ESTIMATION OF MULTICHANNEL TVAR PARAMETERS FROM NOISY OBSERVATIONS BASED ON AN EVOLUTIVE METHOD

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ABSTRACT

The multichannel time-varying autoregressive (M-TVAR) model is used in various applications such as radar processing, mobile communications and econometric time series. In that case, the M-TVAR parameter matrices are usually estimated by extending the evolutive method, initially proposed by Grenier in the 80ies for the scalar case. In that case, the time-varying coefficients of each matrix are expressed as weighted combinations of pre-defined basis functions. However, the estimation of the M-TVAR parameter matrices from noisy observations is a still key issue to be addressed. In this paper, we propose an effective evolutive method to estimate M-TVAR matrices from noisy observations. We suggest viewing the weight estimation from noisy observations as an errors-in-variables issue. Despite its high computational cost, the approach has the advantage of also providing the covariance matrix of both the driving process and the additive noise. Simulation results point out the relevance of the approach.

1. INTRODUCTION

The Multichannel autoregressive (M-AR) model is a useful tool in various applications, especially for the EEG signal analysis and for radar signal processing [1]-[4]. However, this usually requires the estimation of the corresponding autoregressive parameter matrices. They can be obtained by solving the multichannel Yule-Walker (YW) equation, the multichannel Levinson algorithm or the Nutall Strand method. Kalman filtering could be also considered. When only noisy observations are available, M-AR parameter estimation from noisy observation has been discussed and some methods have been proposed in [5], [6] and [7].

However, in the non-stationary case, the so-called multichannel time-varying autoregressive (M-TVAR) process is more suited as it is defined by time-varying parameter matrices. It has been applied for the econometric time series and for the seismic signal analysis [8], [9]. In this literature, the M-TVAR parameter matrices can be estimated by means of the evolutive method (also known as deterministic regression method) which was mainly used by Grenier in the 80ies for the scalar TVAR parameter estimations [10]. In that case, the time-varying coefficients of each matrix are expressed as weighted combinations of pre-defined basis functions such as power of time, Legendre polynomials, or Fourier functions. Usually, the basis choice can be based on *a priori* properties of the signal. Therefore, the TVAR parameter estimation becomes a stationary identification issue because one has to estimate the weights by using the observations of the process. Standard Least Square (LS) methods can be used. However, when the M-TVAR process is disturbed by an additive measurement noise, using the above approaches leads to an estimation bias. To our knowledge, very few approaches deal with this issue. Recursive methods based on Extended Kalman filtering, iterative Kalman filter and Sigma Point Kalman filters (SPKF) such as the unscented Kalman filter (UKF) and the central difference Kalman filter (CDKF) could be considered, but they require the covariance matrix of both the additive noise and the driving process of the M-TVAR process.

In this paper, we suggest using an evolutive method based on an errors-in-variable (EIV) approach to estimate the weights from noisy observations by using the Frisch scheme [11]-[13]. Despite its high computational cost, it has the advantage of also providing the estimation both the unknown variance of the driving process and the M-TVAR parameters. The relevance of the proposed method is then studied by means of a comparative study with standard Yule-Walker methods.

The remainder of the paper is organized as follows: section 2 deals with the problem statement. Then, in section 3, the way to estimate the weights from noisy observations is presented. Simulations results and comments are then provided.

2. LS WEIGHT ESTIMATION FOR MULTICHANNEL TVAR PROCESS

A. Noise-free Case (usually addressed in the literature):

Let $\underline{x}(k)$ be a real M-channel and P-th order TVAR process vector defined as follows:

$$\underline{x}(k)=\begin{bmatrix} x^1(k) \\ x^2(k) \\ \vdots \\ x^M(k) \end{bmatrix} = -\sum_{i=1}^{P} A_i(k-i)\,\underline{x}(k-i)+\underline{u}(k) \tag{1}$$

where $A_i(k-i)$ are the $M\times M$ TVAR parameter matrices:

$$A_i(k-i)=\begin{bmatrix} a_i^{11}(k-i) & a_i^{12}(k-i) & \cdots & a_i^{1M}(k-i) \\ a_i^{21}(k-i) & a_i^{22}(k-i) & \cdots & a_i^{2M}(k-i) \\ \vdots & \vdots & \ddots & \vdots \\ a_i^{M1}(k-i) & a_i^{M2}(k-i) & \cdots & a_i^{MM}(k-i) \end{bmatrix}$$

where $i=1,\cdots,P$, and $\underline{u}(k)$ is assumed to be the $M\times 1$ driving process vector in which each element is a zero-mean stationary white noise with variance σ_u^2. The autocorrelation of $\underline{u}(k)$ hence satisfies:

$$E\left[\underline{u}(k)\underline{u}^T(k-l)\right]=\begin{cases} \sigma_u^2 I_{M\times M} & l=0 \\ 0_{M\times M} & l\neq 0 \end{cases}$$

where $I_{M\times M}$ is the identity matrix of size M and $0_{M\times M}$ is a $M{\times}M$ matrix of zeros. In the following, the TVAR coefficients $a_i^{lm}(k-i)$ $(l=1\cdots M, m=1\cdots M)$ are assumed to be expressed by means of pre-defined basis functions:

$$a_i^{lm}(k-i) = \underline{\beta}_i^{lm^T} \underline{f}(k-i) \tag{2}$$

where $\underline{f}(k-i) = [f_1(k-i) \quad f_2(k-i) \quad \cdots \quad f_N(k-i)]^T$ are the basis function vectors of size N and $\underline{\beta}_i^{lm} = [\beta_{i1}^{lm} \quad \beta_{i2}^{lm} \quad \cdots \quad \beta_{iN}^{lm}]^T$ are the weight vectors corresponding with the coefficients of the l^{th} row and the m^{th} column of the i^{th} AR matrices $A_i(k-i)$.

Introducing the $M\times NM$ weight matrices θ_i $(i=1,\cdots,p)$ as follows:

$$\theta_i = \begin{bmatrix} \underline{\beta}_i^{11^T} & \underline{\beta}_i^{12^T} & \cdots & \underline{\beta}_i^{1M^T} \\ \underline{\beta}_i^{21^T} & \underline{\beta}_i^{22^T} & \cdots & \underline{\beta}_i^{2M^T} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\beta}_i^{M1^T} & \underline{\beta}_i^{M2^T} & \cdots & \underline{\beta}_i^{MM^T} \end{bmatrix}$$

and the $MN\times M$ matrix $F(k-i)$ as:

$$F(k-i) = \begin{bmatrix} \underline{f}(k-i) & 0_{N\times1} & \cdots & 0_{N\times1} \\ 0_{N\times1} & \underline{f}(k-i) & \cdots & 0_{N\times1} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{N\times1} & 0_{N\times1} & \cdots & \underline{f}(k-i) \end{bmatrix}$$

the TVAR parameter matrices can be rewritten as:

$$A_i(k-i) = \theta_i F(k-i) \qquad (i=1,\cdots,p) \tag{3}$$

Substituting equ. (3) into equ. (2), and by denoting $\underline{X}(k-i) = F(k-i)\underline{x}(k-i)$, one obtains:

$$\begin{aligned} x(k) &= -\sum_{i=1}^{P} \theta_i F(k-i)\underline{x}(k-i) + \underline{u}(k) \\ &= -\sum_{i=1}^{P} \theta_i \, \underline{X}(k-i) + \underline{u}(k) \end{aligned} \tag{4}$$

Let Θ^P denote the extended weight matrix defined as $\Theta^P = [\theta_1 \quad \theta_2 \quad \cdots \quad \theta_P]$. Equ. (4) can be rewritten as follows:

$$\underline{x}(k) = -\Theta^P \begin{bmatrix} \underline{X}(k-1) \\ \underline{X}(k-2) \\ \vdots \\ \underline{X}(k-P) \end{bmatrix} + \underline{u}(k) \tag{5}$$

