Living in Symbiosis: On Achievable Rates of an Oblivious Interference System and a Cognitive System

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***Abstract*—We analyze achievable rates of a setup involving a primary two user interference channel and a secondary cognitive point-to-point-system. The primary nodes are oblivious in the sense that they are un-aware of the existence of the secondary system, and thus do not change their strategy. Given the messages of the primary nodes are available a-priori at the secondary transmitter, we show that the secondary user can communicate with non-zero rate while not deteriorating (at least) the rates of the primary nodes. Thus, they are able to live in symbiosis. For the noisy interference regime, the rates achievable for the cognitive user can even be expressed in analytical form.**

I. Introduction

A cognitive system is a secondary communications system that can share the same transmission medium with a primary system, while causing little or no harm to the performance of the primary system. This is possible by allowing the 'smart' cognitive transmitter to gather information about its surrounding. This gathered information is then used to adapt the transmission strategy of the cognitive secondary system such that, if activated, it minimizes the rate loss at the primary while achieving non-zero rate at the secondary system.

Cognitive systems have attracted increasing research attention recently. Several setups have been studied including the cognitive interference channel (CIC) and the interference channel with a cognitive relay. The capacity region of the CIC was characterized for both strong interference in [1] and weak interference [2], [3]. The interference channel with a cognitive relay was studied in [4], [5] where capacity inner and outer bounds were given. In the aforementioned setups, it is assumed that the cognitive node knows the message of the primary node a-priori. Moreover, it is assumed that the primary is aware of the existence of the secondary system, and thus can also change its strategy accordingly.

In this paper, we consider a primary interference channel (IC) and a cognitive point-to-point communications system sharing the same medium. The resulting setup resembles a 3-user IC, with the difference that one transmitter is cognitive and the other transmit-receive pairs are not aware of its existence. We call this model the 3-user CIC with oblivious primary nodes (3-user CIC-OP). The primary nodes constitute a 2-user IC which is oblivious to the secondary system, and thus behaves as if the cognitive system is not present.

The best known communications scheme for the IC is the Han-Kobayashi (HK) [6] scheme. While the original scheme is quite involved, a simplified version was shown in [7] to achieve the capacity of the IC within 1 bit. For this reason, we stick to this strategy at the primary system. The secondary transmitter knows the primary transmitters' messages a-priori. This non-causal knowledge of the messages at the cognitive nodes was discussed in [8]. For instance, the cognitive node can have knowledge of the primary messages in a retransmission phase, when the primary receivers could not correctly decode their messages while the secondary cognitive node could, or when the primary and the secondary systems exchange the messages using a separate cooperation channel. Now the secondary system can act as relay for the primary system, while transmitting its own message to the secondary receiver. We propose an achievable scheme that uses a HK scheme at the primary, and Costa's dirty paper coding (DPC) [9] at the secondary. Using this scheme, an achievable rate region is given.

Then, we discuss the co-existence of the primary and the secondary systems. We say that the primary and the secondary systems are able to co-exist if the secondary is able to communicate with non-zero rate while causing no performace deterioration at the primary system. We study the co-existance rate, which we define as the maximum rate achievable by the secondary system while causing no performance deterioration for the primary system. We show that using the congitive transmitter as a relay for the primary system enlarges the achievable rate region of the primary system. In the same time, the secondary transmitter can establish reliable communication with the secondary receiver.

The sum-capacity of the IC under noisy interference is known from [10]–[12]. Given that the IC has noisy interference, and has its strategy fixed to treating interference as noise, we obtain the co-existence rate that does not decrease the sum-capacity of the primary system.

The rest of the paper is organized as follows. In Section II, we introduce the 3-user CIC-OP. In Section III, an achievable scheme is proposed and its corresponding achievable rate region is obtained. In Section IV, the co-existence of the IC with the cognitive system is studied, and in Section V, the special case of noisy interference is analyzed. Finally, we conclude with Section VI.

II. SYSTEM MODEL

We consider a secondary (cognitive) system sharing the same communications medium with a primary system. The primary system is a symmetric IC, and the secondary is a point to point channel as shown in Figure 1.

