NEIGHBOR-$d_{\min}$ PRECODER FOR THREE DATA-STREAM MIMO SYSTEMS

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ABSTRACT

A new precoding strategy, which considers not only the minimum Euclidean distance but also the number of neighbors providing the distance $d_{\min}$, is proposed. Firstly, a new parameterized form of Neighbor-$d_{\min}$ precoder is presented, where all angles containing the phase are assumed to be zero to reduce the number of difference vectors providing $d_{\min}$. The optimization problem is, therefore, less complex and the new precoder has less distinct expressions. When a rectangular QAM modulation is used at the transmitter, the general Neighbor-$d_{\min}$ precoder for three independent data-streams is proposed. This precoder has only three expressions, which allocate power on one, two, and three virtual subchannels, respectively. Simulation results over Rayleigh fading channels confirm a significant bit-error-rate improvement of the new precoder in comparison with other traditional precoding strategies.

1. INTRODUCTION

When the channel state information (CSI) is available at the transmitter, the linear precoding technique can be used to significantly improve the performance of a MIMO system. The transmitted symbols are pre-multiplied by a precoding matrix which optimizes various criteria such as maximizing the received signal-to-noise ratio (SNR) [1], minimizing the mean square error (MSE) [2], or maximizing the minimum singular value of the channel matrix (max-$\lambda_{\min}$) [3].

An important set of linear precoding techniques is known as diagonal precoders. The precoding matrix is, then, diagonal and leads to power allocation strategies. The authors in [4], [5] proposed a non-diagonal precoder which maximizes the minimum Euclidean distance ($d_{\min}$) between two received symbols. This precoder obtains a significant bit-error-rate (BER) improvement in comparison with diagonal precoders, especially when ML detection is used at the receiver. Unfortunately, the max-$d_{\min}$ solution is only available for a small number of independent data-streams and low-order QAM modulations. By decomposing the transmit channel into 2×2 eigen-subchannels, the authors in [6] presented a suboptimal precoder for large MIMO systems. Another precoding scheme with similar structure, named X-Codes, is proposed in [7]. These precoders achieve higher diversity gains compared to diagonal precoders, but they are only optimized for pair subchannels. An alternative suboptimal solution is also proposed in [8], but it is only suitable for quasi-stationary MIMO channels.

One should note that not only the minimum distance but also the average number of neighbors providing it has an important role in reducing the bit-error-rates. In the previous work [9], we proposed a new minimum distance based precoder, named as Neighbor-$d_{\min}$. Although the expressions of the new precoding strategy are simpler than those of max-$d_{\min}$ precoder, the simulation results confirm a slight BER improvement of the new precoder in comparison with the traditional max-$d_{\min}$ solution. In the present paper, we derive a general parameterized form of the Neighbor-$d_{\min}$ precoder. Thanks to this representation, the optimized solution for three independent data-streams is proposed. The new precoder has only three expressions, which depend on the order of the QAM modulation used at the transmitter.

The paper is organized as follows. In Section 2, we describe the MIMO virtual channel representation and the impact of the number of vectors providing the minimum Euclidean distance on the BER performance. The parameterization of the Neighbor-$d_{\min}$ precoder is presented in Section 3. We propose, in Section 4, three Neighbor-$d_{\min}$ precoding matrices for three-dimensional virtual systems using rectangular QAM-modulations. The performances of our new precoder in terms of minimum distance and bit-error-rate are discussed in Section 5. Finally, the conclusion is given in Section 6.

2. SYSTEM OVERVIEW

2.1 Virtual channel representation

Let us consider a MIMO system with n_T transmit, n_R receive antennas and b independent data-streams over a Rayleigh fading channel. The basic system model is defined by

$$\mathbf{y} = \mathbf{GHFs} + \mathbf{G}\nu, \tag{1}$$

where $\mathbf{s}$ and $\mathbf{y}$ are respectively the $b\times 1$ transmitted and $b\times 1$ received symbols vectors, ν is the $n_R\times 1$ additive Gaussian noise vector, $\mathbf{H}$ is the $n_R\times n_T$ channel matrix, $\mathbf{F}$ is the $n_T\times b$ precoding matrix, and $\mathbf{G}$ is the $b\times n_R$ decoding matrix.

