PERFORMANCE ANALYSIS OF WL ALAMOUTI RECEIVERS FOR REAL-VALUED CONSTELLATIONS IN MULTIUSER CONTEXT

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ABSTRACT

Several Interference Cancellation (IC) schemes have been developed during this last decade for wireless networks to mitigate the effect of intra-network interferences, when each user is equipped with multiple antennas and employs Space Time Block Code (STBC) at transmission. However, these IC techniques all require multiple antennas at reception, which remains a challenge at the handset level due to cost and size limitations. In this context, a receiver able to perform Single and Multiple Antenna IC (SAIC/MAIC) for users using real-valued constellations (such as ASK constellations) and Alamouti scheme at transmission has been introduced very recently. The purpose of this paper is to highlight its great interest in multiuser context by analyzing its mechanism and deriving its performance in terms of Signal to Interference plus Noise Ratio (SINR) and Symbol Error Rate (SER).

1. INTRODUCTION

Orthogonal STBC, and the Alamouti scheme [1] in particular, are of particular interest in Multiple-Input Multiple-Output (MIMO) systems since they achieve full spatial diversity over fading channels and are decoded from linear processing at the receiver. Nevertheless, due to the expensive spectral resource, increasing network capacity requires the development of IC techniques allowing several users to share the same spectral resources without impacting the transmission quality. In this context several IC schemes have been developed during this last decade, where each user is equipped with multiple antennas and employs STBC at transmission (see [2–4] and references therein). However, these IC techniques all require multiple antennas at reception, which remains a challenge at the handset level due to cost and size limitations. For this reason, low complexity Single Antenna Interference Cancellation (SAIC) techniques [5–7], operational in GSM handsets, have been developed recently for single antenna and single carrier users using real-valued modulations or complex filtering of real-valued modulations, by using a widely linear (WL) filtering [8] at reception. Extension to multiple antennas at reception is called Multiple Antenna Interference Cancellation (MAIC) technique. As SAIC technology remains of great interest for 4G wireless networks, an extension of this technology to Orthogonal Frequency Division Multiplex transmission using one transmit antenna and the real-valued Amplitude Shift Keying (ASK) modulation has been presented very recently in [9]. Despite of the fact that ASK modulation is less power efficient than a corresponding complex QAM modulation, additional degrees of freedom are available and can be exploited for interference suppression at the receiver. Besides, it has been reported in [10] for DS-CDMA transmission and later in [9] for OFDM links, that transmission using real-valued data symbols with a WL receiver can lead to a higher spectral efficiency in multiuser context than using a complex symbol alphabet with linear receivers. A WL MMSE receiver extending SAIC/MAIC concept to users using real-valued constellations and Alamouti scheme at transmission has been recently introduced in [11, 12]. This receiver may be used for many applications such as 4G communication networks and military ad hoc networks. The mechanism and performances of this WL MMSE receiver have not yet been analyzed. In this paper we first analyze its behavior in the multiuser context, together with the conventional Alamouti receiver [1]. We then derive and analyze its SINR and SER performances in the multiuser context and compare them with the receiver usually used in the literature, highlighting the great interest of this WL MMSE receiver.

2. PROBLEM STATEMENT

2.1 Hypotheses

We consider a radio communication system that employs a real-valued constellation and the well-known Alamouti scheme [1] with $M = 2$ transmit antennas and N receive antennas. We denote by T the symbol period. We assume either flat fading propagation channels and a single-carrier waveform or, equivalently, OFDM with frequency selective propagation channels, then considering the system sub-carrier by sub-carrier after the Discrete Fourier Transform. Assuming in addition the channels invariant over at least two successive symbol periods, $(2n-1)T$ and $2nT$ respectively, the observation vector over these two symbol periods can then be written as:

$$\begin{cases} \mathbf{x}_1(n) = \mu_1 a_{2n-1}\mathbf{h}_1 + \mu_2 a_{2n}\mathbf{h}_2 + \mathbf{b}_1(n) \\ \mathbf{x}_2(n) = -\mu_1 a_{2n}\mathbf{h}_1 + \mu_2 a_{2n-1}\mathbf{h}_2 + \mathbf{b}_2(n) \end{cases}, \quad (1)$$

where: $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ are the $N \times 1$ observation vectors at symbol periods $(2n-1)T$ and $2nT$ respectively, the quantities a_n are i.i.d real-valued random variables corresponding to the transmitted symbols, $\mu_{i\,(i=1,2)}$ is a real scalar which controls the power of the two transmitted signals received by the array of antennas, $\mathbf{h}_{i\,(i=1,2)}$, such that $\mathbb{E}[\mathbf{h}_i^H\mathbf{h}_i] = N$, is the normalized propagation channel vector between transmit antenna i and the receive array of antennas ($\mathbf{A}^H$ is the conjugate transpose of $\mathbf{A}$), $\mathbf{b}_1(n)$ and $\mathbf{b}_2(n)$ are the sampled total noise vector at sample times $(2n-1)T$ and $2nT$ respectively, potentially composed of intra-network interferences, external interferences (not generated by the network itself) and background noise.