Transmitters 1 and 2 have messages $m_1 \in \mathcal{M}_1$ and $m_2 \in \mathcal{M}_2$ to be delivered to receivers 1 and 2 respectively. The secondary user has message $m_3 \in \mathcal{M}_3$ to be delivered to the secondary receiver. Moreover, the secondary user is cognitive in the sense that it knows m_1 and m_2 non-causally. The rates of the messages are denoted $R_1 = \frac{1}{n}\log(|\mathcal{M}_1|)$, $R_2 = \frac{1}{n}\log(|\mathcal{M}_2|)$ and $R_3 = \frac{1}{n}\log(|\mathcal{M}_3|)$ where n is the code length. Transmitter $j \in \{1,2\}$ encode m_j to X_j^n such that an average power constraint P is satisfied, i.e.,

$$\frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_{ji}^2] \leq P. \tag{1}$$

It is assumed that the primary system is oblivious to the secondary system, i.e. it does not know about its existence, and thus, does not change its transmit-receive strategy. For this reason, the secondary transmitter can help the primary only by sending the same signals X_1^n and X_2^n scaled according to its power constraint.

In this sense, the secondary transmitter acts as a cognitive amplify-and-forward relay for the primary system. In the same time, the secondary transmitter wants to communicate with its respective receiver. Therefore, it forms

$$X_3^n = \alpha_1 X_1^n + \alpha_2 X_2^n + \alpha_3 \hat{X}_3^n, \tag{2}$$

where $\hat{X}_3^n$ is the codeword, with average power P, corresponding to the message m_3. It holds that $\alpha_1^2+\alpha_2^2+\alpha_3^2 \leq 1$ so that the power constraint at the secondary transmitter is satisfied.

As a result, the received signals are given by

$$Y_1 = \tilde{h}_{11}X_1 + \tilde{h}_{21}X_2 + \tilde{h}_r\hat{X}_3 + Z_1, \tag{3}$$
$$Y_2 = \tilde{h}_{12}X_1 + \tilde{h}_{22}X_2 + \tilde{h}_r\hat{X}_3 + Z_2, \tag{4}$$
$$Y_3 = (h_s+\alpha_1)X_1 + (h_s+\alpha_2)X_2 + \alpha_3\hat{X}_3 + Z_3, \tag{5}$$

where the noise terms Z_1, Z_2, and Z_3 are of zero mean and unit variance, i.e. $Z_j \sim \mathcal{N}(0,1)$ and

$$\tilde{h}_{11} = (1+h_r\alpha_1), \quad \tilde{h}_{21} = (h_c + h_r\alpha_2), \tag{6}$$
$$\tilde{h}_{22} = (1+h_r\alpha_2), \quad \tilde{h}_{12} = (h_c + h_r\alpha_1), \tag{7}$$
$$\tilde{h}_r = h_r\alpha_3. \tag{8}$$

Thus, the secondary transmitter helps the primary system by changing its channel parameters as given in (3) and (4). The primary system observes channels $\tilde{h}_{11}$, $\tilde{h}_{21}$, $\tilde{h}_{12}$ and $\tilde{h}_{22}$ instead of 1 and h_c.

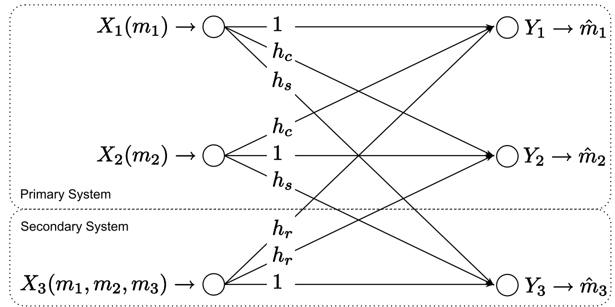

Fig. 1. A secondary cognitive system and a primary IC communicating using the same medium. The resulting model is the 3 user cognitive IC with oblivious primary nodes (CIC-OP).

The achievable rate region is the set of rate tuples (R_1, R_2, R_3) such that the transmit receive pairs can communicate reliably with a small error probability as n increases. In the following section, we discuss a transmission strategy for the CIC-OP, and state its corresponding achievable rate region.