If the full CSI is known at both the transmitter and receiver, the channel can be diagonalized by using the virtual transformation [4]. The precoding and decoding matrices are, then, decomposed as $\mathbf{F} = \mathbf{F}_v\mathbf{F}_d$ and $\mathbf{G} = \mathbf{G}_d\mathbf{G}_v$, and the virtual MIMO channel is represented as

$$\mathbf{y} = \mathbf{G}_d\mathbf{H}_v\mathbf{F}_d\mathbf{s} + \mathbf{G}_d\nu_v, \tag{2}$$

where $\mathbf{H}_v$ is the $b\times b$ virtual channel matrix, $\nu_v = \mathbf{G}_v\nu$ is the $b\times 1$ transformed additive Gaussian noise vector. It is noted that $\mathbf{H}_v$ is a diagonal matrix with diagonal entries σ_i sorted in decreasing order, i.e. $\mathbf{H}_v = \mathrm{diag}(\sigma_1,\ldots,\sigma_b)$.

When an ML detection is considered at the receiver, the decoding matrix $\mathbf{G}_d$ has no effect on the performance. Hence, $\mathbf{G}_d$ is consequently assumed, in this paper, to be an identity matrix of size b. The virtual system model can be now simplified as

$$\mathbf{y} = \mathbf{H}_v\mathbf{F}_d\mathbf{s} + \nu_v. \tag{3}$$

The $b \times b$ precoding matrix $\mathbf{F}_d$ is designed under the power constraint

$$\text{trace}\{\mathbf{F}_d \mathbf{F}_d^*\} = E_s, \tag{4}$$

where E_s is the average transmit power.

2.2 Minimum Euclidean distance based precoder

Let us define N_i as the number of distances $\bar{d}_{ij}$ such that $\bar{d}_{ij} = d_{\min}$. The minimum Euclidean distance $d_{\min}$ of the received constellation is defined by

$$d_{\min}^2 = \min_{s_k, s_l \in S, s_k \neq s_l} \|\mathbf{H}_v \mathbf{F}_d (\mathbf{s}_k - \mathbf{s}_l)\|^2. \tag{5}$$

The average error probability can be simplified as [9]

$$\begin{aligned} P_e &\approx \frac{1}{M} \sum_{i=1}^{M} \text{N}_i\, Q\left(\frac{\bar{d}_{\min}}{2\sqrt{N_0}} \times \sqrt{E_s}\right) \\ &\approx \text{N}_{d_{\min}}\, Q\left(\frac{\bar{d}_{\min}}{2\sqrt{N_0}} \times \sqrt{E_s}\right), \end{aligned} \tag{6}$$

where M is the number of all possible transmitted vectors $\mathbf{s}$, and $\text{N}_{d_{\min}} = \frac{1}{M}\sum_{i=1}^{M} \text{N}_i$. It is obvious that we have to optimize not only the minimum Euclidean distance but also the number of neighbors providing the distance $d_{\min}$ to improve the BER performance of a MIMO system. For this reason, the new precoding strategy considered in this paper is called as Neighbor-$d_{\min}$ precoder.

3. PARAMETERIZATION OF THE NEIGHBOR-$d_{\min}$ PRECODING MATRIX

Our objective is to parameterize the precoding matrix $\mathbf{F}_d$ which satisfies the power constraint. By using a singular value decomposition (SVD), the matrix $\mathbf{F}_d$ can be factorized as

$$\mathbf{F}_d = \mathbf{A} \mathbf{\Sigma} \mathbf{B}^*, \tag{7}$$

where $\mathbf{A}$ and $\mathbf{B}^*$ are $b \times b$ unitary matrices, and $\mathbf{\Sigma}$ is a $b \times b$ diagonal matrix with nonnegative real numbers on the diagonal. $\mathbf{\Sigma}$ can be regarded as a scaling matrix, whereas $\mathbf{A}$ and $\mathbf{B}^*$ can be viewed as rotation matrices.