All along this paper, $\mathbf{R_v}$ and $\mathbf{C_v}$ are the correlation matrices defined by $\mathbf{R_v} = \mathbb{E}_c[\mathbf{v}\mathbf{v}^H]$, $\mathbf{C_v} = \mathbb{E}_c[\mathbf{v}\mathbf{v}^T]$, where $\mathbb{E}_c(\cdot)$ is the conditional expected value with respect to the channel vectors of the sources and where ^{T} means transpose. Note that, in order to simplify the notations, we may not always mention the dependency in n of the variables.

2.2 Observation models

We first recall the classical observation model; most of Alamouti receivers currently available for IC of intra-network interferences in fact exploit the information contained in the $2N \times 1$ observation vector $\overline{\mathbf{x}}(n)$, defined by $\overline{\mathbf{x}} = [\mathbf{x}_1^T, \mathbf{x}_2^H]^T$ (see e.g. [2, 4]). We introduce the $2N \times 1$ vectors $\mathbf{a}(n) = [a_{2n-1}, a_{2n}]^T$, $\overline{\mathbf{b}} = [\mathbf{b}_1^T, \mathbf{b}_2^H]^T$, $\mathbf{g}_1 = \sqrt{\pi_a/\pi_s}[\mu_1\mathbf{h}_1^T, \mu_2\mathbf{h}_2^H]^T$ and $\mathbf{g}_2 = \sqrt{\pi_a/\pi_s}[\mu_2\mathbf{h}_2^T, -\mu_1\mathbf{h}_1^H]^T$, where $\pi_s = \pi_a(\mu_1^2 + \mu_2^2)/2$, with $\pi_a = \mathbb{E}[a_n^2]$, is the mean power of each useful symbol per receive antenna. Introducing the $2N \times 2$ matrix $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2]$, system (1) can be written in the following classical compact form:

$$\begin{aligned}\overline{\mathbf{x}}(n) &= \sqrt{\pi_s/\pi_a}\,(a_{2n-1}\mathbf{g}_1 + a_{2n}\mathbf{g}_2) + \overline{\mathbf{b}}(n)\\ &= \sqrt{\pi_s/\pi_a}\,\mathbf{G}\mathbf{a}(n) + \overline{\mathbf{b}}(n).\end{aligned} \tag{2}$$

We now present the extended observation model. We first introduce the $2N \times 1$ vectors $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T]^T$, $\mathbf{b} = [\mathbf{b}_1^T, \mathbf{b}_2^T]^T$, $\mathbf{f}_1 = \sqrt{\pi_a/\pi_s}[\mu_1\mathbf{h}_1^T, \mu_2\mathbf{h}_2^T]^T$ and $\mathbf{f}_2 = \sqrt{\pi_a/\pi_s}$ $[\mu_2\mathbf{h}_2^*, -\mu_1\mathbf{h}_1^*]^T$. We then introduce the $4N \times 1$ vectors $\tilde{\mathbf{f}}_1 = [\mathbf{f}_1^T, \mathbf{f}_1^H]^T$, $\tilde{\mathbf{f}}_2 = [\mathbf{f}_2^T, \mathbf{f}_2^H]^T$, $\tilde{\mathbf{b}} = [\mathbf{b}^T, \mathbf{b}^H]^T$ and $\tilde{\mathbf{x}} = [\mathbf{x}^T, \mathbf{x}^H]^T$. With $4N \times 2$ matrix $\tilde{\mathbf{F}} = [\tilde{\mathbf{f}}_1, \tilde{\mathbf{f}}_2]$ extended observation vector $\tilde{\mathbf{x}}(n)$ becomes

$$\begin{aligned}\tilde{\mathbf{x}}(n) &= \sqrt{\pi_s/\pi_a}\big(a_{2n-1}\tilde{\mathbf{f}}_1 + a_{2n}\tilde{\mathbf{f}}_2\big) + \tilde{\mathbf{b}}(n)\\ &= \sqrt{\pi_s/\pi_a}\,\tilde{\mathbf{F}}\mathbf{a}(n) + \tilde{\mathbf{b}}(n).\end{aligned} \tag{3}$$

Models (2) and (3) describe the equivalent reception at time nT_b, where $T_b = 2T$ is the duration of a block of two symbols, of two uncorrelated sources (a_{2n-1} and a_{2n}) by a virtual array of N_e antennas, $N_e = 2N$ for (2) and $N_e = 4N$ for (3). The two sources mentioned are associated with the linearly independent virtual channel vectors $\mathbf{g}_1$ and $\mathbf{g}_2$ (2) and $\tilde{\mathbf{f}}_1$ and $\tilde{\mathbf{f}}_2$ (3) respectively, and corrupted by a total noise.

