NEW TRENDS IN PASSIVE LOCALIZATION BY MULTIARRAY NETWORK

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ABSTRACT

We focus on the problem of passive localization of radio-sources, solved by means of several separated multiarrays (base stations) linked with a central process unit. Traditionally, the procedure relies on two distinct steps: intermediate parameters are first estimated (like angles-of-arrival or times-of-arrival), then the position is deduced thanks to these transmitted intermediate parameters. New strategies estimating the position directly by considering the whole base station network as a unique sensor network appeared recently and offered promising results. To the best of our knowledge, the characterization of their performances has not yet been established in a deterministic signal context. In this article the model is developed and the corresponding Cramer-Rao bound is expressed. This bound represents a useful tool to quantify what new one step approaches can offer in terms of optimal performances. Numerical study compare here the proposed bound to the performances of traditional 2 steps and recent one step approaches. We show that for one source existing techniques performs well and equivalently, but the two sources case underline the gap between 2 steps and 1 step approaches. Moreover, we underline the role played by the time-bandwidth product.

1. INTRODUCTION

Passive emitters geolocalization has always received great attention by the signal processing community over the last decades. This problem is commonly solved by means of several arrays of sensors (base stations) linked with a central process unit (CPU). We focus here on the line-of-sight (LOS) context.

Traditionally, the source localization relies on two steps [2]: intermediate parameters, often called measurements (for example the angles-of-arrival (AOAs) as considered in this article) are first estimated on each base station separately. In a second step, the position is then deduced thanks to a localization algorithm (see [9, 6] for example) using all transmitted intermediate parameters at the CPU.

This common strategy suffers from limitations and is clearly suboptimal [2]. First, solving the problem by means of a multiple step strategy is often suboptimal [11]. As the first step does not take into account the fact that the received signals at the stations come from the same emitters this strategy is suboptimal. Moreover in passive multi-emitter context, the problem is ambiguous since one has to identify among all available measurements, which subset of measurements characterize each source, leading to "data association" problems [11]. Moreover, the performance and the number of resolvable emitters on each station are locally limited by the number of sensors of the considered station. This is the reason why an approach based on a one step strategy, gathering the received signals of all available stations, that directly provides the source location, appears of great interest.

Recent algorithms [1, 3] estimate the position starting directly from all the received signals collected on all base stations in a direct "1 step" procedure. The direct position determination (DPD) [1] is based on a filter banks, whereas [3] relies on a alternative spatio-temporal approach. For such processing all received signals should be available directly at the CPU in order to compute the covariance matrix of the whole multiarray network. As it imposes stronger practical requirement, it underlines the particular need of a performance characterization, in order to quantify the gain compared to conventional strategies that require less technological means. For that, the Cramer-Rao bound [10] appears to be a useful benchmark to quantify their optimality.

The comparison between one step and two steps procedures, in the location strategies attracts much interest recently [2, 4]. The stochastic CRB for passive localization by multiarray network for the wideband signal case can be found in [7] and in [5] for the multiple sources case. In [1] the CRB was also provided for gaussian sources and under the assumption that the signals at the output of the Fourier Transform are uncorrelated. To the best of our knowledge, the general case of deterministic wideband signals has never been treated. This paper proposes a general CRB for passive localization of multiple sources in a deterministic signal context and to compare it with the main existing algorithms.

The outline of the paper lies as follows : the signal model is introduced in section 2. Section 3 focuses on the CRB calculation. Section 4 presents some numerical results that underline the potential gain compared to conventional strategies.

