ML-ESTIMATOR AND CRLB ON CARRIER-FREQUENCY OFFSET IN TDM RECEIVERS FOR DIRECTION FINDING

A. Richter and N. Kashyap

Aalto University,
Department of Signal Processing and Acoustics,
PO. Box 13000, 00076 Aalto, Finland
phone: +(358) 50 5213339, email: andyric(at)ieee.org

ABSTRACT

Recently *switched* antenna array, sometimes also called Time Division Multiplexing (TDM), receivers have found their way in systems suitable for, e.g. indoor positioning. Such systems use low-power and low-cost transmitters and receivers, having significant differences in their local reference clocks. Consequently, the received signal in the base-band carries a carrier-frequency offset (CFO). This CFO has to be estimated from the observed signal and compensated for. In this paper Cramér-Rao lower bounds on the CFO for two data models of switched antenna array receivers signals are derived. Furthermore a closed form Maximum Likelihood Estimator (MLE) for the CFO and the antenna data is proposed. The derived CRLBs are used to study the statistical performance of the maximum likelihood estimator.

1. INTRODUCTION

The concept of switched antenna (array) receivers has been used in communication systems for at least 80 years [1]. Initially, switched antenna receivers have been used to exploit the spatial diversity of radio channels and later also to utilize their directivity [2] to increase performance of communication systems. Such systems switch between the receive antennas with a very low frequency, where small is understood in terms of the signal bandwidth. In such systems the CFO estimation/correction can be carried out in the same way as in single channel receivers.

Recently, time division multiplexing (TDM) has been also proposed for multi-antenna wireless receivers, to reduce hardware cost, see [3] and the references therein. In contrast to diversity receivers, TDM multi-antenna receivers switch between the antenna ports within a fraction of a symbol duration. In general, multi-antenna communication receivers, used in SIMO or MIMO systems, suffer from carrier frequency offsets as well as their single channel predecessor. A comprehensive review of CFO correction techniques for MIMO communication systems can be found in, e.g. [4].

The concept of switched antenna array receivers has been also in use for over a decade in SIMO or MIMO radio channel sounders [5, 6]. Here a switched antenna array receiver measures the outputs of an antenna array, in a short sequence. The switching times are typically in the order of some hundreds of nanoseconds up to some microseconds, i.e., the switching time is chosen to be larger than the maximum observable multipath delay of the measured radio channels. Radio channel sounders use sophisticated reference oscillators at the transmitter and the receiver, such as rubidium references. For such systems the carrier frequency offsets are so small that they can been neglected.

In [7], a single channel direction finding system is proposed using antenna switching and phased locked loops. In this system the carrier frequency offset has been corrected using a PLL.

Recently, the switched antenna array receiver concept has been used in low-power and low-cost systems, e.g. for indoor positioning [8]. Such systems use low-cost reference oscillators with low accuracy and stability. Consequently, the received signal in the base-band contains a carrier frequency offset. A carrier frequency offset has a significant impact on the antenna data vector (ADV), i.e., the (sequentially) observed output of the antenna array. To compensate the carrier frequency offset, the antenna switching sequence has to be chosen such that the carrier frequency offset and the ADV can be estimated with low variance form observations.

Due to the knowledge of the authors the problem of joint CFO and ADV estimation in switching antenna receivers for angle of arrival based positioning has not been studied so far. Especially, the impact of the switching sequence on the ADV estimates has not been studied in the literature. Nevertheless, the underlying estimation problem is notably similar to the joint CFO and channel estimation problem studied in [4, 9, 10, 11]. I.e. similar expressions for the Cramér-Rao Lower Bound are obtained. On the other hand, results on the training sequence design for channel estimation cannot be applied to the switching sequence design problem at hand.

In this paper, We introduce two data models for the signals, observed by a switched antenna array receiver. Furthermore, we derive a closed form (search free) maximum likelihood estimator for the CFO and the ADV. We propose also an approximate MLE having low computational complexity, jet exhibiting very good statistical performance in Monte-Carlo simulations. We derive the Cramér-Rao Lower Bound on the model parameters and study the impact of different switching sequences on the bound as well as on the MLEs.