2.3 Total noise model

We subsequently assume that the total noise $\mathbf{b}(n)$ is composed of P synchronous internal interferers, corresponding to other Alamouti users of the network with the same rectilinear modulation as the useful signal. The channel vector between antenna 1 (resp. 2) of interferer i (with $i = 1, \ldots, P$), and the receiver is denoted by $\mu_{2i+1}\mathbf{h}_{2i+1}$ (resp. $\mu_{2i+2}\mathbf{h}_{2i+2}$), defined similarly to $\mu_1\mathbf{h}_1$ (resp. $\mu_2\mathbf{h}_2$). The vectors $\overline{\mathbf{b}}(n)$ and $\tilde{\mathbf{b}}(n)$ can then be written as

$$\overline{\mathbf{b}}(n) = \sum_{i=1}^{P} \sqrt{\pi_i/\pi_a}\,\mathbf{G}_i\mathbf{e}_i(n) + \overline{\mathbf{b}}_v(n), \tag{4}$$

$$\tilde{\mathbf{b}}(n) = \sum_{i=1}^{P} \sqrt{\pi_i/\pi_a}\,\tilde{\mathbf{F}}_i\mathbf{e}_i(n) + \tilde{\mathbf{b}}_v(n), \tag{5}$$

where $\pi_i = \pi_a(\mu_{2i+1}^2 + \mu_{2i+2}^2)/2$ is the mean power of each interfering symbol per receive antenna for interferer i, and where $\mathbf{e}_i(n) = [e_{i,2n-1}, e_{i,2n}]^T$, $e_{i,n}$ corresponding to the symbols transmitted by interferer i. Similarly to $\mathbf{G}$ and $\tilde{\mathbf{F}}$, $\mathbf{G}_i = [\mathbf{g}_{2i+1}, \mathbf{g}_{2i+2}]$ and $\tilde{\mathbf{F}}_i = [\tilde{\mathbf{f}}_{2i+1}, \tilde{\mathbf{f}}_{2i+2}]$, where vectors $\mathbf{g}_{2i+1}$, $\mathbf{g}_{2i+2}$, $\tilde{\mathbf{f}}_{2i+1}$, $\tilde{\mathbf{f}}_{2i+2}$ are defined similarly to $\mathbf{g}_1$, $\mathbf{g}_2$, $\tilde{\mathbf{f}}_1$, $\tilde{\mathbf{f}}_2$ respectively. Vectors $\overline{\mathbf{b}}_v(n)$ and $\tilde{\mathbf{b}}_v(n)$ respectively correspond to the background noise vectors in $\overline{\mathbf{x}}(n)$ and $\tilde{\mathbf{x}}(n)$. The background noise is supposed circular white Gaussian with variance σ^2.

2.4 Widely Linear Filtering and problem formulation

Time invariant and linear filtering of $\overline{\mathbf{x}}(n)$ and $\tilde{\mathbf{x}}(n)$ are respectively defined by the input-output relations $\mathbf{y}(n) = \overline{\mathbf{w}}^H\overline{\mathbf{x}}(n)$ and $\mathbf{y}(n) = \tilde{\mathbf{w}}^H\tilde{\mathbf{x}}(n)$, where $\overline{\mathbf{w}}$ and $\tilde{\mathbf{w}}$ are $2N \times 1$ and $4N \times 1$ complex vectors respectively. These input-output relations describe what we hereafter call a partially WL and a fully WL filtering of $\mathbf{x}(n)$ respectively.

The purpose of this paper is to analyze the behavior and performance of both partially and fully WL MMSE receivers.

