=2(\mathbf{w}(n+1)-\mathbf{w}(n))+\mathbf{U}(n)\mathbf{\Lambda} \tag{7}$$

where, $\mathbf{U}(n) = [\mathbf{u}_M^T(n)\ \mathbf{u}_{M-1}^T(n)\ \cdots\ \mathbf{u}_1^T(n)]$, $\mathbf{u}_i(n) = [u_i(nM)\ u_i(nM-1)\ \cdots\ u_i(nM-N+1)]$ and $\mathbf{\Lambda}=[\lambda_M\ \lambda_{M-1}\ \cdots\ \lambda_1]^T$.

$$\mathbf{w}(n+1)-\mathbf{w}(n)=\frac{1}{2}\mathbf{U}(n)\mathbf{\Lambda} \tag{8}$$

when, $\mathbf{\Lambda} = 2(\mathbf{U}^T(n)\mathbf{U}(n))^{-1}\mathbf{e}(n)$ and $\mathbf{e}(n) = [e_{M,D}(n)\ e_{M-1,D}(n)\ \cdots\ e_{1,D}(n)]^T$. The update equation becomes

$$\mathbf{w}(n+1)=\mathbf{w}(n)+\frac{1}{2}\mu\mathbf{U}(n)\mathbf{U}^T(n)\mathbf{U}(n))^{-1}\mathbf{e}(n) \tag{9}$$

where μ is a step-size parameter.

If the frequency responses of the analysis filters do not significantly overlap, the "pre-whitening" effect of SAF will guarantee the decorrelation of input signals. In a mathematical approach, the matrix product $\mathbf{U}^T(n)\mathbf{U}(n)$ can be simplified as a diagonal matrix. Finally, the update equation of NSAF [9] can be written as

$$\mathbf{w}_{\mathrm{NSAF}}(n+1)=\mathbf{w}(n)+\mu\sum_{i=1}^{M}\frac{\mathbf{u}_i^T(n)}{\|\mathbf{u}_i(n)\|^2}e_{i,\mathrm{D}}(n). \tag{10}$$

Geometrical representation of the decorrelated input signals is the orthogonality of the input hyperplane in Fig. 2. SAF will update the tap-weight to the intersection of input hyperplanes ($\mathbf{w}(n+1)$) and the simplified update will update M times separately ($\mathbf{w}_{\mathrm{NSAF}}(n+1)$). It is easily figured out that the two update results are almost the same in the geometric interpretation.

Table 1: Computational complexity of NSAF and SS-NSAF

	Conventional NSAF	SS-NSAF
Multiplication	$2N+3ML$	$(1+r)N+3M_eL_e$
Division	-	M_e
Comparison	-	$2\log_2 M_e$

2.2 Optimization of Selecting the Subband Filters

Our objective is to optimize the subband selection by employing only a subset of extended subbands at every iteration while increasing the usage of subbands. From this, we expect that the Selective Subband (SS-NSAF) algorithm has a fixed rate of subband usage to achieve a similar level of computational complexity to that of conventional NSAF. Suppose that the NSAF algorithm has an increased number of subbands $M_e > M$ and we select K subbands out of M_e subbands. There will be a subset of subbands that maximize the convergence rate with a restricted number of subbands. Let $T_K = \{t_1, t_2, \cdots, t_K\}$ denote K–subset (subset with K members) of $\{1, 2, \cdots, M_e\}$. That is, t_i denotes the index of the selected subbands. Since the reduction of the computational complexity and the convergence rate depend on the relative size of K and M_e, we introduce the selection ratio $r = K/M_e$. For $r = 1, M_e = M$, the SS-NSAF becomes identical to the conventional NSAF in (10). If the selection ratio r is large, the computational complexity would increase but the convergence performance would improve. So as not to increase computational complexity additionally, r would be fixed as a dependent number of M_e. First, we have to know the computational complexity of conventional algorithm which is illustrated in [10]. Computational complexity of proposed algorithm, which is shown in 1, can be acquired from the equation.An appropriate r is easily calculated from the inequality of the computational complexity in Table 1

$$r \le 1-\frac{3(M_eL_e-ML)}{N}. \tag{11}$$

The following part will describe the method that minimizes the convergence performance deterioration with the fixed r and M_e.