The paper is structured as follows. In Section 2, the data models are introduced. The Cramér-Rao Lower Bounds on the model parameters are derived, in Section 3. The proposed ML-estimator and a low complexity approximate MLE for the CFO and ADV are derived in Section 4. In Section 5, the CRLB is compared with the performance of the Maximum-Likelihood estimator (MLE). Section 6 concludes the paper.

2. DATA MODELS

The physical setup is illustrated in Figure 1. We have only one receiver, measuring each antenna in an order described by a predetermined switching sequence. The switching sequence is contained in the switching matrix $\mathbf{S}$. The rows of $\mathbf{S}$ denote time-instants and columns denote antennas. For example, $\{\mathbf{S}\}_{32}$ would be 1 if the second array element is measured at the third time-instant. The receiver switches between antennas every T_s seconds and down-converts the received signal to baseband. It is assumed, that the transmit signal is a constant wave signal (CW) while the observation

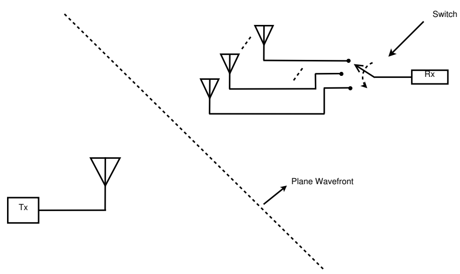

Figure 1: Physical Model

is acquired. This is a realistic assumption, in fact a similar system configuration is used in the angle based positioning system described in, e.g. [8].

2.1 Data Model I

The data model introduced in this section is solely designed to study the impact of the used switching sequence and the CFO on the variance of unbiased direction of arrival estimates. We assume that the receive antenna array is a uniform linear array (ULA) with L elements. The centre-element is considered to be the reference antenna. Furthermore, the channel is restricted to be a single path AWGN channel. The received signal $\mathbf{y} \in \mathbb{C}^{\mathbf{N}\times\mathbf{1}}$ is given by

$$\begin{aligned} \mathbf{y} &= \mathbf{\Phi}(\Delta\omega)\mathbf{S}\mathbf{a}(\phi) + \mathbf{w} &(1)\\ &= \mathbf{s}(\boldsymbol{\theta}) + \mathbf{w}, &(2)\end{aligned}$$

where $\mathbf{\Phi}(\Delta\omega) \in \mathbb{C}^{N\times N}$ is a diagonal matrix with diagonal elements $\mathrm{e}^{-\mathrm{j}\Delta\omega n \mathrm{T_s}}$, n is the sampling index, $\Delta\omega$ is the carrier frequency offset (CFO), $\mathbf{S}$ is the switching matrix, and $\mathbf{a}(\phi)$ is the steering vector. Observe that, in contrast to the joint CFO and channel estimation problem, the matrix $\mathbf{S}$ is orthogonal but not necessarily Toeplitz. An attenuation of the TX-signal has been omitted in this model, adding a weight does not provide additional insight into the estimation problem for a *single*-path channel. The Direction of Arrival (DoA) is contained in the steering vector as $\cos(\phi)$, if the angles are measured from the end-fire direction. To simplify the calculation of the Cramér-Rao bound we make a mapping $f : \psi \rightarrow kd\cos(\phi)$, where k is the wave-number and d is the distance between the elements of the antenna array[1]. The mapping $f : \psi \rightarrow kd\cos(\phi)$ is an one-to-one mapping for angles $0 \leq \phi \leq \pi$, provided d is smaller than $\lambda/2$, where λ is the wavelength of the signal. The vector $\mathbf{w}$ describes i.i.d. circular, complex, normal distributed observation noise, i.e. $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2\mathbf{I})$. The switching matrix $\mathbf{S}$ is an $N \times L$ matrix. The parameter vector to be estimated is

$$\boldsymbol{\theta} = [\ \Delta\omega \quad \psi\ ]^T. \tag{3}$$

In the following it is assumed that $N = k\cdot L$, i.e., the antenna array outputs are sampled k times, each.