3. MMSE RECEIVERS

3.1 Receivers definition

We first recall in this section the WL MMSE receiver introduced in [11], jointly with the usual WL MMSE receiver used in the literature. The fully WL MMSE (F-WL-MMSE) receiver introduced for interference cancellation in [11] [1] is based on a fully WL MMSE filter, while the Partially WL MMSE (P-WL-MMSE) receiver usually used in the literature is based on a partially WL MMSE filter. The outputs of the P-WL-MMSE and the F-WL MMSE receivers for symbol a_{2n-1} are respectively [11]: $\overline{z}_1(n) = \mathrm{Re}\{\overline{\mathbf{w}}_1^H\overline{\mathbf{x}}(n)\}$ and $\tilde{z}_1(n) = \mathrm{Re}\{\tilde{\mathbf{w}}_1^H\tilde{\mathbf{x}}(n)\}$, where $\overline{\mathbf{w}}_1(n) = \sqrt{\pi_a\pi_s}\,\mathbf{R}_{\overline{\mathbf{x}}}^{-1}\mathbf{g}_1$ and $\tilde{\mathbf{w}}_1(n) = \sqrt{\pi_a\pi_s}\,\mathbf{R}_{\tilde{\mathbf{x}}}^{-1}\tilde{\mathbf{f}}_1$.

Note that in the case of internal interferences, the F-WL-MMSE filter $\tilde{\mathbf{w}}_1$ is collinear to the ML filter $\mathbf{R}_{\tilde{\mathbf{b}}}^{-1}\tilde{\mathbf{f}}_1$ [11]. Hence $\tilde{\mathbf{w}}_1 = k\mathbf{R}_{\tilde{\mathbf{b}}}^{-1}\tilde{\mathbf{f}}_1$, where k is a real constant. Similarly the P-WL-MMSE filter $\overline{\mathbf{w}}_1$ is collinear to $\mathbf{R}_{\overline{\mathbf{b}}}^{-1}\mathbf{g}_1$. Besides, elementary algebraic manipulations give $\tilde{\mathbf{w}}_1 = [\mathbf{w}_1^T, \mathbf{w}_1^H]^T$, hence $\tilde{\mathbf{w}}_1^H\tilde{\mathbf{x}} = 2\mathrm{Re}\{y_1\}$, with $y_1 = \mathbf{w}_1^H\mathbf{x}$.

We also consider the conventional Alamouti receiver [1] as a reference. It corresponds to both the P-WL-MMSE and the F-WL-MMSE receivers in the absence of interference (i.e. for $\mathbf{R}_{\tilde{\mathbf{b}}} = \sigma^2\mathbf{I}$). The output of the conventional Alamouti receiver is then given by $z_1(n) = \mathrm{Re}\{\mathbf{w}_{conv,1}^H\mathbf{x}(n)\}$, where $\mathbf{w}_{conv,1} = \sqrt{\pi_a\pi_s}\,\mathbf{f}_1$.

We have naturally similar results for the outputs z_2, $\overline{z}_2$ and $\tilde{z}_2$ of the receivers for the estimation of symbol a_{2n}.

3.2 Interference rejection capability

As previously explained, the filters introduced are associated with a virtual array of N_e virtual antennas. The degrees of freedom available to reject the interferences contained in the total noise is then $N_e - 2$, as one degree of freedom is used to keep one useful symbol and another degree of freedom is used to reject the other useful symbol, which is an interference for the first one. Besides, each internal interferer i generates 2 interferences, in both $\overline{\mathbf{b}}(n)$ and $\tilde{\mathbf{b}}(n)$, thus we

[1] Note that a similar fully WL MMSE receiver has already been introduced in [13] but for equalization purposes

eventually obtain that the maximum number of processed interferers verifies

$$P = N-1 \quad \text{for the P-WL-MMSE receiver,}$$
$$P = 2N-1 \quad \text{for the F-WL-MMSE receiver.}$$

Hence the SAIC ($N=1$) capability of the F-WL-MMSE, illustrated in [11].

4. RECEIVERS STRUCTURE ANALYSIS

The behavior of the SAIC concept has been explained in the SISO (Single-In Single-Out) case in [6]. The receiver in [6] performs an optimal WL filtering which can be seen as a rotation of the constellations followed by a projection on the I axis in the case of strong interferences. The rotation puts the interferer constellation on the Q axis and the interference is therefore canceled by the projection (see figures 4 and 5 of [6]). In this section we give a similar interpretation of the SAIC/MAIC Alamouti concept in the case of one strong interference ($P=1$).

4.1 Interference robustness of the F-WL-MMSE

In this section we prove that the F-WL-MMSE receiver is properly canceling the internal interferences for the estimation of symbol a_{2n-1} at high INR (Interference to Noise Ratio). As we consider only $P=1$ interferer, we can easily derive $\mathbf{R}_{\tilde{\mathbf{b}}}^{-1}$ by using the inversion lemma and thus $\mathbf{w}_1$:

$$\mathbf{w}_1 = \frac{k}{\sigma^2}\left(\mathbf{f}_1 - \frac{2\varepsilon_1}{1+2\varepsilon_1}\left(\frac{\tilde{\mathbf{f}}_3^H\tilde{\mathbf{f}}_1}{\|\tilde{\mathbf{f}}_3\|^2}\mathbf{f}_3 + \frac{\tilde{\mathbf{f}}_4^H\tilde{\mathbf{f}}_1}{\|\tilde{\mathbf{f}}_4\|^2}\mathbf{f}_4\right)\right), \qquad (6)$$

where $\varepsilon_1 = \|\mathbf{f}_3\|^2\pi_1/\sigma^2$.

