IMPACT OF CARRIER FREQUENCY OFFSET IN COOPERATIVE PHASE SHIFT BEAMFORMING

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ABSTRACT

We study the effects of residual carrier frequency offset (CFO) on the performance of cooperative phase shift (PSB) beamforming schemes. Cooperative beamforming may provide the diversity and the power gains of multi-antenna systems and is therefore seen as a promising technology for the next generation wireless communication systems. Considering the distributed nature of cooperative communications, the carrier frequency of each transmitter deviates from the desired one. In this paper, we present the derivations of the signal-to-noise ratio and diversity gains in the cooperative PSB scheme in presence of residual CFOs. It is shown that even with frequency synchronization prior to transmission, residual CFOs may still significantly degrade the performance due to the in-phase misalignment of the incoming streams at the destination. Furthermore, we show that because of the residual CFOs the collaborative beamforming scheme may perform worse than its non-collaborative counter-part. The results and the accuracy are illustrated through simulations.

1. INTRODUCTION

Nowadays, wireless communication devices are often equipped with multiple antennas. Such a configuration enables the use of multiple-input multiple-output (MIMO) techniques for exploiting the spatial domain and therefore, improves the performance of wireless systems, e.g., in terms of transmission data rate, coverage area or energy efficiency. However, the cost associated with each antenna element, the size of the wireless devices, as well as the correlation between antennas limit the ability of a single device to exploit the spatial domain.

To cope with these limitations, an alternative consists in employing multiple transmitters to cooperatively transmit to a single destination, hence creating a virtual array of antennas. This type of scheme is known as cooperative (or distributed) transmissions and has recently drawn much attention. Cooperative space time block coding (CSTBC) may be used to exploit the spatial diversity [4], when no channel state information (CSI) is available at all the transmitting nodes. In some scenarios, the CSI can be made available at the transmitters, e.g., by exploiting the reciprocity property of the channel or through a feedback from the receiver. In such a case, cooperative beamforming techniques may be employed at the transmitters to shape the transmit information and increase the performance at the destination node.

Carrier frequency offset (CFO) is caused by two effects: the mobility of the wireless devices, i.e., the Doppler effect, and the non-ideality of the local oscillator embedded in each wireless transceiver. It is a major source of impairment in orthogonal frequency division multiplexing (OFDM) schemes and must be compensated to obtain acceptable performance [1]. In cooperative communications, each stream originates from a distinct source. The receiver needs therefore to cope with multiple carrier frequency offsets, which is much worse than that of point-to-point communications. Because the impact translates into a non-constructive addition of the incoming streams, it is difficult to estimate or compensate it at the receiver. The primary method for mitigating the effects of CFO consists then in compensating the frequency offset prior to transmission, e.g., through a handshake between the sources and the destination [5]. Methods for the estimation or the compensation of the multiple CFOs at the receiver, which requires a different algorithm to that of point-to-point communications where single CFO compensation methods apply, have also been proposed [3].

In practical scenarios, the perfect synchronization of the wireless devices is very challenging and a residual carrier frequency offset (RFO), e.g., typically up to 0.3 to 1 ppm in WLAN standards, is to be expected after synchronization [8]. It is therefore of interest to measure the effects of residual CFO in cooperative communications. Literature includes the study of the effects of residual CFO on the bit error rate (BER) performance of cooperative space-frequency block code systems [11] and the analysis of frequency offset through simulations for cooperative multi-user MIMO systems [2]. It is also treated for space-time code systems [6]. Nevertheless, to the best of our knowledge, literature does not evaluate the effects of residual CFO, in terms of power gain and diversity gain loss for cooperative beamforming schemes where the transmitters share the same time and frequency resources for transmitting a common data, on the contrary to STBC schemes.

The outline of this paper is as follows. In Section 2, the system model of the cooperative scheme with perfect synchronization is introduced. The derivations of the average SNR gain and the diversity gain are given in Section 2.1 and 2.2, respectively. In Section 3, the system model is defined for multiple CFOs from the different transmitters and derivations for the average SNR and diversity gains are presented. Simulations in Section 4 show the performance of the cooperative scheme for both the ideal case, i.e., perfect

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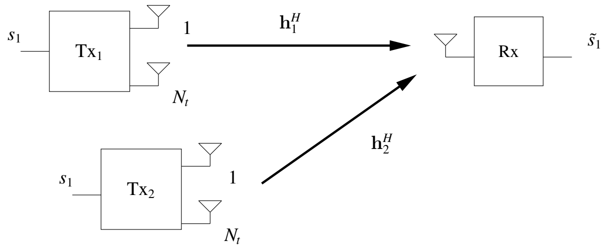

Figure 1: System model of a cooperative scheme in flat-fading channels.

synchronization, and when residual CFO is present. These results are discussed together with the proposed analytical derivations. Section 5 concludes our work.