2. SIGNAL MODELING AND PROBLEM FORMULATION

We focus on the problem of locating multiple emitters on L stations composed of N_l $(1 \leq l \leq L)$ sensors. Let us denote $N = \sum_l N_l$ the total amount of sensors. Let $\mathbf{x}_l(t)$ be the observation vector of the lth station, and let us consider M emitters whose unknown signals (complex envelopes) are denoted $s_m(t)$ $(1 \leq m \leq M)$. Assuming classically that the signals are narrowband on each station, we have

$$\mathbf{x}_l(t) = \sum_{m=1}^{M} \rho_{l,m} \mathbf{a}_l(\theta_l(\mathbf{p}_m)) s_m(t - \tau_l(\mathbf{p}_m)) + \mathbf{n}_l(t), \quad (1)$$

where $\mathbf{p}_m$ denotes the $D \times 1$ coordinates vector of the mth emitter. The attenuation $\rho_{l,q}$ is an unknown complex parameter standing for the channel effect. The steering vector $\mathbf{a}_l(\theta_l(\mathbf{p}))$ is the sensor response of the station l which norm

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is $\sqrt{N_l}$. It depends on the angle of arrival on the lth station θ_l, seen here as a function of the position $\mathbf{p}$ directly. For notational convenience we will consider $\mathbf{a}_l(\mathbf{p}) \triangleq \mathbf{a}_l(\theta_l(\mathbf{p}))$. The delay $\tau_l(\mathbf{p}_m)$ is the relative time of arrival of the mth signal on the lth station, the origin being arbitrary chosen on the first station

$$\tau_l(\mathbf{p}_m) \triangleq \frac{||\mathbf{p}_m - \mathbf{p}(l)|| - ||\mathbf{p}_m - \mathbf{p}(1)||}{c}, \quad (2)$$

where $\mathbf{p}(l)(1 \leq l \leq L)$ is the coordinates vector of the lth station.

2.1 Data model

Denoting $\{s_m(\omega_k), 1 \leq k \leq T\}$ the discrete Fourier coefficients of the signal $s_m(t)$ over T samples we have :

$$s_m(t) = \frac{1}{T}\sum_{k=1}^{T} s_m(\omega_k)e^{j\omega_k t}, \; 1 \leq t \leq T, \quad (3)$$

with

$$\omega_k = 2\pi\frac{k-1}{T}. \quad (4)$$

Assuming now that T is large enough so that we have

$$\left|\frac{F_e}{T}\tau_l(\mathbf{p}_m)\right| \ll 1, \quad (5)$$

where F_e is the sampling frequency, then we can make the following narrowband assumption:

$$s_m(t - \tau_l(\mathbf{p}_m)) \approx \frac{1}{T}\sum_{k=1}^{T} s_m(\omega_k)e^{j\omega_k t}e^{-j\omega_k\tau_l(\mathbf{p}_m)}. \quad (6)$$

Hence, we have

$$\mathbf{x}_l(t) = \frac{1}{T}\sum_{m=1}^{M}\rho_{l,m}\mathbf{a}_l(\mathbf{p}_m)\sum_{k=1}^{T} e^{-j\omega_k\tau_l(\mathbf{p}_m)}s_m(\omega_k)e^{j\omega_k t} + \mathbf{n}_l(t) \quad (7)$$

and considering the following stacked vector

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_1^T(t) & \dots & \mathbf{x}_L^T(t) \end{bmatrix}^T, \quad (8)$$

we have

$$\mathbf{x}(t) = \frac{1}{T}\sum_{m=1}^{M}\sum_{k=1}^{T}\mathbf{u}_k(\mathbf{p}_m,\boldsymbol{\rho}_m)s_m(\omega_k)e^{j\omega_k t} + \mathbf{n}(t), \; 1 \leq t \leq T \quad (9)$$

with

$$\mathbf{u}_k(\mathbf{p}_m,\boldsymbol{\rho}_m) = \frac{\sqrt{N}}{\sqrt{\sum_l N_l|\rho_{l,m}|^2}}\begin{bmatrix} \rho_{1,m}\mathbf{a}_1(\mathbf{p}_m)e^{-j\omega_k\tau_1(\mathbf{p}_m)} \\ \vdots \\ \rho_{L,m}\mathbf{a}_L(\mathbf{p}_m)e^{-j\omega_k\tau_L(\mathbf{p}_m)} \end{bmatrix}. \quad (10)$$