Let us first consider the specific case where $\tilde{\mathbf{f}}_1$ belongs to the hyperplane spanned by $\tilde{\mathbf{f}}_3$ and $\tilde{\mathbf{f}}_4$, i.e. in the absence of both phase and spatio-temporal discrimination between the useful signal and the interference. Then $\cos^2\tilde{\gamma} = 1$, where $\tilde{\gamma}$ is the angle between $\tilde{\mathbf{f}}_1$ and the hyperplane spanned by $\tilde{\mathbf{f}}_3$ and $\tilde{\mathbf{f}}_4$. Besides $\mathbf{w}_1$ reduces to $\mathbf{w}_1 = \frac{k}{\sigma^2}\frac{\mathbf{f}_1}{1+2\varepsilon_1}$, which is proportional to $\mathbf{w}_{conv,1}$: the F-WL-MMSE receiver reduces to the Conventional Alamouti receiver whose robustness is studied in the next section.

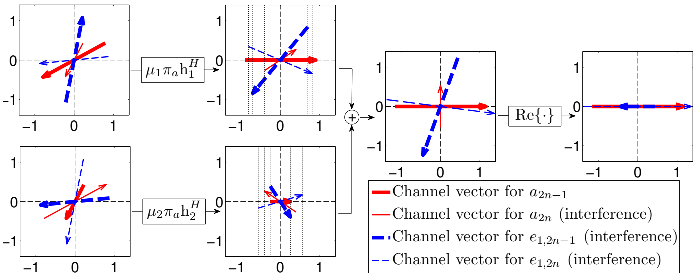
We subsequently suppose that $\cos^2\tilde{\gamma} < 1$, which implies a discrimination between the useful signal and the interference. We first derive the contribution of the useful signal a_{2n-1} in $\tilde{z}_1 = \tilde{\mathbf{w}}_1^H\tilde{\mathbf{x}}$:

$$\tilde{\mathbf{w}}_1^H\tilde{\mathbf{f}}_1 = \frac{k}{\sigma^2}\|\mathbf{f}_1\|^2\left(\sin^2\tilde{\gamma} + \frac{\cos^2\tilde{\gamma}}{1+2\varepsilon_1}\right).$$

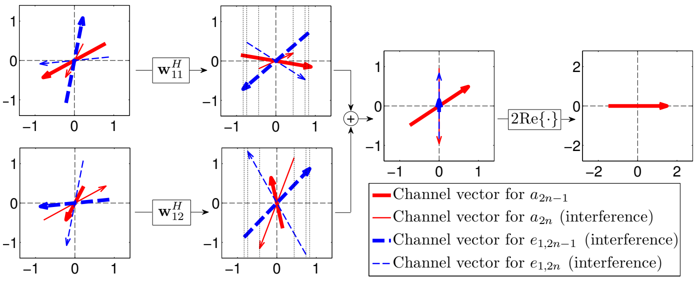
Note that $\cos^2\tilde{\gamma} = (|\tilde{\mathbf{f}}_3^H\tilde{\mathbf{f}}_1|^2 + |\tilde{\mathbf{f}}_4^H\tilde{\mathbf{f}}_1|)/(\|\tilde{\mathbf{f}}_1\|^2\|\tilde{\mathbf{f}}_3\|^2)$. For $\varepsilon_1 \gg 1$ we then have $\tilde{\mathbf{w}}_1^H\tilde{\mathbf{f}}_1 \simeq \frac{k}{\sigma^2}\|\mathbf{f}_1\|^2\sin^2\tilde{\gamma}\,(\neq 0)$.