2.2 Data Model II

In the second data model we generalize the first data model and remove the constraints on the structure of the antenna array output signal. Therefore, we define an Antenna Data Vector (ADV) $\mathbf{x} \in \mathbb{C}^{L\times 1}$, which implicitly contains information about the array geometry and the Channel State Information (CSI). The received signal $\mathbf{y} \in \mathbb{C}^{\mathbf{kL}\times\mathbf{1}}$ is now given by

$$\begin{aligned} \mathbf{y} &= \mathbf{\Phi}(\Delta\omega)\mathbf{S}\mathbf{x} + \mathbf{w} &(4)\\ &= \mathbf{s}(\boldsymbol{\theta}) + \mathbf{w}. &(5)\end{aligned}$$

Here $\mathbf{x}$ can be understood as the output of an ideal multi antenna receiver, using L identical chains for down-conversion to the base-band[2]. The parameter vector $\boldsymbol{\theta}$ is given by

$$\boldsymbol{\theta} = [\ \Delta\omega \quad \mathbf{x}_R \quad \mathbf{x}_I\ ]^T, \tag{6}$$

where $\mathbf{x}_R = \Re(\mathbf{x})$ and $\mathbf{x}_I = \Im(\mathbf{x})$.

3. DERIVATION OF THE CRLB

In the following we derive the Cramér-Rao Lower Bound [13] for the model parameters of both data models.

3.1 Data Model I

The probability distribution function of $\mathbf{y}$ is

$$f_{\mathbf{Y}}(\mathbf{y}|\theta) = \frac{1}{\pi^{kL}\sigma_w^{2kL}}\mathrm{e}^{-\frac{1}{\sigma_w^2}(\mathbf{y}-\mathbf{s}(\boldsymbol{\theta}))^H(\mathbf{y}-\mathbf{s}(\boldsymbol{\theta}))} \tag{7}$$

The Fisher information matrix is derived using expressions enumerated in [6, p. 54]. We have

$$\begin{aligned}\mathbf{D}(\theta) &= \frac{\partial}{\partial\boldsymbol{\theta}^T}\mathbf{s}(\theta)\\ &= \begin{bmatrix} \frac{\partial}{\partial\Delta\omega}\mathbf{s}(\theta) & \frac{\partial}{\partial\psi}\mathbf{s}(\theta)\end{bmatrix}\\ &= [\ -jT_s\mathbf{\Xi}\mathbf{\Phi}(\Delta\omega)\mathbf{S}\mathbf{a}(\psi) \quad -j\mathbf{\Phi}(\Delta\omega)\mathbf{S}\mathbf{Z}\mathbf{a}(\psi)\ ],\end{aligned}$$

which is also

$$\mathbf{D}(\theta) = -j\mathbf{\Phi}(\Delta\omega)[\ T_s\mathbf{\Xi}\mathbf{S}\mathbf{a}(\psi) \quad \mathbf{S}\mathbf{Z}\mathbf{a}(\psi)\ ], \tag{8}$$

where $\mathbf{\Xi}$ and $\mathbf{Z}$ are diagonal matrices with diagonal elements $-\frac{kL-1}{2},\ldots,\frac{kL-1}{2}$ and $-\frac{L-1}{2},\ldots,\frac{L-1}{2}$, respectively. The Fisher information matrix is given by[3]

$$\mathbf{J}(\theta) = \frac{2}{\sigma_w^2}\cdot\Re\{\mathbf{D}(\theta)^H\mathbf{D}(\theta)\}. \tag{9}$$

Substituting (8) in (9), and using $\mathbf{Q} = (\mathbf{\Xi})^2$, equation (10) is obtained for the FIM.