So that $\mathbf{u}_k(\mathbf{p}_m,\boldsymbol{\rho}_m)$ has a constant norm equal to N. We can then define the following Signal-to -noise ratio SNR):

$$SNR_m = 10log_{10}\left(\frac{\sum_t |s_m(t)|^2}{\sigma^2}\right) \quad (11)$$

$$= 10log_{10}\left(\frac{\sum_k |s_m(\omega_k)|^2}{T\sigma^2}\right). \quad (12)$$

Let $\psi_{l,m}$ and $\phi_{l,m}$ be the modulus and phase of $\rho_{l,m} = \psi_{l,m}e^{j\phi_{l,m}}$, respectively. Let us denote $\bar{s}$ and $\check{s}$ the real and imaginary part of the complex number s, respectively. The unknown parameter vector is then

$$\eta = \begin{bmatrix} \mathbf{p}_1^T & \dots & \mathbf{p}_M^T & \psi_1^T & \dots & \psi_M^T & \phi_1^T & \dots & \phi_M^T & \mathbf{s}^T \end{bmatrix}^T, \quad (13)$$

with

$$\psi_m = \begin{bmatrix} \psi_{1,m} & \dots & \psi_{L,m} \end{bmatrix}^T, \quad (14)$$

$$\phi_m = \begin{bmatrix} \phi_{1,m} & \dots & \phi_{L,m} \end{bmatrix}^T, \quad (15)$$

$$\mathbf{s} = \begin{bmatrix} \mathbf{s}^T(\omega_1) & \dots & \mathbf{s}^T(\omega_T) \end{bmatrix}^T, \quad (16)$$

$$\mathbf{s}(\omega_k) = \begin{bmatrix} \mathbf{s}_1^T(\omega_k) & \dots & \mathbf{s}_M^T(\omega_k) \end{bmatrix}, \quad (17)$$

$$\mathbf{s}_m(\omega_k) = \begin{bmatrix} \bar{s}_m(\omega_k) & \check{s}_m(\omega_k) \end{bmatrix}. \quad (18)$$

As we consider the deterministic signal case the Fourier coefficients are now part of the unknown parameter vector. Denoting

$$\mathbf{m}(\eta,t) = \frac{1}{T}\sum_{m=1}^{M}\sum_{k=1}^{T}\mathbf{u}_k(\mathbf{p}_m,\boldsymbol{\rho}_m)s_m(\omega_k)e^{j\omega_k t} \quad (19)$$

the model can be finally written as :

$$\mathbf{x}(t) = \mathbf{m}(\eta,t) + \mathbf{n}(t), \; 1 \leq t \leq T. \quad (20)$$

2.2 Problem formulation

The problem lies in estimating the position $\mathbf{p}_m$ in (9) given T samples of $\mathbf{x}(t)$. In order to evaluate the Cramer-rao bound of the considered estimation problem, we assume that :

- $\mathbf{n}_l(t)$ is an additive centered Gaussian noise, assumed temporally white and circular, such that

$$E\left[\mathbf{n}_l(t)\mathbf{n}_l^H(t)\right] = \sigma^2\mathbf{I}_{N_l}, \quad (21)$$

$$E\left[\mathbf{n}_l(t)\mathbf{n}_{l'}^T(s)\right] = \mathbf{0}_{N_l \times N_{l'}}, \quad (22)$$

  where $E[.]$ denotes the mathematical expectation. We also denote $\mathbf{I}_N$ the $N \times N$ identity matrix and $\mathbf{0}_{N \times M}$ a $N \times M$ matrix composed of zeros. δ stands for the Kronecker symbol :

$$\delta_{mn} = \begin{cases} 1 & if\; m = n \\ 0 & else \end{cases}. \quad (23)$$

  Since it is well-known that the noise variance estimation does not influences the performances of the other parameters for such a deterministic signal model [8], σ^2 does not appear in the unknown parameter vector η.
- The number of sources is assumed to be known.
- The signals are assumed deterministic.