It is noted that the form of the precoding matrix $\mathbf{F}_d$ depends on the channel characteristics. The authors in [10] showed that we can find a precoder $\mathbf{F}_d$ which do not contain the rotation matrix $\mathbf{A}$ such that performance function is not changed.

Proposition 1: If $\mathbf{A}$ is assumed to be an identity matrix, the Euclidean distances provided by two any difference vectors are kept equal by changing only the scaling matrix $\mathbf{\Sigma}$ and retaining the rotation matrix $\mathbf{B}^*$.

Proof: see Appendix A.

The numerical approach shows that the optimized constellation at the receiver is always obtained when some difference vectors provide the minimum Euclidean distances. According to the proposition above, we can conclude that not only the complexity of the optimization but also the number of precoding expressions is reduced if the matrix $\mathbf{A}$ has no influence on the precoding matrix. The parameterized form of the Neighbor-$d_{\min}$ precoder is then

$$\mathbf{F}_d = \mathbf{\Sigma} \mathbf{B}^*. \tag{8}$$

The power constraint in (4) can be rewritten as

$$\text{trace}\{\mathbf{F}_d \mathbf{F}_d^*\} = \text{trace}\{\mathbf{\Sigma}\mathbf{\Sigma}^*\} = E_s. \tag{9}$$

This power constraint is then replaced by the following decomposition

$$\mathbf{\Sigma} = \sqrt{E_s}\, \text{diag}\{\cos\psi_1, \sin\psi_1 \cos\psi_2, .., \sin\psi_1 \sin\psi_2 \sin\psi_{b-1}\}. \tag{10}$$

Theorem: Any matrix $\mathbf{B}^*$, which belongs to the b-dimensional unitary matrix group $U(b)$, can be factorized into an ordered product of $2b-1$ matrices of the following form [11]

$$\mathbf{B}^* = \mathscr{D}_1^{b-1}\, \mathscr{O}_2^{b-2}\, \mathscr{D}_2^{b-2} \ldots \mathscr{O}_{b-1}^{1} \mathscr{D}_{b-1}^{1}\, \mathscr{O}_b \mathscr{D}_b, \tag{11}$$

where $\mathscr{D}_b$ is a diagonal matrix of the form $\mathscr{D}_b = \text{diag}\{e^{i\varphi_1}, \ldots, e^{i\varphi_b}\}$ with $\varphi_i \in [0, 2\pi], i = 1, \ldots, b$ arbitrary phases, $\mathscr{D}_{b-k}^k$ is the same diagonal matrix with first $b-k$ entries equal to unity, i.e. $\mathscr{D}_{b-k}^k = \text{diag}\{1_{b-k}, e^{i\varphi'_1}, \ldots, e^{i\varphi'_k}\}$.

The orthogonal matrices $\mathscr{O}_b$ ($\mathscr{O}_{b-k}^k$) is a product of $b-1$ ($b-k-1$) matrices of the form

$$\mathscr{O}_b = J_{1,2}\, J_{2,3} \ldots J_{b-2,b-1}\, J_{b-1,b} \tag{12}$$

where $J_{i,i+1}$ are $b \times b$ rotation matrices given by

$$J_{i,i+1} = \begin{pmatrix} \mathbf{I}_{i-1} & 0 & 0 & 0 \\ 0 & \cos\theta_i & \sin\theta_i & 0 \\ 0 & -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 0 & \mathbf{I}_{b-i-1} \end{pmatrix}, \tag{13}$$

where I_i is an identity matrix of size i.