$$\mathbf{J}(\boldsymbol{\theta}) = \frac{2}{\sigma_w^2}\begin{bmatrix} T_s^2\mathbf{a}(\psi)^H\mathbf{S}^T\mathbf{Q}\mathbf{S}\mathbf{a}(\psi) & T_s\mathbf{a}(\psi)^H\mathbf{S}^T\mathbf{\Xi}\mathbf{S}\mathbf{Z}\mathbf{a}(\psi)\\ T_s\mathbf{a}(\psi)^H\mathbf{Z}\mathbf{S}^T\mathbf{\Xi}\mathbf{S}\mathbf{a}(\psi) & \mathbf{a}(\psi)^H\mathbf{Z}^T\mathbf{S}^T\mathbf{S}\mathbf{Z}\mathbf{a}(\psi)\end{bmatrix} \tag{10}$$

The Cramér-Rao Lower Bound is the inverse of the FIM, i.e.

$$CRLB(\theta_i) = \left[\mathbf{J}^{-1}(\boldsymbol{\theta})\right]_{ii}.$$

This bound is used to study the influence of $\mathbf{S}$ on the lower bound of unbiased estimates of $\Delta\omega$ and ψ, in Section 5.

[1] For further details on ideal ULA steering vector modelling the reader is referred to [12]

[2] Observe that the assumption of L identical down-conversion chains is *not* realistic in practice, especially for low cost and low power receivers.

[3] $()^H$ denotes the hermitian transpose.

3.2 Data Model II

For Data Model II, the probability distribution function is given by (7) as well. From (4), we have

$$\mathbf{D}(\theta) = \left[\begin{array}{ccc} \mathbf{\Phi}'(\Delta\omega)\mathbf{S}\mathbf{x} & \mathbf{\Phi}(\Delta\omega)\mathbf{S} & -j\mathbf{\Phi}(\Delta\omega)\mathbf{S} \end{array}\right] \quad (11)$$

$$= \mathbf{\Phi}(\Delta\omega)\left[\begin{array}{ccc} \mathbf{\Xi}\mathbf{S}\mathbf{x} & \mathbf{S} & -j\mathbf{S} \end{array}\right], \quad (12)$$

where $\mathbf{\Xi}$ is, as before, a diagonal matrix with diagonal elements $-\frac{kL-1}{2}, \ldots, +\frac{kL-1}{2}$. Using (9), we get

$$\mathbf{J}(\boldsymbol{\theta}) = \frac{2}{\sigma_w^2}\begin{pmatrix} \mathbf{x^H S^T Q S x} & -\mathbf{x}_I^T\mathbf{S}^T\mathbf{\Xi}\mathbf{S} & \mathbf{x}_R^T\mathbf{S}^T\mathbf{\Xi}\mathbf{S} \\ -\mathbf{S}^T\mathbf{\Xi}\mathbf{S}\mathbf{x}_I & \mathbf{S^T S} & \mathbf{0} \\ \mathbf{S}^T\mathbf{\Xi}S\mathbf{x}_R & \mathbf{0} & \mathbf{S^T S} \end{pmatrix}. \quad (13)$$

The Cramér-Rao lower bound is the inverse of the FIM, i.e.

$$CRLB(\theta_i) = \left[\mathbf{J}^{-1}(\boldsymbol{\theta})\right]_{ii}.$$

Using the matrix inversion lemma, we can derive the CRLB for $\Delta\omega$ in closed form. The CRLB is

$$CRLB(\Delta\omega) = \frac{\sigma_w^2}{2}\left(\mathbf{x^H S^T \Xi P_s^\perp \Xi S x}\right)^{-1}, \quad (14)$$

where $\mathbf{P}_s^\perp$ is the projector on the null-space of $\mathbf{S}$, defined as

$$\mathbf{P}_s^\perp = \mathbf{I} - \mathbf{S}\mathbf{S}^+,$$

using the pseudo-inverse $\mathbf{S}^+$ of $\mathbf{S}$.