Postmultiplying both side of equ. (5) by the matrix $[\underline{X}^T(k-1) \quad \underline{X}^T(k-2) \cdots \quad \underline{X}^T(k-P)]$, this leads to:

$$r_X^P = -\Theta^P R_X^P \tag{6}$$

where $r_X^P = E\left[\underline{x}(k)[\underline{X}^T(k-1) \quad \cdots \quad \underline{X}^T(k-p)]\right]$,

and $R_X^P = E\left[\begin{bmatrix} \underline{X}(k-1) \\ \vdots \\ \underline{X}(k-P) \end{bmatrix} [\underline{X}^T(k-1) \quad \cdots \quad \underline{X}^T(k-p)]\right]$

<u>Remark:</u> Equ. (6) can be regarded as the M-TVAR version of the YW equations.

B. Noisy Case (addressed in this paper):

The M-TVAR process is assumed to be disturbed by an additive zero-mean white noise vector $\underline{b}(k)$.

$$\underline{y}(k) = \underline{x}(k) + \underline{b}(k) \tag{7}$$

where the autocorrelation of $\underline{b}(k)$ is assumed to be given by:

$$E\left[\underline{b}(k)\underline{b}^T(k-l)\right] = \begin{cases} \sigma_b^2 I_{M\times M} & l=0 \\ 0_{M\times M} & l\neq 0 \end{cases}$$

In the equation above, σ_b^2 is the unknown (scalar) variance. In addition, the additive noise $\underline{b}(k)$ is assumed to be uncorrelated with the driving process $\underline{u}(k)$, i.e., $E\left[\underline{b}(k)\underline{u}^T(l)\right] = 0_{M\times M}$ for all k and l.

Then, given (7), the modified observation and noise vectors $\underline{Y}(k-i) = F(k-i)\underline{y}(k-i)$ and $\underline{B}(k-i) = F(k-i)\,\underline{b}(k-i)$ satisfy the following relationship for $(i=1,\cdots,P)$:

$$\begin{aligned} \underline{Y}(k-i) &= F(k-i)\left(\underline{x}(k-i) + \underline{b}(k-i)\right) \\ &= \underline{X}(k-i) + \underline{B}(k-i) \end{aligned} \tag{8}$$

Since the additive noise is zero-mean and white and the M-TVAR process and the additive noise vector are independent, substituting equ. (7) and equ. (8) into equ. (6), leads to:

$$\Theta^P = -r_Y^P (R_Y^P - R_b^P)^{-1} \tag{9}$$

where $r_Y^P = E\left[\underline{y}(k)[\underline{Y}^T(k-1) \quad \cdots \quad \underline{Y}^T(k-P)]\right]$,

and for ($\alpha = Y$ or B):

$$R_\alpha^P = E\left[\begin{bmatrix} \underline{\alpha}(k-1) \\ \vdots \\ \underline{\alpha}(k-P) \end{bmatrix} [\underline{\alpha}^T(k-1) \quad \cdots \quad \underline{\alpha}^T(k-P)]\right]$$

If one introduces the following matrix:

$$R_B^P(\beta) = \beta \begin{bmatrix} F(k-1)F^T(k-1) & \cdots & 0_{NM\times NM} \\ \vdots & \ddots & \vdots \\ 0_{NM\times NM} & \cdots & F(k-P)F^T(k-P) \end{bmatrix} \tag{10}$$

where β is any (positive) value, the covariance matrix of $\underline{B}(k-i)$ can be expressed as follows:

$$E\left[\underline{B}(k-i)\underline{B}^T(k-i)\right] = \sigma_b^2 F(k-i)F^T(k-i), \tag{11}$$

Then, given the three above equations, $R_B^P = R_B^P\left(\sigma_b^2\right)$, the size of which is $NMP \times NMP$.

Let $\Theta(y)$ be the estimate of the weights when one does not compensate the influence of the additive noise, i.e., $\Theta(y) = -r_Y^P (R_Y^P)^{-1}$, the weight estimations would satisfy:

$$\Theta(y) R_Y^P = \Theta(y)\left(R_X^P + R_B^P(\sigma_b^2)\right) = -\,r_Y^P = -r_X^P \tag{12}$$

Using equ. (6), equ. (12) and the matrix inversion lemma[1], this means that:

$$\Theta(y) = \Theta^P - \Theta^P R_B^P(\sigma_b^2)(R_Y^P)^{-1}, \tag{13}$$

So, the estimation bias is equal to $\Theta^P R_B^P(\sigma_b^2)(R_Y^P)^{-1}$.