Remark: The angles that parameterize $\mathscr{O}_b$ are denoted as $\theta_1, \ldots, \theta_{b-1}$, then the angles of $\mathscr{O}_{b-1}^1$ are $\theta_b, \ldots, \theta_{2b-3}$, etc. and the last angle entering $\mathscr{O}_2^{b-1}$ will be $\theta_{b(b-1)/2}$. The matrix $\mathscr{O}_{b-k}^k$ has the same structure as $\mathscr{O}_b$

$$\mathscr{O}_{b-k}^k = \begin{pmatrix} \mathbf{I}_k & 0 \\ 0 & \mathscr{O}_{b-k} \end{pmatrix}. \tag{14}$$

It is realized that if all the phases entering $\mathbf{B}^*$ are zero, i.e. $\varphi_i = 0$, $i = 1, \ldots, b(b+1)/2$, the received constellation will have less distances providing the minimum distance. The property is explained by the non-rotated received constellation when a rectangular Quadrature Amplitude Modulation is used at the transmitter. Therefore, the unitary matrix $\mathbf{B}^*$ can be parameterized as

$$\mathbf{B}^* = \mathscr{O}_2^{b-2}\, \mathscr{O}_3^{b-3} \ldots \mathscr{O}_{b-1}^{1}\, \mathscr{O}_b. \tag{15}$$

Thanks to this representation, we are now able to find $(b-1)$ angles ψ_i and $b(b-1)/2$ angles θ_i which give the optimal precoder according to the minimum distance criterion. When b increases, not only the number of parameters but also the received constellation size augments dramatically. For this reason, the optimized solution is now available for only small b virtual channels ($b = 2$). In the following section, we point out the Neighbor-$d_{\min}$ precoder for three-dimensional virtual systems using rectangular QAM-modulations.

4. THREE-DIMENSIONAL NEIGHBOR-$d_{\min}$ PRECODER

A three-dimensional virtual channel can be parameterized as

$$\mathbf{H}_v = \rho \begin{pmatrix} \cos\gamma_1 & 0 & 0 \\ 0 & \sin\gamma_1 \cos\gamma_2 & 0 \\ 0 & 0 & \sin\gamma_1 \sin\gamma_2 \end{pmatrix}, \quad (16)$$

where ρ, γ_1 and γ_2 stand respectively for the channel gain and channel angles. It is noted that the diagonal elements of $\mathbf{H}_v$ are sorted in decreasing order, so $0 \leq \gamma_2 \leq \pi/4$ and $\cos\gamma_2 \leq \text{cotan}\,\gamma_1$.

The unitary matrix $\mathbf{B}^*$ in (15) can be now simplified as

$$\mathbf{B}^* = \begin{pmatrix} c_1 & s_1c_2 & s_1s_2 \\ -s_1c_3 & c_1c_2c_3 - s_2s_3 & c_1s_2c_3 + c_2s_3 \\ s_1s_3 & -c_1c_2s_3 - s_2c_3 & -c_1s_2s_3 + c_2c_3 \end{pmatrix}, \quad (17)$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for $i = 1,..,3$. The angle θ_i corresponds to the scaling of the received constellation while the parameter ψ_i of $\mathbf{\Sigma}$ controls the power allocation on each virtual subchannel.

For a rectangular 4^k-QAM modulation, the transmitted symbols belong to the complex set

$$S = \frac{1}{\sqrt{M_s}} \{a + b\,i\,; a - b\,i\,; -a + b\,i\,; -a - b\,i\}, \quad (18)$$

where $M_s = \frac{2}{3}(4^k - 1)$ and $a, b \in (1, 3, \ldots, 2^k - 1)$.

The expression of the precoding matrix which optimizes $d_{\min}$ for three independent data-streams can be classified into three types which enable power on one, two, or three virtual subchannels.

4.1 Precoder $\mathbf{F}_1$

The precoder is available for high dispersive channels, and can be seen as a max-SNR design that pours power only on the strongest virtual subchannel. In fact, this precoder transforms the rectangular 4^k-QAM signals on three virtual subchannels into a rectangular 4^{3k}-QAM on the first subchannel. The optimized precoding matrix is given by

$$\mathbf{F}_1 = \sqrt{\frac{E_s}{M_1}} \begin{pmatrix} 4^k & 2^k & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (19)$$

where $M_1 = 16^k + 4^k + 1$. The optimized $d_{\min}$ is provided by the difference vector $\frac{1}{\sqrt{M_s}}[0\,0\,2]^T$, and defined by

$$d^2_{\mathbf{F}_1} = \frac{4}{M_sM_1} E_s\rho^2 \cos^2\gamma_1. \quad (20)$$

Although the distance is inferior to the minimum distance obtained by SNR-like max-$d_{\min}$ precoder [5], it has less neighbors providing the distance $d_{\min}$ (4 compared to 5 vectors of SNR-like max-$d_{\min}$ design).