4. MAXIMUM LIKELIHOOD ESTIMATOR

In this section a maximum likelihood estimator for $\Delta\omega$ and $\mathbf{x}$ based on Data model II is derived. The log-likelihood function is

$$\mathcal{L}(\Delta\omega, \mathbf{x}, \sigma^2|\mathbf{y}) = -kL\log(\pi\sigma_w^2) - \frac{1}{\sigma_w^2}\left\|\mathbf{y} - \mathbf{\Phi}(\Delta\omega)\mathbf{S}\mathbf{x}\right\|^2. \quad (15)$$

The maximum likelihood estimate is

$$\left\{\hat{\Delta\omega}, \hat{\mathbf{x}}, \hat{\sigma^2}\right\} = \arg\max_{\Delta\omega,\mathbf{x},\sigma^2} \mathcal{L}(\Delta\omega, \mathbf{x}, \sigma^2|\mathbf{y}). \quad (16)$$

Since $\sigma_w^2 > 0$ the maximizer $\hat{\Delta\omega}, \hat{\mathbf{x}}$ of (16) is independent of σ_w^2. Consequently, the maximization problem (16) can be reduced to

$$\left\{\hat{\Delta\omega}, \hat{\mathbf{x}}\right\} = \arg\min_{\Delta\omega,\mathbf{x}} \left\|\mathbf{y} - \mathbf{\Phi}(\Delta\omega)\mathbf{S}\mathbf{x}\right\|^2. \quad (17)$$

The minimization is quadratic in $\mathbf{x}$ and can be solved in closed form. It is easy to see that the LS-solution of (17) with respect to $\mathbf{x}$, given $\Delta\omega$ is[4]

$$\mathbf{x} = \left(\mathbf{S}^H\mathbf{S}\right)^{-1}\mathbf{S}^H\mathbf{\Phi}^*(\Delta\omega)\,\mathbf{y}. \quad (18)$$

Substituting $\mathbf{x}$ in (17) by (18) yields

$$\hat{\Delta\omega} = \arg\min_{\Delta\omega}\left\|\mathbf{y} - \mathbf{\Phi}(\Delta\omega)\mathbf{S}\left(\mathbf{S}^H\mathbf{S}\right)^{-1}\mathbf{S}^H\mathbf{\Phi}^*(\Delta\omega)\,\mathbf{y}\right\|^2. \quad (19)$$

Introducing the projection on the vector space spanned by $\mathbf{S}$, $\mathbf{P}_S = \mathbf{S}\mathbf{S}^+$, (19) can be written as

$$\hat{\Delta\omega} = \arg\min_{\Delta\omega}\left\|\mathbf{y} - \mathbf{\Phi}(\Delta\omega)\mathbf{P}_S\mathbf{\Phi}^*(\Delta\omega)\,\mathbf{y}\right\|^2. \quad (20)$$

[4] $\mathbf{A}^*$ denotes element-wise conjugation of $\mathbf{A}$.

Observing that $\mathbf{\Phi}(\Delta\omega)\mathbf{\Phi}^*(\Delta\omega) = \mathbf{I}$, and using the projector to the null-space of $\mathbf{S}$, $\mathbf{P}_S^\perp$, the following expression is obtained

$$\hat{\Delta\omega} = \arg\min_{\Delta\omega}\left\|\mathbf{\Phi}(\Delta\omega)\mathbf{P}_S^\perp\mathbf{\Phi}^*(\Delta\omega)\,\mathbf{y}\right\|^2. \quad (21)$$

Expression (21) can be rewritten as

$$\hat{\Delta\omega} = \arg\min_{\Delta\omega}\mathbf{y}^H\mathbf{\Phi}(\Delta\omega)\mathbf{P}_S^\perp\mathbf{\Phi}^*(\Delta\omega)\,\mathbf{y}. \quad (22)$$