The following notations are used in this paper. The vectors and matrices are in boldface letters, vectors are denoted by lower-case and matrices by capital letters. The superscript $(\cdot)^H$ denotes the Hermitian transpose operator, and $(\cdot)^\dagger$ denotes the pseudo inverse. $E[\cdot]$ is the expectation operator, $\mathbf{I}_N$ is an identity matrix of size $(N \times N)$ and $\mathbb{C}^{N\times 1}$ denotes the set of complex vectors of size $(N \times 1)$. We denote by $\mathbf{x} \sim \mathbb{C}N(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$ means that the vector $\mathbf{x}$ of size $N \times 1$ has zero-mean Gaussian distributed independent complex elements with variance σ_n^2.

2. SYSTEM MODEL

We consider a distributed beamforming system where two independent nodes transmit simultaneously to a single receiver. The system model is shown in Fig. 1. Time synchronization is performed at the network level to ensure a negligible timing offset between the transmit streams. In cooperative beamforming, the transmitters must share information about the data to transmit. Each transmitter is equipped with N_t ($N_t \geq 1$) antennas while the receiver has a single antenna.

We consider that only a residual CFO is present at the receiver, e.g., 0.5 part per million (ppm). In such a case, the performance degradation in an 802.11 systems due to the ICI and the SNR loss is negligible compared to that of the impairment caused by the phase rotation of the symbols [2]. Also, phase offset is a considerable source of impairment in a cooperative scheme, for both single-carrier and OFDM systems, due to the non-constructive addition of the incoming streams at the destination. We assume flat fading channels and present the remaining derivations for the single carrier case, as all subcarriers within a same OFDM symbol will undergo a same phase rotation. The channel vector is composed of independent and identically distributed (i.i.d.) Rayleigh fading elements and variance σ_i^2, i.e., $\mathbf{h}_i \sim \mathbb{C}N(\mathbf{0}, \sigma_i^2 \mathbf{I}_N)$. $\mathbf{h}_i$ denotes the channel vector from the transmitter i with $i = 1, 2$.

We denote by $s \in \mathbb{C}^{1\times 1}$ the symbol transmitted from Tx_1 and Tx_2 where $E[ss^H] = 1$. We consider a cooperative beamforming scenario where each transmitter exploits only limited channel knowledge to improve the performance of the destination node, i.e, the information of all the CSIs is not available. We assume that each transmitter has the knowledge of the channels from its own antennas to the receiver. That is Tx_1 has the channel knowledge of $\mathbf{h}_1$ while Tx_2 has the channel knowledge of $\mathbf{h}_2$, for all subcarriers. At the channel output, the received signal at a given subcarrier is denoted by $y \in \mathbb{C}^{1\times 1}$ and can be expressed as

$$y = \left(\mathbf{h}_1^H \mathbf{w}_1 + \mathbf{h}_2^H \mathbf{w}_2\right) s + n \tag{1}$$

where $\mathbf{w}_i \in \mathbb{C}^{N_t \times 1}$ denotes the beamforming vector from the transmitter i. The transmit power at transmitter i is assumed to be 1. The third term in (1), i.e., $n \in \mathbb{C}^{1\times 1}$, is the zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) with variance σ_n^2. We consider a transmit phase shift beamformer (PSB). The PSB beamforming weights are given as

$$\mathbf{w}_i = e^{j\angle \mathbf{h}_i}, \quad i = 1, 2 \tag{2}$$

2.1 Average Power Gain

The average power gain is obtained by taking the expectation of the instantaneous power at the receiver. In the following derivations, we assume a zero-forcing (ZF) equalizer at the receiver. The estimated symbol is given by

$$\begin{aligned} y &= \left(\mathbf{h}_1^H \mathbf{w}_1 + \mathbf{h}_2^H \mathbf{w}_2\right)^\dagger \left(\left(\mathbf{h}_1^H \mathbf{w}_1 + \mathbf{h}_2^H \mathbf{w}_2\right) s + n\right) \\ &= s + \left(\mathbf{h}_1^H \mathbf{w}_1 + \mathbf{h}_2^H \mathbf{w}_2\right)^\dagger n. \end{aligned} \tag{3}$$