Therefore, the purpose of this paper is to propose a method to estimate the weights Θ^P by compensating the influence of the additive noise.

3. EVOLUTIVE METHOD BASED ON AN ERRORS-IN-VARIABLES APPROACH

Introducing the $M \times M(1+NP)$ matrix:

$$\overline{\Theta^P} = [I_{M\times M} \quad \Theta^P\ ] = [I_{M\times M} \quad \theta_1 \ \ \theta_2 \ \cdots \ \theta_P],$$

equ. (5) can be equivalently rewritten as follows:

$$\overline{\Theta^P}\begin{bmatrix} \underline{x}(k) - \underline{u}(k) \\ \underline{X}(k-1) \\ \underline{X}(k-2) \\ \vdots \\ \underline{X}(k-P) \end{bmatrix} = \underline{0}_{M\times 1} \tag{14}$$

By postmultiplying equ. (14) by the matrix $\left[\underline{x}^T(k) - \underline{u}^T(k) \quad \underline{X}^T(k-1) \quad \cdots \quad \underline{X}^T(k-P)\right]$, and taking the expectation, one obtains:

$$\overline{\Theta^P}\, E\left[\begin{bmatrix} \underline{x}(k) - \underline{u}(k) \\ \underline{X}(k-1) \\ \underline{X}(k-2) \\ \vdots \\ \underline{X}(k-P) \end{bmatrix} [\underline{x}^T(k) - \underline{u}^T(k) \quad \underline{X}^T(k-1) \quad \cdots \quad \underline{X}^T(k-P)]\right] = 0_{M\times M(1+NP)} \tag{15}$$

Combining equ. (7) and equ. (8), equ. (15) can be rewritten as:

$$\overline{\Theta^P} E\left[\begin{bmatrix} \underline{y}(k) - \underline{b}(k) - \underline{u}(k) \\ \underline{Y}(k-1) - \underline{B}(k-1) \\ \vdots \\ \underline{Y}(k-P) - \underline{B}(k-P) \end{bmatrix} [\underline{x}^T(k) - \underline{b}^T(k) - \underline{u}^T(k) \quad \underline{Y}^T(k-1) - \underline{B}^T(k-1) \quad \cdots \quad \underline{Y}^T(k-P) - \underline{B}^T(k-P)]\right] \tag{16}$$

$$= 0_{M\times M(1+NP)}$$

At that stage, let us have a look at some components of the matrix of the equation above. Hence, as $\underline{b}(k-j)$ is a zero-mean vector, one has for j>0

$$E\left[\left(\underline{Y}(k-j) - \underline{B}(k-j)\right)\left(\underline{y}^T(k) - \underline{b}^T(k) - \underline{u}^T(k)\right)\right]$$

$$= F(k-j) E\left[\left(\underline{y}(k-j) - \underline{b}(k-j)\right)\left(\underline{y}^T(k) - \underline{b}^T(k) - \underline{u}^T(k)\right)\right]$$

$$= F(k-j)\, E\left[\underline{y}(k-j)\underline{y}^T(k)\right]$$

and similarly:

$$E\left[\left(\underline{y}(k) - \underline{b}(k) - \underline{u}(k)\right)\left(\underline{Y}^T(k-j) - \underline{B}^T(k-j)\right)\right]$$

$$= E\left[\left(\underline{Y}(k-j) - \underline{B}(k-j)\right)\left(\underline{y}^T(k) - \underline{b}^T(k) - \underline{u}^T(k)\right)\right]^T$$

$$= E\left[\underline{y}(k)\underline{y}^T(k-j)\right] F^T(k-j)$$

Therefore, the relationship (16) can be expressed as:

$$\overline{\Theta^P}\left(\overline{R_Y^P} - \begin{bmatrix} (\sigma_u^2 + \sigma_b^2) I_{M\times M} & 0_{M\times MNP} \\ 0_{MNP\times M} & R_B^P(\sigma_b^2) \end{bmatrix}\right) = 0_{M\times M(1+NP)} \tag{17}$$

where $\overline{R_Y^P} = \begin{bmatrix} E\left[\underline{y}(k)\underline{y}^T(k)\right] & r_Y^P \\ {r_Y^P}^T & R_Y^P \end{bmatrix}$