3. CRB CALCULATION

The CRB is a useful benchmark that provide a lower bound on the variance of unbiased estimators. For its calculation we first need the expression of the Fisher information matrix (FIM) :

$$\mathbf{J}(\eta) = E\left[\frac{\partial L(\mathbf{x}|\eta)}{\partial\eta}\left(\frac{\partial L(\mathbf{x}|\eta)}{\partial\eta}\right)^T\right], \quad (24)$$

where $L(\mathbf{x}|\eta)$ is the log-likelihood of the considered problem that depends on the data $\mathbf{x}$ and the unknown parameter vector η. In our case, the observation vector is Gaussian with a parametrized mean and its covariance matrix is $\sigma^2 \mathbf{I}_{N\times N}$. According to a well known formula ([10] equation (8.34), p.927) we directly have the expression of the FIM :

$$[\mathbf{J}(\eta)]_{ij} = \frac{2}{\sigma^2}\sum_{t=1}^{T} Re\left\{\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \eta_i}\frac{\partial \mathbf{m}(\eta,t)}{\partial \eta_j}\right\}, \qquad (25)$$

where $Re\{.\}$ stands for the real part. In the sequel we will denote α a $M\times 1$ vector whose mth element α_m can be either x_m, $y_m, \psi_{l,m}$ or $\phi_{l,m}$ $(1 \leq m \leq M)$ where $\mathbf{p}_m = (x_m, y_m)^T$ denotes the emitter coordinates vector. For the sake of brevity we will also denote $\mathbf{u}_{k,m} \triangleq \mathbf{u}_k(\mathbf{p}_m, \rho_m)$. Straightforward calculation leads to

$$\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \alpha_m} = \frac{1}{T}\sum_{k=1}^{T}\frac{\partial \mathbf{u}_{k,m}}{\partial \alpha_m} s_m(\omega_k) e^{j\omega_k t} \qquad (26)$$

$$\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \bar{s}_m(\omega_k)} = \frac{1}{T}\mathbf{u}_{k,m} e^{j\omega_k t} \qquad (27)$$

$$\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \breve{s}_m(\omega_k)} = \frac{1}{T} j\mathbf{u}_{k,m} e^{j\omega_k t} \qquad (28)$$

But since we have:

$$\sum_{t=1}^{T} e^{j(\omega_{k'}-\omega_k)t} = T\delta_{kk'}, \qquad (29)$$

we can write the following FIM blocks:

$$\sum_{t=1}^{T} Re\left\{\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \alpha_m}\frac{\partial \mathbf{m}(\eta,t)}{\partial \alpha_n}\right\} = \frac{1}{T} Re\left\{\sum_{k=1}^{T}\frac{\partial \mathbf{u}_{k,m}^H}{\partial \alpha_m}\frac{\partial \mathbf{u}_{k,n}}{\partial \alpha_n} \times s_m^*(\omega_k) s_n(\omega_k)\right\} \qquad (30)$$

$$\sum_{t=1}^{T} Re\left\{\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \alpha_m}\frac{\partial \mathbf{m}(\eta,t)}{\partial \bar{s}_n(\omega_k)}\right\} = Re\left\{\frac{\partial \mathbf{u}_{k,m}^H}{\partial \alpha_m}\mathbf{u}_{k,n} s_m^*(\omega_k)\right\}, \qquad (31)$$

$$\sum_{t=1}^{T} Re\left\{\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \alpha_m}\frac{\partial \mathbf{m}(\eta,t)}{\partial \breve{s}_n(\omega_k)}\right\} = -Im\left\{\frac{\partial \mathbf{u}_{k,m}^H}{\partial \alpha_m}\mathbf{u}_{k,n} s_m^*(\omega_k)\right\}, \qquad (32)$$