The symbol error is denoted as $e = \left(\mathbf{h}_1^H \mathbf{w}_1 + \mathbf{h}_2^H \mathbf{w}_2\right)^\dagger n$. We then obtain the following instantaneous output SNR,

$$\gamma = \frac{E\left[ss^H\right]}{E\left[ee^H\right]}, \tag{4}$$

where

$$E\left[ee^H\right] = \sigma_n^2 \left(\left(\mathbf{h}_1^H \mathbf{w}_1 + \mathbf{h}_2^H \mathbf{w}_2\right)^H \left(\mathbf{h}_1^H \mathbf{w}_1 + \mathbf{h}_2^H \mathbf{w}_2\right)\right)^{-1}. \tag{5}$$

As the inverse term being a scalar, and $\mathbf{h}_i^H \mathbf{w}_i = \sum_{n=1}^{N_t} |h_n|$, the output SNR is

$$\gamma = \frac{1}{\sigma_n^2} \left(\sum_{n=1}^{N_t} |h_{1,n}| + \sum_{n=1}^{N_t} |h_{2,n}|\right)^2. \tag{6}$$

Averaged over the channel realizations, the SNR again can be expressed as

$$\begin{aligned} E[\gamma] = & \frac{1}{\sigma_n^2} \Bigg(E\left[\sum_{n=1}^{N_t} |h_{1,n}|^2\right] + E\left[\sum_{n=1}^{N_t} |h_{2,n}|^2\right] \\ & + \sum_{i=1}^{2} \sum_{n=1}^{N_t} \sum_{\substack{m=1 \\ m\neq n}}^{N_t} E\left[|h_{i,n}|\right] E\left[|h_{i,m}|\right] \\ & + 2 \sum_{n=1}^{N_t} \sum_{m=1}^{N_t} E\left[|h_{1,n}|\right] E\left[|h_{2,m}|\right] \Bigg), \end{aligned} \tag{7}$$

where $|h_{i,n}|$ is a Rayleigh distributed random variable, the sum term $\sum_{n=1}^{N_t} |h_{i,n}|^2$ is therefore a Chi-square distributed random variable with $2 \times N_t$ degrees of freedom [7]. If we take $\sigma_i^2 = 1$, the resulting average SNR gain compared to that of the single-input single-output (SISO) case can therefore be expressed as follows

$$G_{coop} = 10 \log_{10} \frac{4N_t + \pi N_t (N_t - 1) + \pi N_t^2}{4N_t}. \tag{8}$$

Equation (8) denotes the power gain obtained from cooperative transmission schemes.

2.2 Diversity gain

The diversity order d represents the number of independent copies of the information that are collected at the receiver.

$$d = \lim_{SNR\to\infty} -\frac{\log_{10} Pe\,(SNR)}{\log_{10} SNR} \tag{9}$$

where $Pe(SNR)$ denotes the error probability as a function of the SNR. For a BPSK modulation $Pe(SNR)$ can be expressed as

$$Pe = \int_0^\infty Q\left(\sqrt{2\gamma}\right) p(\gamma) d\gamma \tag{10}$$

where $p(\gamma)$ denotes the probability density function of γ. The PDF of the equivalent channel is hence needed, in the present scheme it is expressed as

$$p(\gamma) = p\left(\frac{1}{\sigma_n^2}\left(\sum_{n=1}^{N_t} |h_{1,n}| + \sum_{n=1}^{N_t} |h_{2,n}|\right)^2\right). \tag{11}$$

This is the sum of Rayleigh distributed channels. No closed-form solution of Pe is known for the general case. However, this result is equivalent to that of the equal gain combining (EGC) scheme and Zhang [9] derived the closed form solution for scenarios where $N_r = 2$ and $N_r = 3$. This result can be straightforwardly translated to our cooperative case. Following this result, the diversity gain of the cooperative scheme with one antenna at each transmitter and one antenna at the receiver, the number of transmitters$\times N_t \times N_r$, i.e., $2\times 1\times 1$, can be expressed as

$$Pe = \frac{1}{2}\left[1 - \frac{\sqrt{\gamma(\gamma+2)}}{\gamma+1}\right]. \tag{12}$$

Based on this equation, the diversity order of the $2\times 1\times 1$ scheme is $d = 2$. For multiple transmit antennas ($N_t > 1$) at the transmitters, the diversity gain can be computed numerically according to the results given in [10].