4.2 Precoder $\mathbf{F}_2$

The optimized precoder which enables power on first and second virtual subchannels ($\psi_2 = 0$) may have many expressions. To simplify the form of $\mathbf{F}_2$, we present, herein, the most important expression of $\mathbf{F}_2$. The expression is available when there is a large dispersion between the two first subchannels and the third subchannel. For rectangular QAM modulations, a numerical approach shows that the minimum distance is provided by five difference vectors: $\check{\mathbf{x}}_1 = \frac{1}{\sqrt{M_s}}[0,2,0]^T$, $\check{\mathbf{x}}_2 = \frac{1}{\sqrt{M_s}}[0,2(k\text{-}1),\text{-}2]^T$, $\check{\mathbf{x}}_3 = \frac{1}{\sqrt{M_s}}[0,2k,\text{-}2]^T$, $\check{\mathbf{x}}_4 = \frac{1}{\sqrt{M_s}}[2,\text{-}2(M_2\text{-}k\text{+}1),2(k\text{-}1)]^T$, and $\check{\mathbf{x}}_5 = \frac{1}{\sqrt{M_s}}[2,\text{-}2M_2,2k]^T$, where $M_2 = 2^k - 1$.

Let us note $d^2_{\check{x}_i}$ as the corresponding distance of $\check{\mathbf{x}}_i$ with $i = 1,..,5$. By solving the system of equations $d^2_{\check{x}_1} = d^2_{\check{x}_2} = d^2_{\check{x}_3} = d^2_{\check{x}_4} = d^2_{\check{x}_5}$, we obtain all constant angles of the matrix $\mathbf{B}^*$ (confirmed by Proposition 1). The optimized angles (in radians) of $\mathbf{B}^*$ are described in Tab. 1, while the angle ψ_1 which depends on the channel angles γ_1 and γ_2 is defined by

$$\psi_{1|(\gamma_1,\gamma_2)} = \text{atan}\,\frac{\tan\left(\psi_{1|(\pi/4,0)}\right)}{\tan\gamma_1 \cos\gamma_2}. \quad (21)$$

Modulation	θ_1	θ_2	θ_3	$\psi_{1\|(\pi/4,0)}$
4-QAM	0.5083	0.1753	0.9951	0.5066
16-QAM	0.6155	0.7854	0.3876	0.7227
64-QAM	0.5538	1.0216	0.2229	0.8433
256-QAM	0.6690	1.2490	0.0977	0.6331

Table 1: Optimized angles for the precoder $\mathbf{F}_2$

The minimum distance is provided by the difference vector $\frac{1}{\sqrt{M_s}}[0\,2\,0]^T$, and given by

$$d^2_{\mathbf{F}_2} = \kappa\,\frac{2E_s\rho^2}{M_s(2M_2 + 4 - k)}, \quad (22)$$

where κ depends on γ_1 and γ_2 and is defined in (29).

4.3 Precoder $\mathbf{F}_3$

The Neighbor-$d_{\min}$ precoder which pours power on all subchannels also has many expressions. Each expression is available for different variations of the transmit channel. We present, herein, a general precoding matrix for all rectangular QAM-modulations. For every precoder has the form like (8), this precoder provides the highest minimum distance when the channel is small dispersive. The matrix $\mathbf{B}^*$ is then defined by

$$\mathbf{B}^* = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \text{-}1 & \frac{1-\sqrt{3}}{2} & \frac{1+\sqrt{3}}{2} \\ 1 & \frac{-1-\sqrt{3}}{2} & \frac{-1+\sqrt{3}}{2} \end{pmatrix}. \quad (23)$$