Using the matrix-vector exchange

$$\mathbf{\Phi}^*(\Delta\omega)\,\mathbf{y} = \operatorname{diag}\{\mathbf{y}\}\,\mathbf{a}^*(\Delta\omega),$$

where the elements of the vector $\mathbf{a}(\Delta\omega)$ are the diagonal elements of the matrix $\mathbf{\Phi}(\Delta\omega)$, and observing that $\mathbf{S}$ is real, equation (22) can be expressed as

$$\hat{\Delta\omega} = \arg\min_{\Delta\omega}\mathbf{a}^H(\Delta\omega)\operatorname{diag}\{\mathbf{y}\}\,\mathbf{P}_S^\perp\operatorname{diag}\{\mathbf{y}^*\}\,\mathbf{a}(\Delta\omega). \quad (23)$$

As it is well known from root-MUSIC [14], the minimization problem (23), being non-linear in $\Delta\omega$, can be relaxed and rewritten as a polynomial rooting problem. Furthermore, one can apply the same spectral factorization approach proposed in [15], leading to a significant reduction in computational complexity. This is especially true since only one root, the magnitude largest, has to be found from the factored polynomial.

An estimate of $\mathbf{x}$, using $\Delta\hat{\omega}$, can be obtained by (18). Furthermore, an estimate of σ^2 is given by

$$\hat{\sigma}^2 = \frac{1}{(k-1)L}\left\|\mathbf{y} - \mathbf{\Phi}(\Delta\omega)\mathbf{S}\mathbf{x}\right\|^2,$$

which can be useful to estimate, e.g., the current SNR of $\mathbf{x}$. The computational complexity of the maximum likelihood estimator derived so far is largely determined by the complexity of the root finding algorithm used. Stable root finding algorithms have computational complexity $\mathcal{O}((kL)^2)$, [16].

The computational complexity can be reduced using an approximate ML-estimator. Observe that the columns of $\mathbf{S}$ are orthogonal due to the design of the receiver, i.e. only one antenna port is measured at a time. Consequently the matrix $\mathbf{S}^H\mathbf{S}$ is diagonal and so is its inverse. Therefore,

$$\mathbf{Q}_S = \mathbf{S}(\mathbf{S}^H\mathbf{S})^{-\frac{1}{2}} \quad (24)$$

is an orthonormal basis for $\mathbf{S}$. It is easy to see that, using $\mathbf{Q}_S$, equation (23) may be rewritten as

$$\hat{\Delta\omega} = \arg\min_{\Delta\omega}\|\mathbf{y}\|^2 - \left\|\mathbf{a}^H(\Delta\omega)\operatorname{diag}\{\mathbf{y}\}\,\mathbf{Q}_S\right\|^2. \quad (25)$$

Since the norm $\|\mathbf{y}\|$ is independent of $\Delta\omega$ the minimization problem (25) reduces to the maximization problem

$$\hat{\Delta\omega} = \arg\max_{\Delta\omega}\left\|\mathbf{a}^H(\Delta\omega)\operatorname{diag}\{\mathbf{y}\}\,\mathbf{Q}_S\right\|^2. \quad (26)$$

The maximization problem (26) can be solved using a line search over $\Delta\omega$. Since $\mathbf{a}(\Delta\omega)$ is a complex exponential this line search can be efficiently implemented using a FFT. The accuracy of this estimator is limited by the length of the FFT used. Therefore the estimator is an approximate MLE and useful only up to a certain SNR. However, since the SNR of a real receiver is always bounded, this estimator can yield a good trade-off between accuracy and computational complexity, as it is shown in the following section.

5. RESULTS

In the following, we discuss CRLBs obtained for different switching sequences for Data Model I and compare the Maximum Likelihood estimates for Data Model II with the CRLB using Monte Carlo simulations.