$$\sum_{t=1}^{T} Re\left\{\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \bar{s}_m(\omega_k)}\frac{\partial \mathbf{m}(\eta,t)}{\partial \bar{s}_n(\omega_{k'})}\right\} = Re\left\{\mathbf{u}_{k,m}^H \mathbf{u}_{k',n}\right\}\delta_{kk'}, \qquad (33)$$

$$\sum_{t=1}^{T} Re\left\{\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \breve{s}_m(\omega_k)}\frac{\partial \mathbf{m}(\eta,t)}{\partial \breve{s}_n(\omega_{k'})}\right\} = Re\left\{\mathbf{u}_{k,m}^H \mathbf{u}_{k',n}\right\}\delta_{kk'}, \qquad (34)$$

$$\sum_{t=1}^{T} Re\left\{\frac{\partial \mathbf{m}(\eta,t)^H}{\partial \breve{s}_m(\omega_k)}\frac{\partial \mathbf{m}(\eta,t)}{\partial \bar{s}_n(\omega_{k'})}\right\} = Im\left\{\mathbf{u}_{k,m}^H \mathbf{u}_{k',n}\right\}\delta_{kk'}. \qquad (35)$$

$Im\{.\}$denotes here the imaginary part. Then the CRB is deduced by the inverse of the FIM. This CRB can then be used as a benchmark since it represents the best achievable performance for the passive geolocalization of multiple sources in a deterministic signal context.

$$\mathbf{CRB}(\eta) = \mathbf{J}^{-1}(\eta). \qquad (36)$$

4. GEOLOCALIZATION STRATEGIES

We present here three strategies that will be compared to the provided CRB.

4.1 Classical two steps approach

The conventional strategy relies on two steps : in the first step the AOAs are estimated on each station (here thanks to a MUSIC algorithm [10]), the second step estimates the position thanks to a least squares step [9].

4.2 DPD technique

The DPD technique [1] relies on the use of filter banks in order to treat the problem as a narrowband problem. Then an incoherent sum of MUSIC based criteria on each frequency channel is optimized in order to estimate the position directly. The size of the filter bank (unwindowed Fourier Transform) is $K = 4$ in this paper.

4.3 LOST processing

The localization by space-time processing (LOST) [3] relies on the use of a space-time observation vector, where regularly delayed versions of $\mathbf{x}(t)$ are embedded into an extended vector $\mathbf{y}(t) = \begin{bmatrix} \mathbf{x}^T(t) & \ldots & \mathbf{x}^T(t-K-1) \end{bmatrix}^T$ that is used to form a spatio-temporal covariance matrix. The problem can then also be treated as a narrowband problem and the position can be deduced directly. The number of shifts is here $K = 4$.

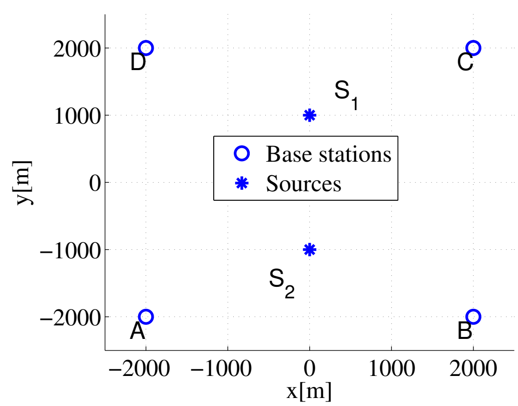

Figure 1: Four base stations A, B, C and D on the corner of a 4×4 km square. The sources are in $S_1(0,1000)$ and $S_2(0,-1000)$, respectively.