III. AN ACHIEVABLE RATE REGION

The best known achievable scheme for the 2-user IC is the HK scheme. It was shown in [7] that a HK scheme achieves the capacity of the 2-user IC within one bit. For this reason, we stick to this scheme at the primary system.

The primary users use rate splitting. Message m_1 is split into a private part m_{1p} to be decoded by the first receiver only, and a common part m_{1c} to be decoded at both receivers. Similarly, m_2 is split into m_{2p} and m_{2c}. The rates corresponding to these messages are R_{1p}, R_{2p}, R_{1c} and R_{2c}, and the total rates are $R_1 = R_{1p} + R_{1c}$ and $R_2 = R_{2p} + R_{2c}$.

Then these messages are encoded into i.i.d. sequences $x_j^n(m_j)$ where $X_j \sim \mathcal{N}(0,P)$, $j \in \{1p, 1c, 2p, 2c\}$. The transmit signal of the transmitter $j \in \{1,2\}$ is

$$x_j^n = \sqrt{\beta}_j x_{jc}^n + \sqrt{\bar{\beta}_j} x_{jp}^n, \tag{9}$$

with $\beta_j \in [0,1]$ and $\bar{\beta}_j = 1 - \beta_j$. The power allocation parameter β_j plays the role of a trade-off parameter between private and common messages. Higher β_j results in higher power for the common message and vice versa.

The secondary user encodes m_3 into an i.i.d. sequence $\hat{x}_3^n(m_3)$ where $\hat{X}_3 \sim \mathcal{N}(0,P)$, and transmits

$$X_3^n = \alpha_1 x_1^n + \alpha_2 x_2^n + \alpha_3 \hat{x}_3^n, \tag{10}$$

where $\sum_{i=1}^{3}\alpha_i^2 \leq 1$ in order to fulfill the power constraint (1). The parameter α_i^2 determines the power dedicated for cooperation with receiver i. We allow α_1 and α_2 to range in $[-1,1]$. Negative values of α_1 and α_2 can allow interference neutralization, e.g. by setting $\alpha_1 = -h_c/h_r$, the interference from tranmitter 1 to receiver 2 can be eliminated.

The signal $\hat{x}_3^n$ is constructed using DPC with respect to the non-causally known interference at the secondary receiver as side information

$$(h_s+\alpha_1)x_1^n + (h_s+\alpha_2)x_2^n. \tag{11}$$

Thus, the secondary receiver can decode m_3 with small probability of error if [9]

$$R_3 \leq \frac{1}{2}\log(1+\alpha_3^2 P). \quad (12)$$

The received signal at receiver 1 can be written as

$$\begin{aligned} y_1^n &= \tilde{h}_{11}x_1^n + \tilde{h}_{21}x_2^n + \tilde{h}_r\hat{x}_3^n + z_1^n \quad (13)\\ &= \tilde{h}_{11}(\sqrt{\bar{\beta}_1}x_{1p}^n + \sqrt{\beta_1}x_{1c}^n)\\ &\quad + \tilde{h}_{21}(\sqrt{\bar{\beta}_2}x_{2p}^n + \sqrt{\beta_2}x_{2c}^n) + \tilde{h}_r\hat{x}_3^n + z_1^n. \quad (14)\end{aligned}$$

Decoding at the primary receiver proceeds as in [7]. First, both common messages are decoded, then they are subtracted from the received signal, and then the respective private message is decoded. Messages m_{1c} and m_{2c} can be decoded at the first receiver with a small probability of error if

$$R_{1c} \leq \frac{1}{2}\log\left(1+\frac{\tilde{h}_{11}^2\beta_1 P}{1+\tilde{h}_r^2 P+\tilde{h}_{11}^2\bar{\beta}_1 P+\tilde{h}_{21}^2\bar{\beta}_2 P}\right) \quad (15)$$

$$R_{2c} \leq \frac{1}{2}\log\left(1+\frac{\tilde{h}_{21}^2\beta_2 P}{1+\tilde{h}_r^2 P+\tilde{h}_{11}^2\bar{\beta}_1 P+\tilde{h}_{21}^2\bar{\beta}_2 P}\right) \quad (16)$$

$$R_{1c}+R_{2c} \leq \frac{1}{2}\log\left(1+\frac{\tilde{h}_{11}^2\beta_1 P+\tilde{h}_{21}^2\beta_2 P}{1+\tilde{h}_r^2 P+\tilde{h}_{11}^2\bar{\beta}_1 P+\tilde{h}_{21}^2\bar{\beta}_2 P}\right). \quad (17)$$