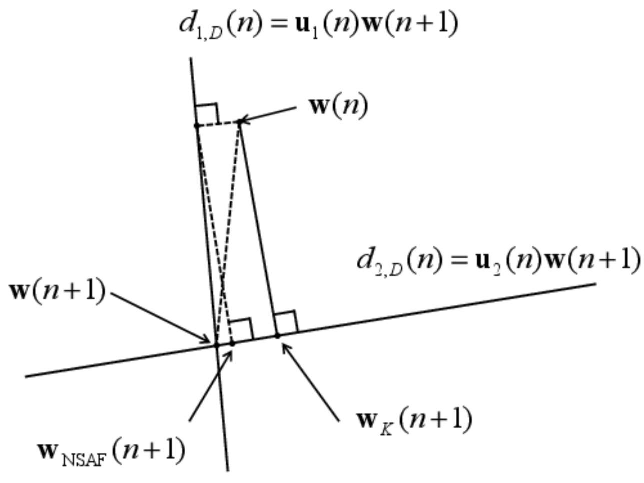

Figure 2: Weight update example of the proposed algorithm for N=2, M_e=2, r=1/2.

Our approach is to seek the subset T_K that minimizes the performance degradation compared to the NSAF using M_e subbands. From (10), we can see the update value is the independent summation of the M-subband. The proposed algorithm updates only K-subbands using T_K. Substituting $\mathbf{U}_K(n) = [\mathbf{u}_{t_1}^T(n) \;\; \mathbf{u}_{t_2}^T(n) \;\; \cdots \;\; \mathbf{u}_{t_K}^T(n)]$ to $\mathbf{U}(n)$ in (10), we can derive the update equation of the proposed algorithm as

$$\mathbf{w}_K(n+1) = \mathbf{w}(n) + \mu \sum_{i=1}^{K} \frac{\mathbf{u}_{t_i}^T(n)}{\|\mathbf{u}_{t_i}(n)\|^2} e_{i,\mathrm{D}}(n). \tag{12}$$

If we assume that the correlation of $\mathbf{u}_i(n)$ vanished because of the pre-whitening effect, from equation (12), we can easily derive

$$\|\mathbf{w}_K(n+1) - \mathbf{w}(n)\|^2 = \mu^2 \sum_{i=1}^{K} \frac{e_{t_i,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_i}(n)\|^2}. \tag{13}$$

For a more qualitative understanding of the optimal subbands selection, we adapt the geometric interpretation in Fig. 2. By the Pythagorean theorem, we can write the squared a posteriori errors for K subbands as

$$\begin{aligned}\|\mathbf{w}_K(n+1) - \mathbf{w}(n+1)\|^2 &= \|\mathbf{w}(n+1) - \mathbf{w}(n)\|^2 \\ &\quad - \|\mathbf{w}_K(n+1) - \mathbf{w}(n)\|^2.\end{aligned} \tag{14}$$

Substituting (13) into (14), we get

$$\begin{aligned}\|\mathbf{w}_K(n+1) - \mathbf{w}(n+1)\|^2 &= \|\mathbf{w}(n+1) - \mathbf{w}(n)\|^2 \\ &\quad -\mu^2 \sum_{i=1}^{K} \frac{e_{t_i,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_i}(n)\|^2}.\end{aligned} \tag{15}$$

As we can see in Fig.2, $\mathbf{w}_K(n+1)$, which makes (13) bigger, is closer to $\mathbf{w}(n+1)$ among M_e candidates of $\mathbf{w}_K(n+1)$. That is, the bigger (13) guarantees the faster convergence speed. In other words, when (13) is the maximum value among all K–subsets and the a priori weight error vector given, the norm square of the a posteriori weight error vector is minimized. To find the fastest solution of K–subsets, the proposed algorithm updates the weight vector so as to maximize the square deviation decrease among all possible K–subsets.