5. NUMERICAL RESULTS

In this section we consider four base stations A, B, C and D all composed of a uniform circular array, with a radius equal to 0.5 wavelength and a number of sensor equal to $N_A = N_B = N_C = N_D$=3. We consider two emitters in S_1 and S_2 illustrated by figure 1. The number of samples is $T = 200$. In this section, $\psi_{l,m}$ is generated thanks to a Gaussian random process (mean=1, std=0.2) and $\phi_{l,m}$ is uniformly distributed onto $[0, 2\pi]$. The noise $\mathbf{n}(t)$ is circular white Gaussian. The signals are generated from a white Gaussian process, so that their bandwidth are equal to the sampling frequency F_e, precised each time. The performances are studied trough the root mean square error (RMSE) of the miss distance defined in meters :

$$RMSE = \sqrt{\frac{1}{M_c}\sum_{k=1}^{M_c}(x_m - \hat{x}_{k,m})^2 + (y_m - \hat{y}_{k,m})^2}, \quad (37)$$

where M_c is the number of Monte-Carlo runs and $\hat{x}_{k,m}$ and $\hat{y}_{k,m}$ denote the kth estimation of the true position (x,y) of the emitter. The corresponding CRB is

$$CRB = \sqrt{CRB(x_m) + CRB(y_m)}, \quad (38)$$

where $CRB(x_m)$ and $CRB(y_m)$ denote the CRB of the corresponding parameter.

We denote τ_{max} the maximal time-delay-of-propagation (TDOA) between the two most separated stations. Note that since the provided CRB rely on (5) when the Time-Bandwidth product becomes large compared to the number of samples this bound becomes irrelevant. Nevertheless, for the considered sensor network, if we assume a sampling frequency $F_e = 300kHz$, for typical GSM signals, and taking $T = 100$ samples the maximum value of the Time-Bandwidth product divided by the number of samples $\frac{F_e}{T} \times \tau_{max}$ is nearly 1 percent. Hence, the approximation in (5) seems rather realistic in practical scenarios.

The calculated CRB is compared in this section to the three previously presented algorithms : a conventional two step strategy (denoted AOA) and two recent one step strategies (DPD and LOST).

5.1 Influence of the number of sources

In this section we choose $F_e = 5kHz$, so that the time-bandwidth product, defined as the emitter bandwidth product by the time of propagation across the sensor network, is here equal to 0.2.

On figure 2 we consider only one source. As we can see, all strategies behaves asymptotically very close to the CRB. For low SNR, one step apoproaches outperforms conventionnal two steps approach. Note that DPD technique is already known to be asymptotically equivalent to a conventional strategy for one source [2].

On figure 3 we consider two sources. in that case existing one step strategies are known to perform better than a conventional approach and the theoretical gap becomes obviously larger in a multiple sources context. It show the relevancy of the existing one step approaches in this context, appeared to be close to the optimal performance.

5.2 Influence of the time-bandwidth (TB) product

Now we examine the influence of the time-bandwidth (TB) product. This products plays a great role since its low values indicates when the signals can be assumed narrowband. We compare here the CRB to the empirical performances of previously cited algorithms for different values of the sampling frequency in presence of one source and two sources in figure 4 and 5, respectively.

As we can see on figure 4, for one source when the time-bandwidth product is small all algorithms behaves equivalently very close to the CRB. But when this product increases, as expected it does not change the performances of the traditional algorithms based on AOA estimation. One can

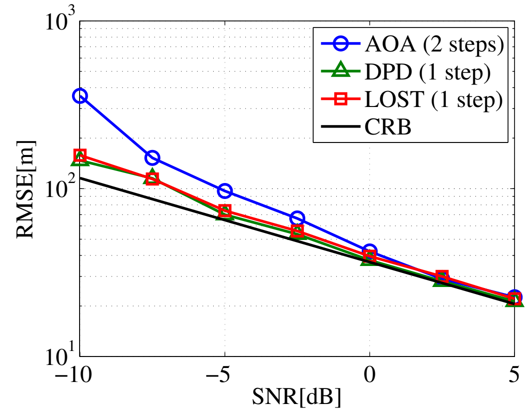

Figure 2: RMSE of the first emitter only, T=200, number of Monte-carlo runs=100, TB product=0.2.