Then, after subtracting the contribution of m_{1c} and m_{2c} from Y_1, the private message m_{1p} can be decoded reliably if

$$R_{1p} \leq \frac{1}{2}\log\left(1+\frac{\tilde{h}_{11}^2\bar{\beta}_1 P}{1+\tilde{h}_r^2 P+\tilde{h}_{21}^2\bar{\beta}_2 P}\right). \quad (18)$$

Similarly, at the second receiver, the following constraints must be satisfied to achieve reliable communication

$$R_{1c} \leq \frac{1}{2}\log\left(1+\frac{\tilde{h}_{12}^2\beta_1 P}{1+\tilde{h}_r^2 P+\tilde{h}_{12}^2\bar{\beta}_1 P+\tilde{h}_{22}^2\bar{\beta}_2 P}\right) \quad (19)$$

$$R_{2c} \leq \frac{1}{2}\log\left(1+\frac{\tilde{h}_{22}^2\beta_2 P}{1+\tilde{h}_r^2 P+\tilde{h}_{12}^2\bar{\beta}_1 P+\tilde{h}_{22}^2\bar{\beta}_2 P}\right) \quad (20)$$

$$R_{1c}+R_{2c} \leq \frac{1}{2}\log\left(1+\frac{\tilde{h}_{12}^2\beta_1 P+\tilde{h}_{22}^2\beta_2 P}{1+\tilde{h}_r^2 P+\tilde{h}_{12}^2\bar{\beta}_1 P+\tilde{h}_{22}^2\bar{\beta}_2 P}\right) \quad (21)$$

$$R_{2p} \leq \frac{1}{2}\log\left(1+\frac{\tilde{h}_{22}^2\bar{\beta}_2 P}{1+\tilde{h}_r^2 P+\tilde{h}_{12}^2\bar{\beta}_1 P}\right). \quad (22)$$

The achievable rate R_j, $j \in \{1,2\}$ is given by the sum of the achievable private and common message rates, i.e. $R_j = R_{jp} + R_{jc}$ satisfying (15)-(22). For given power allocation parameters $(\alpha_1,\alpha_2) \in [-1,1]^2$ and $(\alpha_3,\beta_1,\beta_2) \in [0,1]^3$ such that $\sum_{i=1}^3 \alpha_i^2 \leq 1$, we denote by $\mathcal{R}(\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2)$ the following region

$$\mathcal{R}(\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2) \triangleq \left\{\begin{array}{l}(R_1,R_2,R_3) \in \mathbb{R}_+^3 :\\ \text{for } j \in \{1,2\}\\ R_{jp}, R_{jc} \text{ satisfy (15)-(22)},\\ R_j = R_{jp} + R_{jc}\\ R_3 \leq \frac{1}{2}\log(1+\alpha_3^2 P)\end{array}\right\}$$

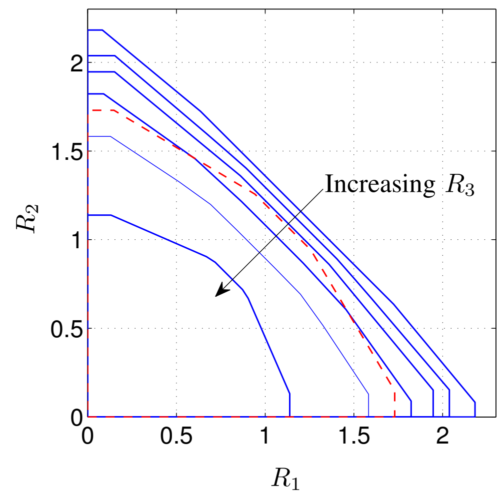

Fig. 2. Slices of the region $\mathcal{R}$ for a CIC-OP with $h_c = 0.5$, $h_r = 0.4$, and $P = 10$ for different achievable values of R_3. Namely $R_3 = \frac{r_3}{2}\log(1+P)$ where $r_3 \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$. Rates are in bits/channel use.