By equalizing three difference distances provided by $\check{\mathbf{x}}_1 = \frac{1}{\sqrt{M_s}}[0,2,0]^T$, $\check{\mathbf{x}}_2 = \frac{1}{\sqrt{M_s}}[0,0,2]^T$, and $\check{\mathbf{x}}_3 = \frac{1}{\sqrt{M_s}}[0,2,\text{-}2]^T$, we obtain

$$\begin{cases} \psi_2 = \text{atan}\,\frac{1}{\tan\gamma_2} \\ \psi_1 = \text{atan}\,\frac{1}{2\tan\gamma_1 \cos\gamma_2 \cos\psi_2} \end{cases} \quad (24)$$

The distance $d_{\min}$ obtained by $\mathbf{F}_3$ is then

$$d^2_{\mathbf{F}_3} = \frac{8E_s\rho^2\cos^2\gamma_1 \sin^2\gamma_1 \cos^2\gamma_2 \sin^2\gamma_2}{4\sin^2\gamma_1 \cos^2\gamma_2 \sin^2\gamma_2 + \cos^2\gamma_1 \sin^2\gamma_2 + \cos^2\gamma_1 \cos^2\gamma_2}. \quad (25)$$

Fig. 1 plots the received constellation provided by the precoder $\mathbf{F}_3$ in the case of 4-QAM. One should note that whenever two received vectors are close on one virtual subchannel, they are distant on the others (e.g. A and B).

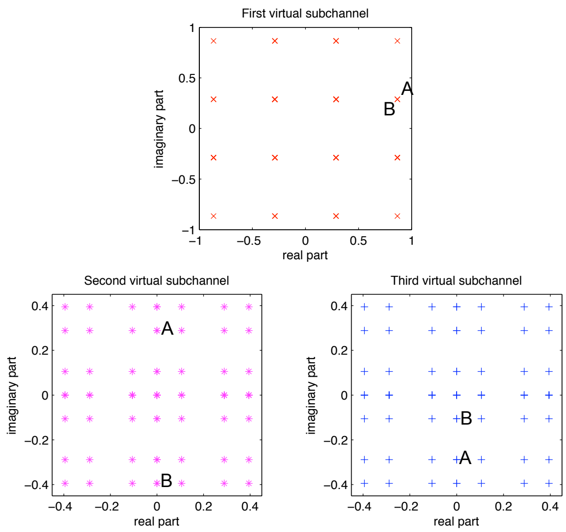

Figure 1: Received constellations provided by precoder $\mathbf{F}_3$ for QPSK modulation.

5. SIMULATION RESULTS

5.1 Range of definition

To improve the BER performance of a MIMO system, we can choose from the three precoding matrices above the precoder which provides the highest minimum Euclidean distance. For a given modulation order, by comparing the three minimum distances in (20), (22), and (25), we obtain the range of definition for each precoder.

The range of definition for QPSK is shown in Fig. 2. It is observed that when the modulation order increases, the normalized minimum distances ($d_{\min}/\sqrt{4E_s\rho^2/M_s}$) provided by $\mathbf{F}_1$ and $\mathbf{F}_2$ are decreased. In other words, two precoder $\mathbf{F}_1$ and $\mathbf{F}_2$ are less used for higher order modulations (the range of definition changes following the arrows in Fig. 2).

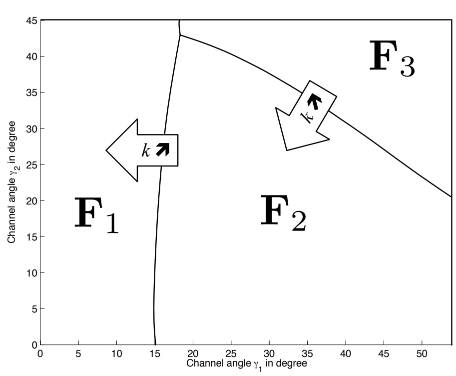

Figure 2: Range of definition for the three precoders $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ using a QPSK modulation. The arrows represent the evolution of the borders when the modulation order increases.