5.1 Data Model I

The Cramér-Rao bound for the CFO and the parameter ψ are shown in Figures 2 and 3. Here we have chosen a simple, symmetric switching sequence given by

$$\mathbf{S} = \begin{bmatrix} \mathbf{I}_L \\ \mathbf{\Pi}_L \end{bmatrix},$$

where $\mathbf{I}_L$ is an $L \times L$ identity matrix and $\mathbf{\Pi}_L$ is an $L \times L$ exchange matrix. Both bounds decrease proportionally with the number of antenna elements.

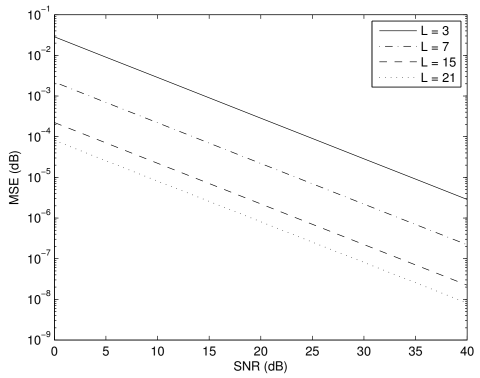

Figure 2: Cramér-Rao bound for the CFO in Data Model I.

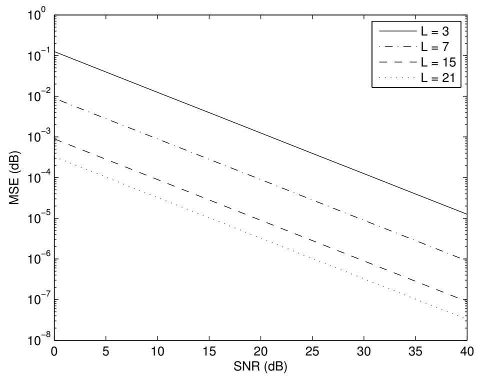

Figure 3: Cramér-Rao bound for ψ in Data Model I.

5.2 Data Model II

For Data Model II the CRLB and the statistical performance of the MLE has been analysed for different switching sequences of the same length. To determine the variance of approximate ML-estimates Monte Carlo simulations have been carried out, using (26). Examples for switching sequences with $L = 3$ and $k = 3$ are shown in Figure 4. The CRLB on the CFO for the four sequences is compared in Figure 5. The bound is changing insignificantly for the sequences. The variance of the estimates for the CFO $\Delta\omega$ using switching sequence a or c, are shown in Figures 6 and 8, respectively. The approximate ML-estimates attain the CRLB for SNRs higher than 10dB. The MSEs for estimates of the antenna data vector $\mathbf{x}$ for the same switching sequences are shown in Figures 7 and 9. The estimates for $\mathbf{x}$ attain the CRLB as well for SNRs higher than 10dB. Figure 6 and 8 show also that the switching sequence used has an impact on the behaviour of the MLE in the threshold region.

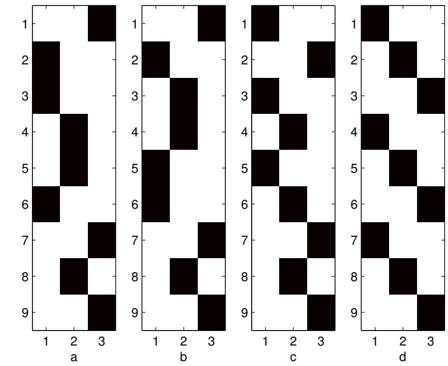

Figure 4: Switching Sequences used for comparison of MLEs and CRLBs.

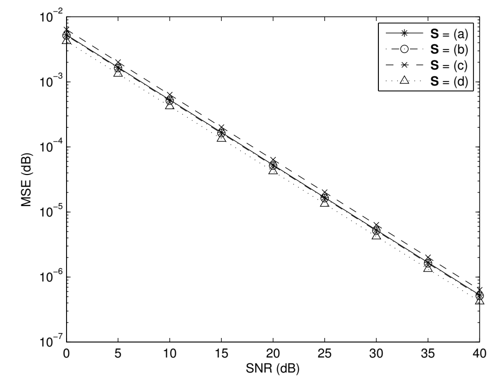

Figure 5: Cramér-Rao Lower Bound for the CFO in Data Model II for different switching sequences.