As a result of this section, we obtain the following theorem.

Theorem 1: Any rate triple $(R_1,R_2,R_3) \in \mathcal{R}$ is achievable in the CIC-OP where

$$\mathcal{R} = \bigcup_{\substack{(\alpha_1,\alpha_2)\in[-1,1]^2\\(\alpha_3,\beta_1,\beta_2)\in[0,1]^3\\ \sum_{i=1}^3 \alpha_i^2 \leq 1}} \mathcal{R}(\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2). \quad (23)$$

The achievable region $\mathcal{R}$ is 3 dimensional. Figure 2 shows the boundaries of $\mathcal{R}$ for different achievable values of R_3, for a CIC-OP with $h_c = 0.5$, $h_r = 0.4$, and $P = 10$. Obviously, achieving higher rate R_3 at the secondary system shrinks the achievable rate region of the primary system. This is a cost that must be paid, since increasing R_3 leads to increasing the interference (noise) power at the oblivious primary receivers. The dashed red region is the achievable region for the primary IC using the HK scheme in the absence of the secondary cognitive system. Notice that the secondary system can achieve a non-zero rate, while enlarging the achievable region of the primary system (namely for $r_3 \in (0, 0.4]$ in this example).

IV. Co-existence of the IC with the cognitive system

The aim of this section is to analyze the achievable rates in the IC in the presence of the cognitive system, and when it is possible to achieve non-zero rate for the cognitive user while not deteriorating the performance of the primary system.

Assume that the primary system is able to communicate reliably with a rate pair (R_1^*, R_2^*) in the absence of the cognitive system. We say that the cognitive system and the primary one are able to co-exist if the cognitive system achieves non-zero rate R_3 while the primary system performance does not decrease compared to (R_1^*, R_2^*). That is, the co-existence condition is

$$R_1 \geq R_1^* \quad (24)$$
$$R_2 \geq R_2^* \quad (25)$$
$$R_3 > 0. \quad (26)$$

The sum-capacity is a criterion that can be used to evaluate the performance of a a given network (cf. [7], [10], [11]). It measures the spectral efficiency achieved by the whole network. Here, we restrict the analysis to the achievable sum-rate $R_s = R_1 + R_2$ in the presence of the cognitive user, and compare to the achievable sum-rate of the IC in the absence of the cognitive user $R_s^* = R_1^* + R_2^*$ using the HK scheme.

By restricting to the sum-rate, the co-existence condition becomes

$$R_s \geq R_s^* \quad (27)$$
$$R_3 > 0. \quad (28)$$

We call the maximum rate R_3 that can be achieved by the secondary user while co-existing with the IC the maximum co-existence rate $\bar{R}_3$. In order to bound $\bar{R}_3$, let us fix the following power allocation: $\alpha_1 = \alpha_2 = \alpha$, and $\beta_1 = \beta_2 = \beta$. Then, using (15)-(22), we can show that the following sum-rate is achievable

$$R_s(\alpha, \alpha_3, \beta) = \min\{R_{s1}, R_{s2}, R_{s3}\}, \quad (29)$$

with $\alpha \in [-1, 1]$, $\alpha_3, \beta \in [0, 1]$, and $2\alpha^2 + \alpha_3^2 \leq 1$ where

$$R_{s1} = \log\left(1 + \frac{\tilde{h}_d^2\bar{\beta}P + \tilde{h}_c^2\beta P}{1 + \tilde{h}_r^2 P + \tilde{h}_c^2\bar{\beta}P}\right) \quad (30)$$

$$R_{s2} = \frac{1}{2}\log\left(1 + \frac{\tilde{h}_d^2 P + \tilde{h}_c^2\beta P}{1 + \tilde{h}_r^2 P + \tilde{h}_c^2\bar{\beta}P}\right) + \frac{1}{2}\log\left(1 + \frac{\tilde{h}_d^2\bar{\beta}P}{1 + \tilde{h}_r^2 P + \tilde{h}_c^2\bar{\beta}P}\right) \quad (31)$$

$$R_{s3} = \log\left(1 + \frac{\tilde{h}_d^2 P}{1 + \tilde{h}_r^2 P + \tilde{h}_c^2\bar{\beta}P}\right). \quad (32)$$