At that stage, equ. (17) is defined by the set of variances σ_u^2 and σ_b^2. Hence, the identification approach consists in searching the sets of variances that make the following noise-compensating matrix positive semi-definite:

$$\left(\overline{R_Y^P} - \begin{bmatrix} (\sigma_u^2 + \sigma_b^2) I_{M\times M} & 0_{M\times MNP} \\ 0_{MNP\times M} & R_B^P(\sigma_b^2) \end{bmatrix}\right)^T \geq 0 \tag{18}$$

Then, for each set of variances, the M-TVAR parameter matrices can be obtained by solving the noise compensated YW equation (9) where the estimations of the variance σ_b^2 is considered.

To find the various sets of variances that satisfy the semi-definiteness condition (18), we suggest using the Frisch scheme

[1] If $A = B + CD^T$
then $A^{-1} = B^{-1} - B^{-1}C(I + D^T B^{-1} C)^{-1} D^T B^{-1}$

[11]- [13]. It operates as follows. The idea is to retrieve the values α, and β that make the noise compensated matrix positive semi-definite.

$$\overline{R_Y^P} - C^P(\alpha, \beta) \geq 0$$

with $$C^P(\alpha, \beta) = \begin{bmatrix} \alpha I_{M\times M} & 0_{M\times MNP} \\ 0_{MNP\times M} & R_B^P(\beta) \end{bmatrix}.$$

To determine this set, let us consider the set of candidates $[\alpha_1 \quad \beta_1]$ so that ${\alpha_1}^2 + {\beta_1}^2 = 1$ and given (18), one must necessarily have $\alpha_1 \geq \beta_1$ since $(\sigma_u^2 + \sigma_b^2) > \sigma_b^2$. So, the 2-tuple $[\lambda\alpha = \alpha_1 \quad \lambda\beta = \beta_1]$ makes $\overline{R_Y^P} - C^P(\alpha,\beta)$ semi-definite positive provided that λ is the largest eigenvalue of $\overline{R_Y^P}^{-1} C^P(\alpha_1, \beta_1)$. See appendix A for proof.

The Frisch scheme hence leads to an infinite set of solutions defining a first curve $S_P(\alpha_1, \beta_1)$, with ${\alpha_1}^2 + {\beta_1}^2 = 1$ and $\alpha_1 \geq \beta_1$.

To extract the solutions, we propose to iterate the same process by using another model order q higher than P. In that case, $\overline{R_Y^q}$, $R_B^q(\beta)$ and $C^q(\alpha, \beta)$ are similarly defined as $\overline{R_Y^P}$, $R_B^P(\beta)$ and $C^P(\alpha, \beta)$, but the matrices sizes now depend on q instead of P. Using the Frisch scheme with this new model order leads to a second curve $S_q(\alpha_1, \beta_1)$. Therefore, in theory, the variances to be found belong to both curves.

In practice, the expectation is replaced by the temporal mean over a sliding window. As a consequence, there is no longer an intersection between both curves. Therefore, we suggest estimating the noise variance as follows:

If ${\alpha_1}^2 + {\beta_1}^2 = 1$ and $\alpha_1 \geq \beta_1$, if $\lambda_{q,max}$ denotes the maximum eigenvalue of the matrix $\overline{R_Y^q}^{-1} C^q(\alpha_1, \beta_1)$ and if one introduces the estimated weight matrix Θ^q from the noise compensated YW equations (9) when the model order is set to q and the noise variance is assumed to be $\beta_1 \lambda_{q,max}^{-1}$:

$$\Theta^q = -r_Y^q (R_Y^q - R_B^q(\beta_1 \lambda_{q,max}^{-1}))^{-1}$$

we propose to find the noise variances by minimizing the following criterion:

$$J(\alpha_1, \beta_1) = \left\| \overline{\Theta}^q_{(1:M)\times(1:P)} \left(\overline{R_Y^P} - C^P\left(\alpha_1 \lambda_{q,max}^{-1}, \beta_1 \lambda_{q,max}^{-1}\right) \right) \right\|_2, \quad (19)$$

where $\|\cdot\|_2$ denotes the Frobenius norm and $\overline{\Theta}^q_{(1:M)\times(1:P)} = \left[\mathrm{I}_{M\times M} \quad \Theta^q_{(1:M)\times(1:P)} \right]$.

The procedure to estimate the M-TVAR by using the EIV approach can thus be summarized as follows:

1. Set the initial point (α_1, β_1) under the conditions of ${\alpha_1}^2 + {\beta_1}^2 = 1$ and $\alpha_1 \geq \beta_1$.

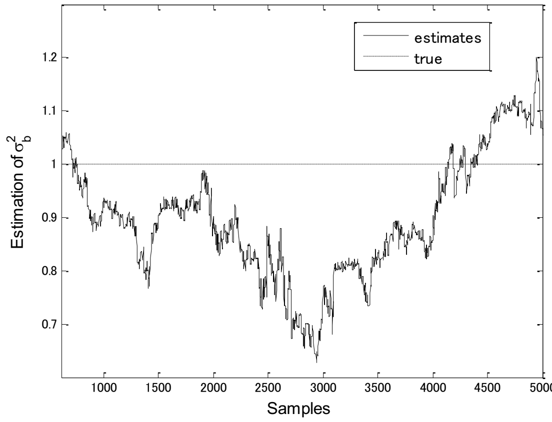

Figure 1: Estimation of noise variance σ_b^2.

2. Construct $C^q(\alpha_1, \beta_1)$ and solve the eigenvalue decomposition of the matrix $\overline{R_Y^q}^{-1} C^q(\alpha_1, \beta_1)$.
3. Obtain Θ^q and construct $\bar{\Theta}^q_{(1:M)\times(1:P)}$.
4. Calculate the criterion (19).
5. Obtain the 2-tuple $(\hat{\alpha}_1, \hat{\beta}_1)$ minimizing $J(\alpha_1, \beta_1)$ by a loop iterating from 2. to 4. with respect to (α_1, β_1).
6. Deduce the estimations of noise variances σ_u^2 and σ_b^2 by $\widehat{\sigma_b^2} = \hat{\beta}_1 \lambda_{p,max}^{-1}$ and $\widehat{\sigma_u^2} = \hat{\alpha}_1 \lambda_{p,max}^{-1} - \hat{\beta}_1 \lambda_{p,max}^{-1}$, respectively.
7. Deduce the extended weight matrix $\overline{\Theta^P}$ by solving the YW equations (18).
8. Deduce the M-TVAR matrices.

4. SIMULATION STUDIES

In this section, the performances of the EIV approach are investigated by simulations and are compared with standard YW method. For a simpler exposition, the number of channels and the order of the simulated M-TVAR process are set to $M = 2$ and $P = 2$ respectively. The basis functions vector of size $N = 2$ is set to $\underline{f}(k) = [1, \ \sin(2\pi k/T)]$ where T denotes the number of samples. The weight matrices are then set to: $\theta_1 = \begin{bmatrix} 0.8 & 0.01 & 0.3 & 0.05 \\ -0.6 & 0.05 & 0.2 & 0.02 \end{bmatrix}$ and $\theta_2 = \begin{bmatrix} 0.5 & 0.05 & 0.7 & 0 \\ -0.2 & -0.01 & 0.4 & 0.01 \end{bmatrix}$.
True values of the variance of the driving process and the additive noise are $\sigma_u^2 = 1$ and $\sigma_b^2 = 0.3$, respectively. The signal-to-noise ratio (SNR) of each TVAR process is between 16 and 17 dB.