6. CONCLUSIONS

We have introduced two models to study the statistical performance of algorithms for CFO estimation in switched antenna array receivers. Cramér-Rao Lower Bounds on the model parameters have been derived. Furthermore, we derived a closed-form MLE for the CFO and ADV and proposed an approximate MLE having low complexity but very good statistical performance for realistic SNRs. Furthermore, we have shown that the MLE attains the CRLBs for high SNRs. Consequently, the CRLB expressions can be an useful tool to optimize switching sequences in the high SNR. region. To optimize switching sequences for low SNRs, i.e. in the threshold region the CRLB is naturally not a suitable measure, since the data models are strongly non-linear in the CFO. To optimize switching in the low SNR region other bounds should be used.

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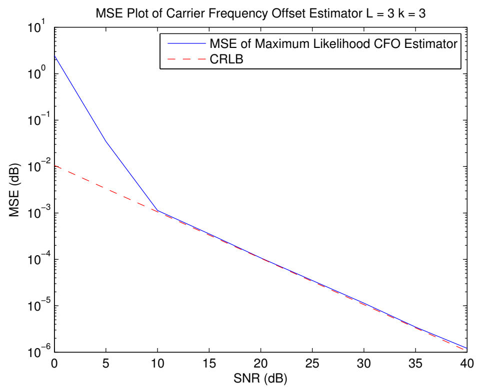

Figure 6: Comparison of the performance of an ML Estimator for the CFO to the CRLB. Switching sequence used is (a)

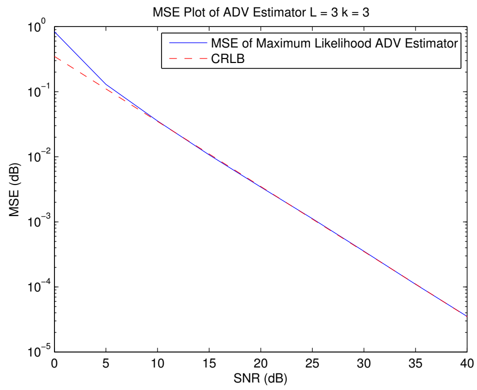

Figure 7: Comparison of the performance of an ML Estimator for the antenna data vector to the CRLB. Switching sequence used is (a)

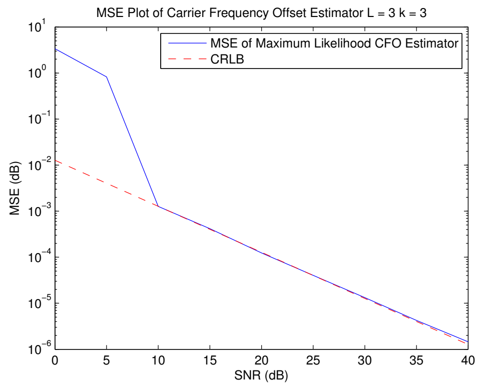

Figure 8: Comparison of the performance of an ML Estimator for the CFO to the CRLB. Switching sequence used is (c)

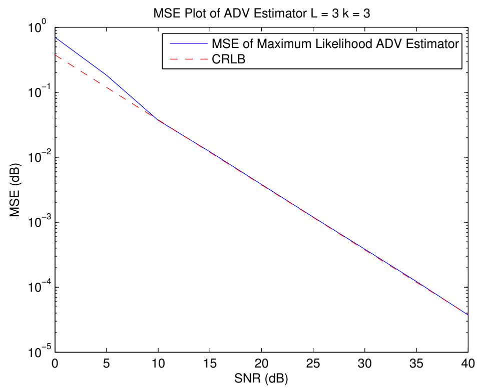

Figure 9: Comparison of the performance of an ML Estimator for the antenna data vector to the CRLB. Switching sequence used is (c)

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