We call the achievable co-existence rate in this case $\widetilde{R}_3$ which is clearly a lower bound for $\bar{R}_3$ due to the restrictions we made on α_j and β_j, i.e. $\bar{R}_3 \geq \widetilde{R}_3$. Any rate R_3 below $\widetilde{R}_3$ is achievable with no harm to the sum-rate of the primary system.

Note that by increasing α_3, we increase R_3. In the same time, this decreases the remaining power at the secondary transmitter that can be used to cooperate with the primary system. Moreover, it increases the noise power at the primary receivers and thus decrease R_s. This can be seen in Figure 3, where we plot R_s versus R_3 for a setup with $h_c = 0.4$, $h_r = 0.2$ and $P = 10$. On the same plot, R_s^* is shown. The intersection between R_s and R_s^* marks an achievable co-existance rate $\widetilde{R}_3$, which is in this case 0.8 bits/channel use. This means that the secondary system can transmit at a rate not less than 0.8 bits per channel use without having any impact on the achievable sum-rate of the primary system.

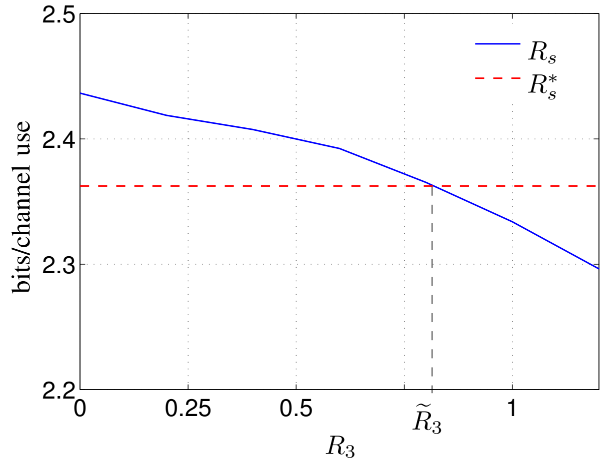

Fig. 3. Achievable rate R_s as a function of R_3 for a CIC-OP with $h_c = 0.4$, $h_r = 0.2$ and $P = 10$. The point where R_s meets R_s^* marks $\widetilde{R}_3$.

In order to find $\widetilde{R}_3$, we need to solve

$$\begin{aligned} \text{maximize} \quad & R_3 \\ \text{subject to} \quad & R_s(\alpha, \alpha_3, \beta) \geq R_s^* \\ & \alpha \in [-1, 1] \\ & \alpha_3 \in [0, 1] \\ & 2\alpha^2 + \alpha_3^2 \leq 1 \\ & \beta \in [0, 1]. \end{aligned} \quad (33)$$

This rate $\widetilde{R}_3$ is a function of h_c, h_r, and P.

V. Noisy Interference

In this section, we consider a special case of the CIC-OP. We consider the CIC-OP where the IC has noisy interference, i.e. the interference is very weak. The sum-capacity of the IC with noisy interference is known from [10]–[12] and given by

$$C_{\Sigma,ni} = \log\left(1 + \frac{P}{1 + h_c^2 P}\right), \quad (34)$$

This, however, can be achieved simply by treating interference as noise. The symmetric IC has noisy interference if [10]

$$h_c(1 + h_c^2 P) \leq 1/2, \quad (35)$$

Suppose that the IC in our CIC-OP has noisy interference, and that the strategy is fixed to treating interference as noise. Thus, no rate splitting is done, and the achievable sum-rate of the primary system in the presence of the cognitive system is then[1]

$$R_{s,ni} = \log\left(1 + \frac{\tilde{h}_d^2 P}{1 + \tilde{h}_r^2 P + \tilde{h}_c^2 P}\right), \tag{36}$$

for some $\alpha \in [-1, 1]$, $\alpha_3 \in [0, 1]$, such that $\alpha_3^2 + 2\alpha^2 \leq 1$. In this case it is possible to express $\widetilde{R}_3$ analytically. Let us find the value of α_3 that solves

$$R_{s,ni} = C_{\Sigma,ni}. \tag{37}$$

for some $\alpha \in [-1, 1]$. The solution of this equation is

$$\alpha_3^2 = \frac{\tilde{h}_d^2(1 + h_c^2 P) - 1 - \tilde{h}_c^2 P}{h_r^2 P} \equiv f(\alpha). \tag{38}$$