3. CFO IN COOPERATIVE BEAMFORMING

From the results given in Section 2, good performance should be expected in cooperative beamforming thanks to the added power and diversity gains. In this section, we discuss the effects from the residual carrier frequency offset on those gains in a cooperative beamforming system.

3.1 System Model with CFOs

Due to different local oscillator components at each transmitter and receiver, information transmission suffers from carrier frequency offset. The combination of the channel with the CFO effect can be equivalently represented by the channel vector multiplied by the complex component $c_i = e^{\phi_i} = e^{i2\pi f_{\Delta,i}t}$, where t is the time index and $f_{\Delta,i}$ denotes the carrier frequency offset of transmitter i. Because of the CFOs, the time coherency of the channel reduces, and the originally quasi-static channel now becomes time varying hence decreasing the performance of the beamforming scheme with static weights.

As introduced above, we assume that the frequency offset is pre-compensated at the transmitters prior to transmission, i.e., only residual CFO is left. Equation (6) can be written as follows

$$\gamma(t) = \frac{1}{\sigma_n^2}\left(c_1(t)\sum_{n=1}^{N_t} |h_{1,n}| + c_2(t)\sum_{n=1}^{N_t} |h_{2,n}|\right)^2. \tag{13}$$

where $c_i(t) = e^{j2\pi f_{\Delta i}t}$ is the complex cfo component.

3.2 Average Power Gain

We compute the average SNR. Following the procedure of Eq. (7) and Eq. (13), we obtain

$$\begin{aligned} E[\gamma(t)] =& \frac{1}{\sigma_n^2} E\Bigg[\sum_{n=1}^{N_t} |h_{1,n}|^2 + \sum_{n=1}^{N_t} |h_{2,n}|^2 \\ &+ \sum_{i=1}^{2}\sum_{n=1}^{N_t}\sum_{\substack{m=1\\ m\neq n}}^{N_t} |h_{i,n}||h_{i,m}| \\ &+ 2\cos(2\pi f_\Delta t)\sum_{n=1}^{N_t}\sum_{m=1}^{N_t} |h_{1,n}||h_{2,m}|\Bigg]. \end{aligned} \tag{14}$$

For simplicity, we assume one of transmitters is perfectly synchronized to receiver. So in (14), f_Δ denotes the carrier frequency offset between the two transmitters. The average SNR is time dependent, i.e., the average gain varies over the transmission period T_p. We hence perform a time averaging computation to obtain the exact average SNR for a given transmission period T_p

$$\begin{aligned} E[\gamma(T_p, f_\Delta)] =& \frac{1}{\sigma_n^2}\Bigg(E\left[\sum_{n=1}^{N_t} |h_{1,n}|^2\right] + E\left[\sum_{n=1}^{N_t} |h_{2,n}|^2\right] \\ &+ \sum_{i=1}^{2}\sum_{n=1}^{N_t}\sum_{\substack{m=1\\ m\neq n}}^{N_t} E\left[|h_{k,n}|\right] E\left[|h_{k,m}|\right] \\ &+ 2\,\mathrm{sinc}(2\pi f_\Delta T_p)\sum_{n=1}^{N_t}\sum_{\substack{m=1\\ m\neq n}}^{N_t} E\left[|h_{1,n}|\right] E\left[|h_{2,m}|\right]\Bigg). \end{aligned} \tag{15}$$

Equation (15) can be equivalently expressed as

$$G_{cfo} = 10\log_{10}\frac{4N_t + N_t(N_t-1)\pi + \mathrm{sinc}(2\pi f_\Delta T_p) N_t^2 \pi}{4N_t}. \tag{16}$$

This result shows that the average power gain degrades following a sinc function with the parameters f_Δ and T_p. We can observe that for a long enough T_p, no power gain is obtained. Moreover, the power gain may be lower than that of the single user case as the output of the sinc function oscillate around zero.