Fig. 1 points out the estimations of the variance of the additive noise over time when using our EIV approach. The proposed approach has good performances. Figure 2 and 3 show the estimations of the evolutions of all weights which are elements of the weight matrices θ_i ($i = 1$ or 2), comparing between EIV approach and the standard YW method. It can be seen that the proposed approach provides estimates that almost vary along the true curves; however biased estimations are obtained when using YW. In our simulations, the mean square errors (MSE) of the weight estimations for YW method and the proposed EIV approach are 0.0055 and 0.0014, respectively.

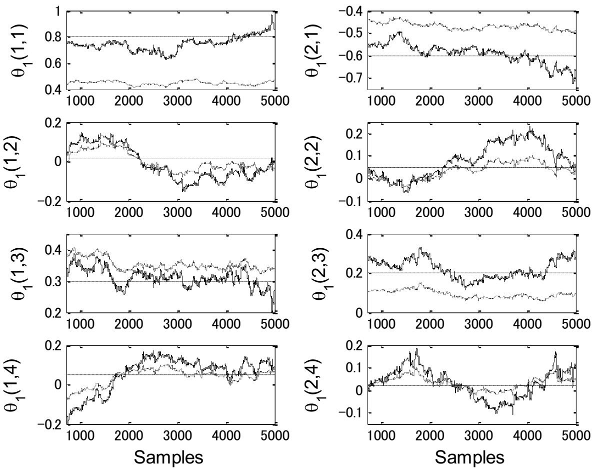

Figure 2: Estimations of weight matrix θ_1
(each line indicates that dash-dot line: true values, dotted line: standard YW method and solid line: EIV approach, respectively).

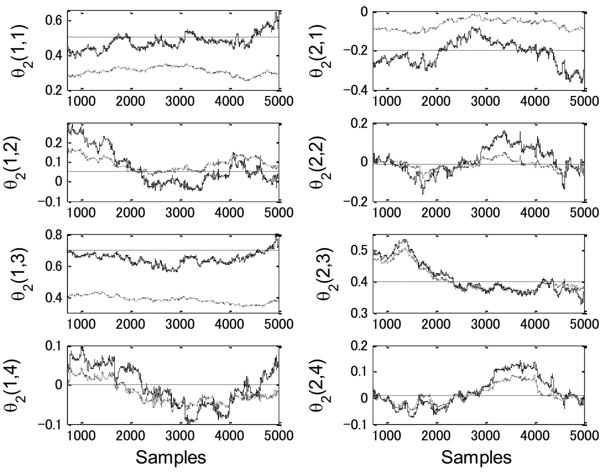

Figure 3: Estimations of weight matrix θ_2
(each line indicates that dash-dot line: true values, dotted line: standard YW method and solid line: EIV approach, respectively).

5. CONCLUSIONS

In this paper, an evolutive method has been proposed to estimate weights corresponding to the M-TVAR parameters from noisy observations. The variances of the additive noise and the driving process are also estimated based on the errors-in-variables method. The extended weight matrix can be estimated by means of the noise compensated YW equations. Therefore, the proposed approach has the advantage of not requiring any *a priori* information on the noise variances.

REFERENCES

[1] J. Russ, D. Casbeer and A. Lee Swindlehurst, "STAP Detection Using Space-Time Autoregressive Filtering," *Proc. of IEEE-Radar conference '04*, pp. 541-545, 2004.

[2] Y. I Abramovich, N. K. Spencer, and M. D. E. Turley, "Time-Varying Autoregressive (TVAR) Models for Multiple Radar Observations," *IEEE Trans. on Signal Processing*, vol. 55, no. 4, pp. 1298-1311, 2007.

[3] T. Locatelli, M. Cursi, D. Liberati, M. Franceschi and G. Comi, "EEG Coherence in Alzheimer's Disease," *Electroencephalogr. Clinical Neurophysiology* 106, pp. 229–237, 1998.

[4] P. J. Franaszczuk and G. K. Bergey, "An Autoregressive Method for the Measurement of Synchronization of Interictal and Ictal EEG Signals," *Biological Cybernetics* 81, 3-9, 1999.