5.2 Performance of Neighbor-$d_{\min}$ precoder

Thanks to the rectangular constellation (see Fig. 1), our new precoder not only optimizes the minimum Euclidean distance but also has less neighbors which provide the distance $d_{\min}$. The normalized minimum distance of the Neighbor-$d_{\min}$ and other precoders are illustrated in Fig. 3. For diagonal precoders, the transmit power is large enough to be allocated on all virtual subchannels. It is observed that the minimum distance provided by the Neighbor-$d_{\min}$ precoder is better than those of WaterFilling, max-$\lambda_{\min}$ [3] and MMSE [2]. Furthermore, unlike diagonal precoders, the minimum distance of Neighbor-$d_{\min}$ precoder is much superior to zero if the virtual channels are large dispersive. When the channels are small dispersive, the minimum distance provided by max-$\lambda_{\min}$ is better than MMSE and Waterfilling but is really outperformed by our new precoder.

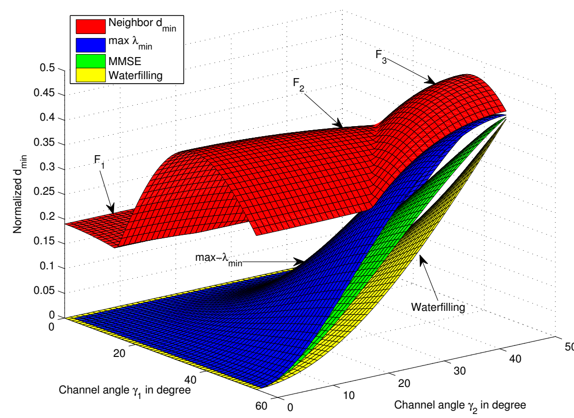

Figure 3: Normalized minimum distance for QPSK.

Let us consider a MIMO-OFDM system with $n_T = 4$ transmit antennas and $n_R = 3$ receive antennas. The transmit channel is Rayleigh fading and the noise is additive white Gaussian. Due to the improvement of the minimum distance and the number of neighbors providing $d_{\min}$, a large enhancement of BER performance is expected. Fig. 4 illustrates the BER performance with respect to SNR for QPSK modulation. It is obvious that the Neighbor-$d_{\min}$ precoder has a significant BER enhancement compared to diagonal precoders. A gain of about 8 dB is observed (at high SNR) in comparison with the SNR-like max-$d_{\min}$ precoder [5].

6. CONCLUSION

We presented, herein, a new precoding strategy which considers not only the minimum Euclidean distance in the received constellation but also the number of neighbors providing $d_{\min}$. The general parameterization of the new precoder is described in the first part of this paper. By using a singular value decomposition, the precoding matrix can be factorized as the product of a scaling matrix $\mathbf{\Sigma}$ and two rotation matrices $\mathbf{A}$ and $\mathbf{B}^*$. To reduce the complexity of the optimization and the number of precoding expressions, the unitary matrix $\mathbf{A}$ is assumed to be an identity matrix. The unitary $\mathbf{B}^*$ is then factorized into a product of $b-1$ matrices which do not contain the phase parameters. This parameterized form can reduce the number of difference vectors providing the mini-

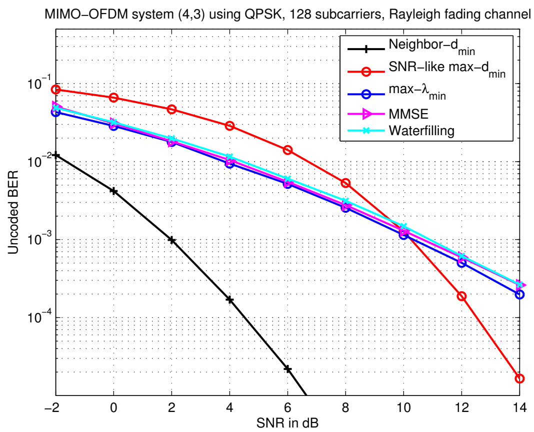

Figure 4: Uncoded BER for MIMO-OFDM system (4,3) using QPSK modulation.

mum distance when a rectangular QAM is considered at the transmitter.