Notice that $f(\alpha)$ is quadratic in α, $f(0) = 0$, and that $\left.\frac{df}{d\alpha}\right|_{\alpha=0} \neq 0$ (except if $h_r = 0$). Thus, there exist $\alpha \in [-1, 1]$ such that $\alpha_3^2 = f(\alpha) \in [0, 1]$. In order to maximize R_3, we need to maximize α_3^2 such that

$$\alpha_3^2 + 2\alpha^2 \leq 1. \tag{39}$$

Let α^- and α^+ be the roots of the following equation

$$f(\alpha) = 1 - 2\alpha^2. \tag{40}$$

Then, it can be shown that $\alpha_3^2 = f(\alpha) \leq 1 - 2\alpha^2$ if $\alpha \in [\alpha^-, \alpha^+]$, and that $\alpha^- \leq 0$ and $\alpha^+ \geq 0$. As a result, the maximum α_3^2 such that (37) and (39) are satisfied is given by

$$\bar{\alpha}_3^2 = \max_{\alpha \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \cap [\alpha^-, \alpha^*]} f(\alpha), \tag{41}$$

and the corresponding maximum R_3 is

$$\widetilde{R}_3 = \frac{1}{2}\log(1 + \bar{\alpha}_3^2 P). \tag{42}$$

In Figure 4, we plot $\widetilde{R}_3$ versus h_r for a CIC-OP with noisy interference where $P = 10$. The cognitive user can achieve $\widetilde{R}_3$ while not decreasing the sum-capacity of the primary system. Interestingly, the $\widetilde{R}_3$ is monotonically decreasing with h_r, i.e. the secondary user has to invest qualitatively much more to ensure that the primary rates are not deteriorating, which leaves less for $\widetilde{R}_3$ as h_r increases.

VI. Conclusion

We studied a model where a secondary cognitive system shares the same medium with a primary 2-user interference channel. The primary nodes are oblivious in the sense that they are un-aware of the existence of the secondary system, and thus do not change their strategy. The secondary system is cognitive in the sense that it knows the messages of the primary system a-priori, making it possible to even increase the primary system's rate with the help of the secondary user. We show that indeed co-existence is possible, i.e. the secondary user can achieve a non-zero rate while ensuring that the rates of the primary nodes is not deteriorating. Interestingly, as the channel gain from the secondary transmitter to the primary receivers increases, the negative impact of an active cognitive user gets larger, forcing the latter one to decrease the rate such that lossless co-existence is possible.

[1]This can be achieved using our scheme by setting $\beta = 0$ in (29)

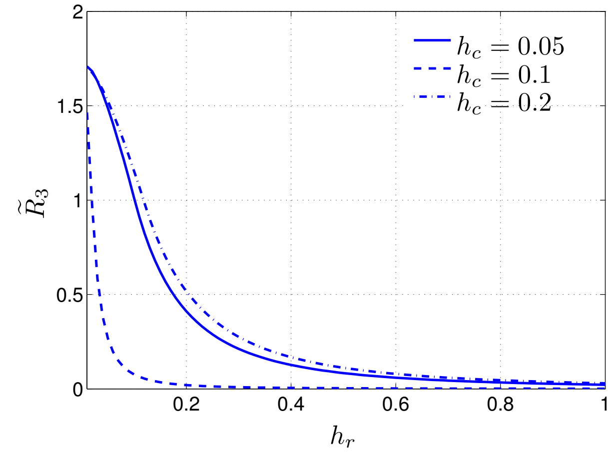

Fig. 4. The achievable co-existence rate $\widetilde{R}_3$ as a function of h_r for a CIC-OP with noisy interference where $P = 10$.

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