[5] K. Hasan, J. Hossain and A. Haque, "Parameter Estimation of Multichannel Autoregressive Processes in Noise," *Signal processing*, vol. 83, pp. 603-610, 2003.

[6] A. Mahmoudi and M. Karimi, "Estimation of the Parameters of Multichannel Autoregressive Signals From Noisy Observations," *Signal Processing*, vol. 88, issue 11, pp. 2777-2783, 2008.

[7] J. Petitjean, E. Grivel, W. Bobillet and P. Roussilhe, "Multichannel AR Parameter Estimation From Noisy Observations as an Errors-In-Variables Issue," *Proc. of EURASIP-EUSIPCO 2008*, pp. 25-29 August 2008.

[8] W. Gersch and G. Kitagawa, "A Time Varying Multivariate Autoregressive Modeling of Econometric Time Series," *Statistical Reserch Division, Bureau of the ensus, U.S. Department of Commerce*, Technical Paper 9, 1983.

[9] X. Q. Jiang and G. Kitagawa, "A Time Varying Coefficient Vector AR Modeling of Nonstationary Covariance Time Series," *Signal Processing*, vol. 33, issue 3, pp. 315-331, 1993.

[10] Y. Grenier, "Time-Dependent ARMA Modeling of Nonstationary Signals," *IEEE Trans. on Acoust. Speech Signal Processing*, vol. ASSP 31, no. 4, pp. 899–911, 1983.

[11] S. Beghelli, R. Guidorzi, and U. Soverini, "The Frisch Scheme in Dynamic System Identification," *Automatica*, vol. 26, pp. 171-176, 1990.

[12] R. Diversi, U. Soverini, and R. Guidorzi, "A New Estimation Approach for AR Models in Presence of Noise," *Proc. of 16th IFAC World Congress*, Jul. 2005.

[13] W. Bobillet, R. Diversi, E. Grivel, R. Guidorzi, M. Najim, and U. Soverini, "Speech Enhancement Combining Optimal Smoothing and Errors-In-Variables Identification of Noisy AR Process," *IEEE Trans. on Signal Processing,* vol. 55, n. 2, pp. 5564-5578, 2007.

APPENDIX A

Let OM_1 be defined by $[\alpha_1 \quad \beta_1]$ such as $\alpha_1^2+\beta_1^2=1$ with $\beta_1>0$, $\alpha_1>0$, and $\alpha_1 \geq \beta_1$. Defining the relation $\mathrm{OM}_1 = \lambda\ \mathrm{OM}$ by $[\lambda\alpha=\alpha_1 \quad \lambda\beta=\beta_1]$, then let us look at the semi-definite compensated matrix:

$$\overline{R_Y^P} - C^P(\alpha,\beta) = \overline{R_Y^P} - \begin{bmatrix} \alpha & 0 & \dots & 0 \\ 0 & & & \\ \vdots & & R_B^P(\beta) & \\ 0 & & & \end{bmatrix} \geq 0$$

As $\overline{R_Y^P}$ is definite positive, $\overline{R_Y^P}^{-1}$ exists and one has:

$$I - \overline{R_Y^P}^{-1} C^P(\alpha,\beta) \geq 0.$$

This is equivalent to:

$$I - \frac{\overline{R_Y^P}^{-1}}{\lambda} C^P(\alpha_1,\beta_1) \geq 0$$

At that stage, let us introduce $\{\lambda_{P,i}\}_{i=1,\dots,M(1+NP)}$ the eigenvalues of $\overline{R_Y^P}^{-1} C^P(\alpha_1,\beta_1)$. If $I - \frac{\overline{R_Y^P}^{-1}}{\lambda} C^P(\alpha_1,\beta_1) \geq 0$, its eigenvalues are positive or null. As one must satisfy $\left\{1-\frac{\lambda_i}{\lambda}\right\}_{i=1,\dots,M(1+NP)} \geq 0$, λ is necessarily set to the largest eigenvalue $\lambda_{\mathrm{P,max}}$ of $\overline{R_Y^P}^{-1} C^P(\alpha_1,\beta_1)$.