$$\arg\max_{T_K} \sum_{i=1}^{K} \frac{e_{t_i,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_i}(n)\|^2} \tag{16}$$

It is easily noticed that (16) is the sum of the square deviation decrease of each subband. To select the set T_K that maximizes the convergence rate of the proposed algorithm, we first perform reordering of the subbands according to the size of $e_{t_i,\mathrm{D}}^2(n)/\|\mathbf{u}_{t_i}(n)\|^2$ for each subband.

$$\sum_{i=1}^{M_e} \frac{e_{t_i,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_i}(n)\|^2} = \frac{e_{t_1,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_1}(n)\|^2} + \frac{e_{t_2,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_2}(n)\|^2} + \cdots + \frac{e_{t_{M_e},\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_{M_e}}(n)\|^2} \tag{17}$$

for $T_M = \{t_1, t_2, \cdots, t_{M_e}\}$. By selecting the subbands which have the biggest $e_{t_i,\mathrm{D}}^2(n)/\|\mathbf{u}_{t_i}(n)\|^2$ in a row in T_M, we can say that the proposed algorithm selected the optimal subbands that maximize the convergence speed. Thus, the MSD decrease of the proposed algorithm will be

$$\sum_{i=1}^{K} \frac{e_{t_i,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_i}(n)\|^2} = \frac{e_{t_1,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_1}(n)\|^2} + \frac{e_{t_2,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_2}(n)\|^2} + \cdots + \frac{e_{t_K,\mathrm{D}}^2(n)}{\|\mathbf{u}_{t_K}(n)\|^2}. \tag{18}$$

The proposed NSAF incorporates the selection of the subband to select the subset of subbands associated with K values for every update, which makes the maximum value of (18) with the subset T_K of $\{1, 2, \cdots, M_e\}$. Finally the proposed NSAF update equation is

$$\mathbf{w}_K(n+1) = \mathbf{w}_K(n) + \mu \sum_{i=1}^{K} \frac{\mathbf{u}_{t_i}^T(n) e_{t_i,\mathrm{D}}(n)}{\|\mathbf{u}_{t_i}(n)\|^2}. \tag{19}$$

where $T_K = \{t_1, t_2, \cdots, t_K\}$ and $1 \le K \le M_e$.

For every input sample, the additional computations for the proposed SS-NSAF algorithm are M_e divisions and $2\log_2 M_e$ sorting operations per iteration using the SORTLINE algorithm [17]. Even though division usually requires much more computation than multiplication, accurate and complex divisions are unnecessary because the divisions are just for comparison.

3. EXPERIMENTAL RESULTS

We illustrate the performance of the proposed algorithm by carrying out computer experiments in a channel estimation configuration in which the 1024 tap unknown channel is randomly generated. The adaptive filter and the unknown channel are assumed to have the same number of taps. The input signal $u(i)$ is obtained by filtering a white, zero-mean Gaussian random sequence through a first-order autoregressive system

$$G(z) = \frac{1}{1 - 0.9z^{-1}}. \tag{20}$$

As a result, a highly correlated Gaussian input signal is obtained. The signal-to-noise ratio (SNR) is calculated as

$$\mathrm{SNR} = \log_{10}\left(E\left[y^2(n)\right]/E\left[v^2(n)\right]\right) \tag{21}$$

where $\mathbf{y}(n) = \mathbf{w}^o \mathbf{u}_i(n)$. The measurement noise $v(n)$ is added to $y(n)$ with SNR=30dB. The step size is set to $\mu = 1$.