Due to this parameterization, the general Neighbor-$d_{\min}$ precoder for three-dimensional virtual systems using rectangular QAM-modulations is presented. The proposed precoder has three different expressions which enable power on one, two, and three virtual subchannels, respectively. It is shown that the new max-$d_{\min}$ precoder offers a significant BER improvement in comparison with traditional precoding strategies such as max-$\lambda_{\min}$, water-filling, and minimizing the mean square error. Furthermore, the distribution of three precoders depends on the virtual channel characteristics. The more dispersive the virtual subchannels, the less we use the precoder $\mathbf{F}_1$ and $\mathbf{F}_2$.

A. PROOF OF PROPOSITION 1

Let us denote $\breve{\mathbf{a}}_1$, $\breve{\mathbf{a}}_2$, as two difference vectors which have the same Euclidean distances. These Euclidean distances are given by

$$\begin{cases} d^2_{\breve{\mathbf{a}}_1|\mathbf{H}_v} = \|\mathbf{H}_v\mathbf{\Sigma}\mathbf{B}\breve{\mathbf{a}}_1\|^2 \\ d^2_{\breve{\mathbf{a}}_2|\mathbf{H}_v} = \|\mathbf{H}_v\mathbf{\Sigma}\mathbf{B}\breve{\mathbf{a}}_2\|^2 \end{cases} \quad (26)$$

One should note that $\mathbf{\Sigma}$ is a diagonal matrix with real nonnegative elements, i.e. $\mathbf{\Sigma} = \mathrm{diag}(\phi_1,...,\phi_b)$. When the channel varies from $\mathbf{H}_v = \mathrm{diag}(\sigma_1,...,\sigma_b)$ to $\hat{\mathbf{H}}_v = \mathrm{diag}(\hat{\sigma}_1,...,\hat{\sigma}_b)$, the two distances above can be kept equal by changing only the values of ϕ_i, $i = 1,..,b$. Indeed, we define the diagonal matrix $\hat{\mathbf{\Sigma}}$ with real nonnegative elements such that

$$\hat{\phi}_i\hat{\sigma}_i = \kappa\,\phi_i\sigma_i, \quad (27)$$

where κ is a constant. By substituting ϕ_i into the power constraint in (10), we get

$$\sum_{i=1}^{n}\hat{\phi}_i^2 = \kappa^2\sum_{i=1}^{n}\phi_i^2\left(\frac{\sigma_i}{\hat{\sigma}_i}\right)^2 = E_s \quad (28)$$

or

$$\kappa = \sqrt{\frac{E_s}{\sum_{i=1}^{n}\phi_i^2\sigma_i^2/\hat{\sigma}_i^2}}. \quad (29)$$

The Euclidean distance provided by $\breve{\mathbf{a}}_1$ is then

$$\begin{aligned} d^2_{\breve{\mathbf{a}}_1|\hat{\mathbf{H}}_v} &= \|\hat{\mathbf{H}}_v\hat{\mathbf{\Sigma}}\mathbf{B}\breve{\mathbf{a}}_1\|^2 \\ &= \|\kappa\mathbf{H}_v\mathbf{\Sigma}\mathbf{B}\breve{\mathbf{a}}_1\|^2 \\ &= \kappa^2 d^2_{\breve{\mathbf{a}}_1|\mathbf{H}_v}. \end{aligned}$$

Similarly, we get

$$d^2_{\breve{\mathbf{a}}_2|\hat{\mathbf{H}}_v} = \kappa^2 d^2_{\breve{\mathbf{a}}_2|\mathbf{H}_v}.$$

Since $d^2_{\breve{\mathbf{a}}_1|\mathbf{H}_v} = d^2_{\breve{\mathbf{a}}_2|\mathbf{H}_v}$, we have $d^2_{\breve{\mathbf{a}}_1|\hat{\mathbf{H}}_v} = d^2_{\breve{\mathbf{a}}_2|\hat{\mathbf{H}}_v}$. Consequently, two any difference distances can be kept equal by changing only the matrix $\mathbf{\Sigma}$.

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