We now consider the contribution of the interferences induced by a_{2n}, $e_{1,2n-1}$ and $e_{1,2n}$ in $y_1 = \mathbf{w}_1^H\mathbf{x}$ (we recall that the output of the F-WL-MMSE receiver is given by $z_1 = 2\mathrm{Re}\{y_1\}$). We can show that

$$\begin{cases} \mathbf{w}_1^H\mathbf{f}_2 = \frac{k}{\sigma^2}\beta_2\mathbf{i}, \\ \mathbf{w}_1^H\mathbf{f}_3 = \frac{k}{\sigma^2}(\alpha_3 + \mathbf{i}\beta_3), \\ \mathbf{w}_1^H\mathbf{f}_4 = \frac{k}{\sigma^2}(\alpha_4 + \mathbf{i}\beta_4), \end{cases}$$



(a) Conventional Alamouti receiver



(b) F-WL-MMSE receiver

Figure 1: Constellations variation inside the receivers

with $\beta_2 = \mathrm{Im}\{\frac{2\pi_a\mu_1\mu_2}{\pi_s}\mathbf{h}_1^H\mathbf{h}_2 - \frac{2\varepsilon_1}{1+2\varepsilon_1}\frac{\mathbf{f}_3^T\mathbf{f}_1^*\mathbf{f}_3^H\mathbf{f}_2}{\|\mathbf{f}_3\|^2}\}$, $\alpha_3 = \frac{\mathrm{Re}\{\mathbf{f}_3^H\mathbf{f}_1\}}{1+2\varepsilon_1}$, $\beta_3 = \mathrm{Im}\{\mathbf{f}_1^H\mathbf{f}_3 - \frac{\pi_a\mu_3\mu_4}{\sigma^2}\frac{4\mathrm{Re}\{\mathbf{f}_4^H\mathbf{f}_1\}}{1+2\varepsilon_1}\mathbf{h}_4^H\mathbf{h}_3\}$, $\alpha_4 = \frac{\mathrm{Re}\{\mathbf{f}_4^H\mathbf{f}_1\}}{1+2\varepsilon_1}$, $\beta_4 = \mathrm{Im}\{\mathbf{f}_1^H\mathbf{f}_4 - \frac{\pi_a\mu_3\mu_4}{\sigma^2}\frac{4\mathrm{Re}\{\mathbf{f}_3^H\mathbf{f}_1\}}{1+2\varepsilon_1}\mathbf{h}_3^H\mathbf{h}_4\}$. The contribution of a_{2n} in y_1 is thus purely imaginary. Moreover, for $\cos^2\tilde{\gamma} < 1$ and $\varepsilon_1 \gg 1$, $\mathbf{w}_1^H\mathbf{f}_3$ and $\mathbf{w}_1^H\mathbf{f}_4$ are approximately on the imaginary axis, as their real part is negligible compared to $\tilde{\mathbf{w}}_1^H\tilde{\mathbf{f}}_1$. Hence the contribution of all interferences at high INR ($\varepsilon_1 \gg 1$) is properly canceled after the projection on the I axis: the F-WL-MMSE cancels the multiuser interferences by fully exploiting the real-valued nature of the sources symbols and the Alamouti structure of the signals. More precisely, for $N=1$, the number of degrees of freedom of the F-WL-MMSE receiver corresponds to the phases and moduli of both $\mathbf{w}_{11}$ and $\mathbf{w}_{12}$, where $\mathbf{w}_{11}$ and $\mathbf{w}_{12}$ are the $N\times 1$ vector such that $\mathbf{w}_1 = [\mathbf{w}_{11}^T, \mathbf{w}_{12}^T]^T$. One degree of freedom is used to keep the useful signal while the three degrees of freedom remaining, which can be seen as two rotations and an homothety, allow to cancel the three interferences induced by a_{2n}, $e_{1,2n-1}$ and $e_{1,2n}$.

4.2 Interference robustness of the Conventional Alamouti receiver

The Conventional Alamouti receiver does not reject the signal induced by the interferer. As $\mathbf{w}_{conv,1} = \sqrt{\pi_a\pi_s}\,\mathbf{f}_1$, it only rejects the interference induced by a_{2n} ($\mathbf{f}_1^H\mathbf{f}_2$ is purely imaginary) and compensates the phase of the useful signal ($\mathbf{f}_1^H\mathbf{f}_1$ is purely real).

4.3 Illustration

In this section, we consider the case $N=1$, $P=1$, with a SNR of 0 dB and a INR of 20 dB. We fix all the channels and display them on Fig. 1(a) and 1(b) along the different steps of the estimation of a_{2n-1} by both receivers. These two figures confirm the previously stated results: the Conventional Alamouti receiver only puts the useful signal on the real axis and the interference induced by a_{2n} on the imaginary axis, while the F-WL-MMSE receiver puts the three interferences induced by a_{2n}, $e_{1,2n-1}$ and $e_{1,2n}$ on the imaginary axis, therefore rejecting them properly at the end. We can thus interpret the $\mathbf{w}_{1,conv} = \pi_a[\mu_1 \mathbf{h}_1^T, \mu_2 \mathbf{h}_1^T]^T$ filter as a simple rotation putting the interference a_{2n} on the Q axis. On the opposite the filter $\mathbf{w}_1 = [\mathbf{w}_{11}^T, \mathbf{w}_{12}^T]^T$ associated to the F-WL-MMSE receiver can be seen as two rotations coupled with an homothety as mentioned earlier.