3.3 Diversity gain

As expressed in equation (10), the error probability Pe is required to compute the diversity gain. However, when a residual CFO is present, the error probability Pe becomes time- and CFO- dependent. For a given transmit duration and residual CFO (f_Δ), the error probability $Pe(t)$ for a BPSK modulation can be expressed as

$$Pe(t,f_\Delta) = \int_0^\infty F(t,f_\Delta)p\left(|h_e(t,f_\Delta)|\right)d|h_e(t,f_\Delta)|, \quad 0 \le t \le T_p \tag{17}$$

where $F(t,f_\Delta) = Q\left(\sqrt{2|h_e(t,f_\Delta)|^2 SNR}\right)$. $h_e(t,f_\Delta)$ denotes the equivalent channel at time index t, i.e.,

$$h_e(t,f_\Delta) = \left(c_1(t)\sum_{n=1}^{N_t}|h_{1,n}| + c_2(t)\sum_{n=1}^{N_t}|h_{2,n}|\right) \tag{18}$$

with $c_i = e^{i2\pi f_{\Delta,i}t}$. To compute Pe, the PDF of the equivalent channel h_e is hence required, i.e.,

$$p(h_e(t,f_\Delta)) = p\left(c_1(t)\sum_{n=1}^{N_t}|h_{1,n}| + c_2(t)\sum_{n=1}^{N_t}|h_{2,n}|\right). \tag{19}$$

This is a time- and CFO-dependent sum of complex weighted Rayleigh channels, whose PDF is unfortunately unknown. We approximate this PDF numerically, for different transmission durations (T_p) and different values of residual CFO (f_Δ). The average probability of error is then obtained by

$$P_{e|T_p} = \frac{1}{T_p}\int_0^{T_p} P_e(t)dt. \tag{20}$$

The numerical approximation of the error probability Pe can then be used to compute the diversity order of our cooperative scheme in presence of carrier frequency offset. The computation of the resulting diversity order yields the results presented in the Section 4.1.

4. SIMULATION RESULTS

The cooperative beamforming scenario in simulation is described as in Section 2. The simulations are performed using an 802.11n chain, with a 5GHz carrier frequency and a 20MHz bandwidth. We consider an uncoded OFDM scheme with 64 subcarriers ($N = 64$), an OFDM symbol duration is hence $4\mu s$. The multiple CFOs are assumed known at the receiver where a zero-forcing frequency domain equalizer is applied to mitigate the CFO effects. Assuming a residual CFO, the inter-carrier interference is not the major source of impairment in such a scheme in the low-to-medium SNR region and can be neglected [2]. Perfect time synchronization is also assumed.

4.1 Performance of Cooperative Beamforming with Frequency Offset

In the simulations, we assume 2 transmitters each equipped with a single antenna ($2(1)\times 1$ scheme). The residual CFO difference between them is assumed to be $f_\Delta = 2$ ppm.

4.1.1 Power gain with residual CFO

Following the derivations in Section 3.2, the residual carrier frequency offset introduces a loss in power gain that follows a sinc function. In Fig. 2 we display the BER curves obtained for 3 different transmission durations (T_p), i.e., $18, 25$ and 31 transmit symbols. These values of T_p represent the minimum, a zero and a local maximum of the $\text{sinc}(2\pi f_\Delta T_p)$ function, respectively. We can observe in Fig. 2 that the curves from $T_p = 31$, i.e., the local maximum, achieves a power gain that is higher than that of $T_p = 25$. Similarly, the BER for $T_p = 31$ outperforms that of $T_p = 18$, confirming the result from equation (16).

4.1.2 Diversity gain with residual CFO

Fig. 3 shows the BER curves for various transmission durations. From this figure, we can observe that the diversity order decreases quickly as the number of transmit symbols increases, to finally approach the SISO curve for a long transmit period, e.g., $T_p > 10$ transmit symbols. Moreover, for both the $2\times 1\times 1$ and $2\times 2\times 1$ scenarios, for some values of T_p, e.g., $T_p = 13$ the curves perform worse than that of the SISO case, i.e., the diversity order is lower than 1 ($d < 1$).

As it is not possible to get the closed form of the error probability, we computed the diversity gain through numerical approximations as described in Section 3.3. Fig. 4 shows these results for a CFO of: $0/+1$ppm, $0/+2$ppm and $0/+4$ppm. We can observe that the diversity gain rapidly decreases with the residual CFO. After some time, depending on the value of the residual CFO, the diversity gain oscillates around $d = 1$, meaning that the diversity gain is lost: close to that of a SISO system. This result means that for such residual CFOs cooperative transmissions will lose all the diversity gain unless short packets are used.

4.1.3 Tightness of the analytical results

Fig. 5 aims at showing the accuracy of the results obtained from the proposed analytical derivations. This figure displays the simulated BER curves (i.e., the continuous curves) and the theoretical ones. From the figure, we can observe that the proposed analytical derivations match closely the curves from that of the simulated results.