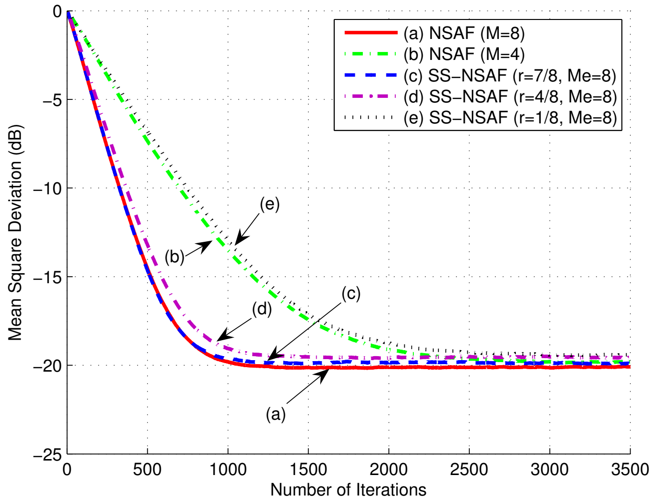

Figure 3: Normalized MSD curves of the NSAF ($M = 4, 8$) and the SS-NSAF ($M_e = 8, r = 1/8, 4/8, 7/8$).

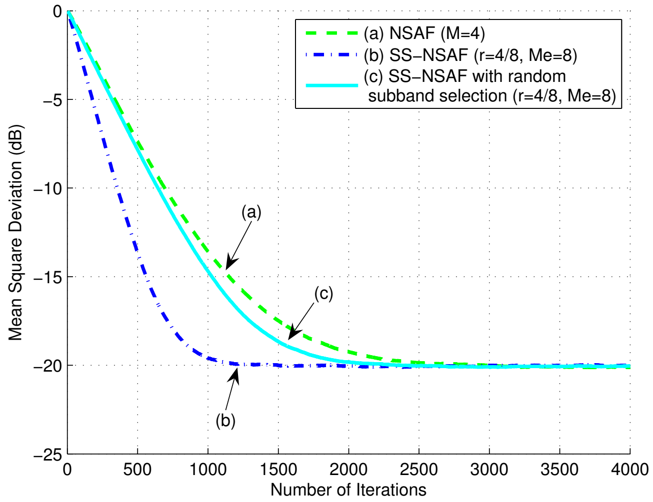

Figure 4: Normalized MSD curves of the conventional NSAF ($M = 4$), the proposed NSAF and random subband selection ($M_e = 8, r = 4/8$).

The experimental results are obtained by taking the ensemble average of normalized MSD, $E\left[\|\mathbf{w}^o - \mathbf{w}(n)\|^2\right] / \|\mathbf{w}^o\|^2$ over 100 independent trials. The cosine-modulated filter banks [16] with subband numbers $M = 4$ and $M_e = 8$ are used in the experiments. The length of the prototype filter L and L_e are 32.

Fig. 3. demonstrates the normalized MSD performance comparison of the proposed SS-NSAF algorithm and conventional NSAF for various selection ratios r with fixed M_e. For the purpose of watching the trade-off, $r = 1/8, 4/8$ and $7/8$ are applied to the proposed NSAF. These proposed SS-NSAF algorithms have different computational complexity based on Table1. The graph of $r = 1/8$, which has the lowest computational complexity, shows that a small number of subbands degrades the convergence speed. The graph of $r = 4/8$ and $r = 7/8$, which have the highest computational complexity, show similar performance to the graph of NSAF ($M = 8$). This result shows that an overly high selection rate is redundant for optimal subband selection.

Fig. 4. and Fig. 5 illustrate the normalized MSD curves versus the number of iterations for the conventional NSAF, the proposed SS-NSAF and an unoptimized selective subband algorithm with colored input signal and white input signal. These three algorithms have almost the same computational complexity. The number of subbands is set to $M_e = 8$ and the selection ratio for the SS-NSAF is set to $r = 4/8$. Both selective algorithms are faster than the conventional NSAF. The selective subband algorithm with random subband selection cannot expect high convergence speed but the proposed algorithm greatly enhanced the convergence rate. This result shows that the optimal subband selection of the proposed algorithm is valid. Fig. 5. shows that the proposed algorithm also performs well with various input signal condition, such as, white input signal.