5. SINR PERFORMANCES

In this section we compute the SINR at the output of the two presented receivers using total noise models (4) and (5) with $P = 1$ interferer. We define the SINR as the ratio between the power of the useful signal and the power of the total noise at the output of the considered receiver.

5.1 P-WL-MMSE receiver

Using the collinearity of $\overline{\mathbf{w}}_1$ and $\mathbf{R}_{\overline{\mathbf{b}}}^{-1}\mathbf{g}_1$ and deriving $\mathbf{R}_{\overline{\mathbf{b}}}^{-1}$ and $\mathbf{C}_{\overline{\mathbf{b}}}$, we obtain the following global SINR :

$$\overline{\text{SINR}} = \frac{2\varepsilon_s(1 - \frac{\varepsilon_1}{1+\varepsilon_1}\cos^2\overline{\gamma}_1)^2}{1 - \frac{\varepsilon_1}{1+\varepsilon_1}\cos^2\overline{\gamma}_1 + \frac{\varepsilon_1}{(1+\varepsilon_1)^2}\text{Re}\{\alpha_{13}^2 + \alpha_{14}^2\}} \quad (7)$$

with $\varepsilon_1 = \|\mathbf{f}_3\|^2\pi_1/\sigma^2$ the ratio between the power of interferer 1 at the receiver and the background noise power, $\varepsilon_s = \|\mathbf{f}_1\|^2\pi_s/\sigma^2$ the ratio between the power of the signal at the receiver and the background noise power, $\alpha_{13} = \mathbf{g}_1^H\mathbf{g}_3/(\|\mathbf{g}_1\|\|\mathbf{g}_3\|)$, $\alpha_{14} = \mathbf{g}_1^H\mathbf{g}_4/(\|\mathbf{g}_1\|\|\mathbf{g}_4\|)$ and $\overline{\gamma}_1$ the angle formed by the vector $\mathbf{g}_1$ and the plane spanned by the interfering vectors $\mathbf{g}_3$ and $\mathbf{g}_4$ ($\cos^2\overline{\gamma}_1 = |\alpha_{13}|^2 + |\alpha_{14}|^2$). With a strong interfer, that is $\varepsilon_1 \gg 1$, the SINR reduces to

$$\overline{\text{SINR}} \simeq 2\varepsilon_s(1 - \cos^2\overline{\gamma}_1). \quad (8)$$

The same approximation is obtained in the case of $P < N$ strong interferers, but where $\overline{\gamma}_1$ is defined as the angle formed by $\mathbf{g}_1$ and the hyperplane spanned by all the interfering vectors $\mathbf{g}_3, \mathbf{g}_4, \ldots, \mathbf{g}_{2P+1}, \mathbf{g}_{2P+2}$.

5.2 F-WL-MMSE receiver

As the F-WL-MMSE filter $\tilde{\mathbf{w}}_1$ is collinear to the ML receiver $\mathbf{R}_{\tilde{\mathbf{b}}}^{-1}\tilde{\mathbf{f}}_1$, the SINR corresponding to $\tilde{z}_1$ can be written as $\widetilde{\text{SINR}}_1 = \pi_s\tilde{\mathbf{f}}_1^H\mathbf{R}_{\tilde{\mathbf{b}}}^{-1}\tilde{\mathbf{f}}_1$. The same expression for the SINR corresponding to $\tilde{z}_2$ can be obtained. Deriving $\mathbf{R}_{\tilde{\mathbf{b}}}^{-1}$ leads to the following global SINR:

$$\widetilde{\text{SINR}} = 2\varepsilon_s\left(1 - \frac{2\varepsilon_1}{1+2\varepsilon_1}\cos^2\tilde{\gamma}_1\right) \quad (9)$$

where $\tilde{\gamma}_1$ is the angle formed by the vector $\tilde{\mathbf{f}}_1$ and the plane spanned by the interfering vectors $\tilde{\mathbf{f}}_3$ and $\tilde{\mathbf{f}}_4$. Supposing a strong interference, that is $\varepsilon_1 \gg 1$, the SINR reduces to

$$\widetilde{\text{SINR}} \simeq 2\varepsilon_s(1 - \cos^2\tilde{\gamma}_1) \quad (10)$$

Again, this approximation holds in the case of $P < 2N$ strong interferers, but where $\tilde{\gamma}_1$ is defined as the angle formed by $\tilde{\mathbf{f}}_1$ and the hyperplane spanned by all the interfering vectors $\tilde{\mathbf{f}}_3, \tilde{\mathbf{f}}_4, \ldots, \tilde{\mathbf{f}}_{2P+1}, \tilde{\mathbf{f}}_{2P+2}$.