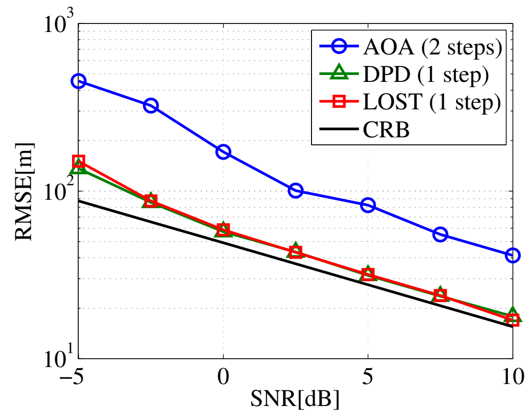

Figure 3: RMSE of the first emitter in presence of a second emitter in S_2, T=200, number of Monte-carlo runs=100, TB product=0.2.

see that the larger the time-bandwidth the lower the CRB, suggesting the fact that the time of arrival plays a bigger role, so that a traditional algorithm that does not exploit the dependency of time delay on the position can't achieve the CRB. For two steps approaches, when this product grows, the narrowband approximation on which they are based on, becomes less and less acceptable leading to a performance breakdown that appears to be stronger for the DPD algorithm than for the LOST technique.

On figure 5 we can see that provided that the TB product is small one step procedures are close to the CRB but when the time-bandwidth product increases new algorithms might be found that can better exploit the TDOA since its influence on the performances becomes stronger. Even for low values of the TB product the traditional approach is clearly suboptimal.

6. CONCLUSION

In this paper we provide the expression of a new deterministic Cramer-Rao bound dedicated to the passive sources localization by multiarray network. The provided CRB can now be used a benchmark in order to quantify what new one step localization procedures can offer compared to the

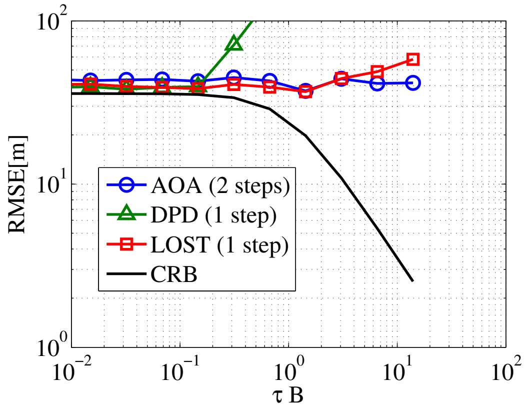

Figure 4: RMSE of the first emitter, SNR=0dB, T=200, number of Monte-carlo runs=100.

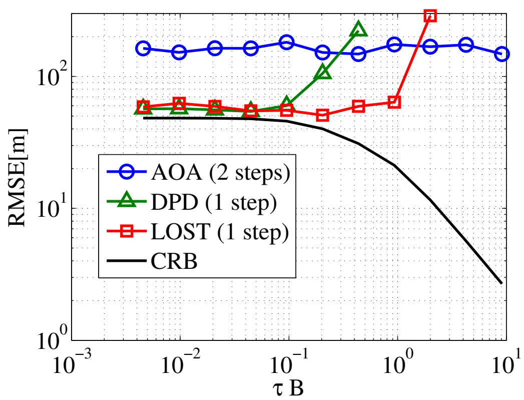

Figure 5: RMSE of the first emitter in presence of a second emitter in S_2, SNR=0dB, T=200, number of Monte-carlo runs=100.

conventional strategies. We showed that for one source all strategies exhibit very close performances but when there is two sources the potential gain increases strongly and existing one step approaches performs better. Finding an optimal and practical exploitation of the original wideband model is still an open question that should attracts interest regarding the potential gain that could be achieved especially when the time-bandwidth product is high.

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