Fig. 6. illustrates the normalized MSD curves versus number of iterations for the proposed SS-NSAF and conventional NSAF for varying channel. The number of subbands is set to $M_e = 8$ and the selection ratio for the SS-NSAF is set to $r = 4/8$. In this figure, the conventional NSAF is robust to varying channel conditions. The proposed algorithm

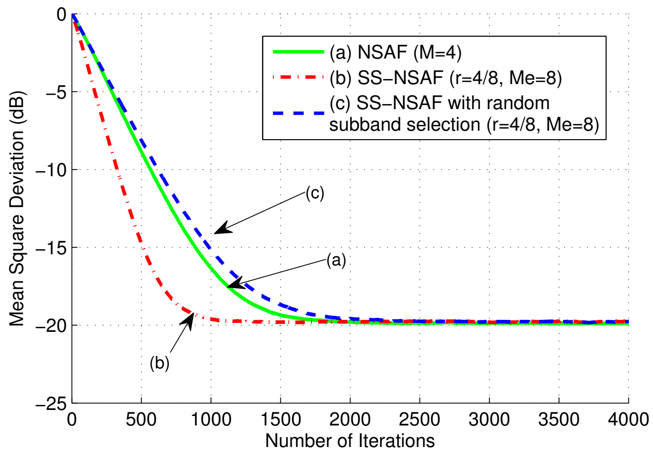

Figure 5: Normalized MSD curves of the conventional NSAF ($M = 4$), the proposed NSAF and random subband selection ($M_e = 8, r = 4/8$) for white input signal.

is also robust to channel estimation though the channel suddenly changes.

A more specific number of computations is as follows. The NSAF algorithm requires 2432 multiplications while the proposed algorithm requires only 2304 multiplications, 8 divisions, and 6 comparisons for every iteration . These numbers show that the proposed algorithm is a little less complex than the conventional algorithm, while the proposed algorithm has much faster convergence speed.

4. CONCLUSIONS

We have presented the SS-NSAF, which has faster convergence speed than the conventional NSAF. The proposed algorithm performed better than the conventional algorithm by the increasing the number of subbands and removing additional computational complexity by selecting an effective subset of subbands at every iteration. Optimization of sub-

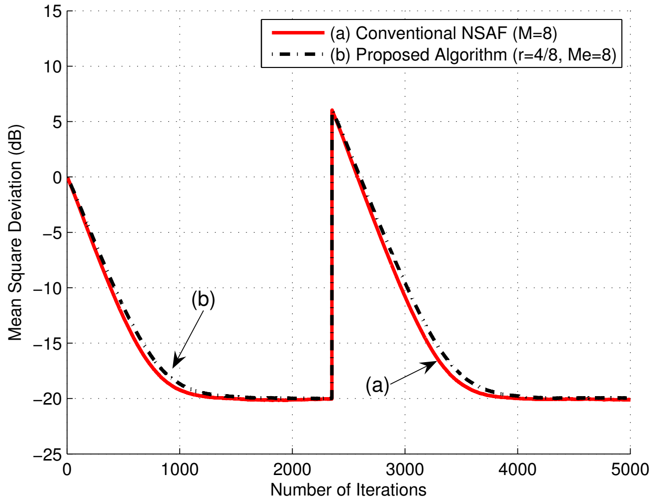

Figure 6: Normalized MSD curves of the conventional NSAF ($M = 8$) and the proposed NSAF ($M_e = 8, r = 4/8$) for varying channel experiment setups.

band selection made it possible for the SS-NSAF to follow the performance of extended NSAF. The optimal selection of subbands was derived from comparing the distance between the extended updated tap-weight and the proposed tap-weight. As a result, the proposed algorithm has comparable convergence performance with slightly less computational complexity than the conventional NSAF.

Acknowledgments

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