5. CONCLUSIONS

In this paper, we study the effects of residual CFO on cooperative phase shift beamforming scheme, where both transmitters transmit the same information to a common destination. Analytical derivations and numerical approximations of the power and the diversity gains introduced by the residual carrier frequency offset were developed. Both analytical and simulated results show that residual carrier frequency offset introduces a significant performance degradation compared to the ideal case. Even with a prior-to-transmission synchronization process of the frequency offset, after a certain transmission time, both the diversity gain and the power gain are lost. These results show that cooperative communications schemes should be constrained to short packet sizes transmissions unless additional efforts are set to reduce the residual CFOs, e.g., through more complex or longer synchronization processes.

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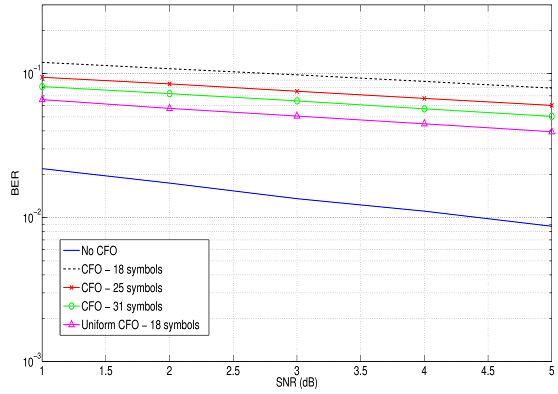

Figure 2: Average power gain fluctuation in the 2(1)x1 cooperative beamforming scheme with residual CFO difference of 2ppm

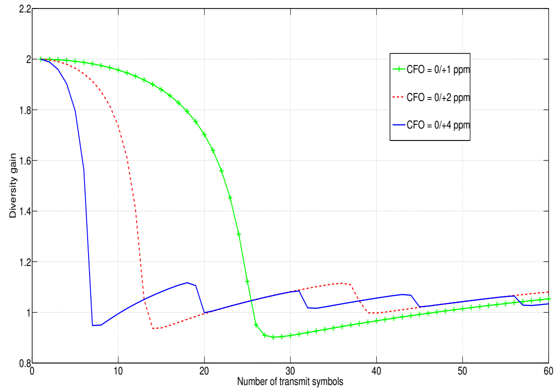

Figure 4: Plot of the diversity gain, for various number of transmit symbols and residual CFOs.

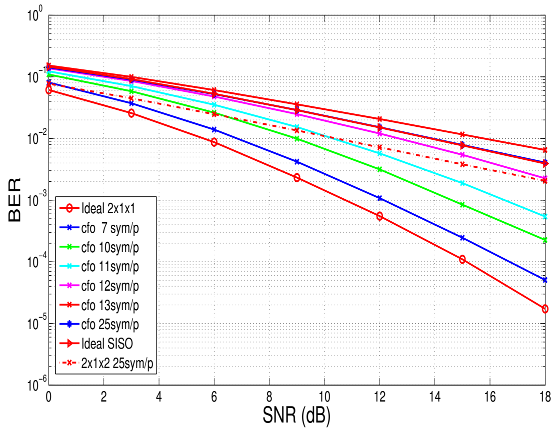

Figure 3: Diversity loss in the 2(1)x1 cooperative beamforming scheme with residual CFO difference of 2ppm

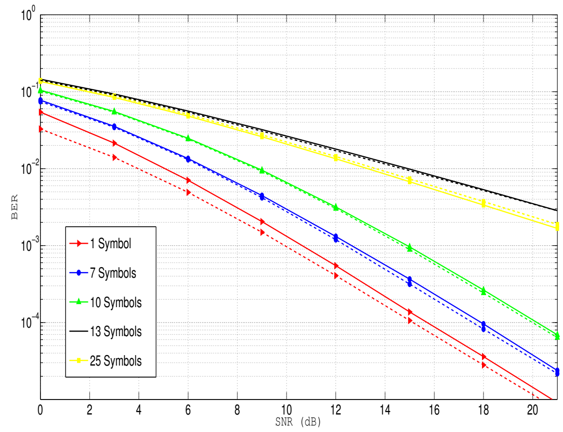

Figure 5: The continuous curves are the simulated results and the dashed lines are obtained from the analytical derivations. The simulations are for the 2x1x1 cooperative beamforming scheme with residual CFO difference of 2ppm and a BSPK modulation.

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