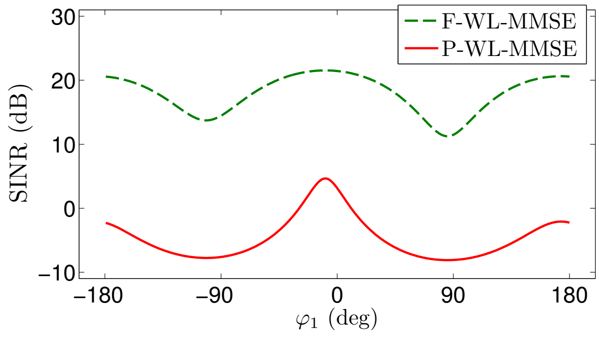


Figure 2: SINR comparison for $N = 1$

5.3 SINR comparison

We now compare the SINRs obtained for the different filters supposing $P = 1$ and a strong interference ($\varepsilon_1 \gg 1$). We note that $\cos^2\tilde{\gamma}_1 = \text{Re}\{\alpha_{13}\}^2 + \text{Re}\{\alpha_{14}\}^2 \leq |\alpha_{13}|^2 + |\alpha_{14}|^2 = \cos^2\overline{\gamma}_1$. Using this inequality and (8), (10) proves that $\widetilde{\text{SINR}} \geq \overline{\text{SINR}}$. This inequality also stands in the P interferers case. The F-WL-MMSE receiver takes into account the phases of $\mathbf{g}_1^H\mathbf{g}_3$ and of $\mathbf{g}_1^H\mathbf{g}_4$, which are dropped in the P-WL-MMSE receiver. Hence the F-WL-MMSE receiver has a higher SINR than the P-WL-MMSE. As the SER is known to decrease with the SINR, we expect the SER to be lower for the F-WL-MMSE than for the P-WL-MMSE.

5.4 Simulation and results

We consider $N = 1$ receiving antenna for the conducted simulation, and the following power ratios: $\varepsilon_s = 13$dB, $\varepsilon_1 = 30$dB. We fix $\mathbf{h}_2$, $\mathbf{h}_3$, $\mathbf{h}_4$ and $|\mathbf{h}_1|$ ($|\mathbf{h}_1| = 0.72$, $|\mathbf{h}_2| = 0.11$, $\arg(\mathbf{h}_2) = -119°$, $|\mathbf{h}_3| = 1.74$, $\arg(\mathbf{h}_3) = 75°$, $|\mathbf{h}_4| = 0.87$, $\arg(\mathbf{h}_4) = 130°$), and display the SINRs of both filters as functions of the angle $\varphi_1 = \arg(\mathbf{h}_1)$ in Fig. 2. Both SINRs vary depending on the interference alignment: the SINRs are maximal when $\mathbf{g}_3$, $\mathbf{g}_4$ are orthogonal to $\mathbf{g}_1$, and they are minimal when $\mathbf{g}_1$,$\mathbf{g}_3$, $\mathbf{g}_4$ are coplanar. A 20 dB difference is observed in this example, confirming the stated inequality $\overline{\text{SINR}} \leq \widetilde{\text{SINR}}$. As the F-WL-MMSE receiver performs SAIC, it outperforms the P-WL-MMSE of the literature in terms of SINR.

Note that when $N = 1$, $\cos^2\overline{\gamma}_1 = 1$ and therefore (7) reduces to $\overline{\text{SINR}} = 2\varepsilon_s/(1 + 2\varepsilon_1\cos^2\tilde{\gamma}_1)$. Nevertheless, comparing this expression with (9) still yields $\overline{\text{SINR}} \leq \widetilde{\text{SINR}}$.

6. SER PERFORMANCES

6.1 SER derivation

We consider $\mathscr{A} = \{\pm1, \pm3, ..., \pm(2L-1)\}$ a $2L$ ASK constellation. Supposing all symbols equally likely, for interferers signals as for the useful signal, we derive the following expression of the SER valid for both receivers:

$$\text{SER} = k_L \sum_{\substack{e_{1,1}, e_{1,2}, \ldots, \\ e_{P,1}, e_{P,2} \in \mathscr{A}}} Q\left(\frac{\sqrt{\text{SNR}}}{\sqrt{\pi_a}} + \sum_{i=1}^{P} \mathbf{u}_i^H \mathbf{e}_i \frac{\sqrt{\text{INR}_i}}{\sqrt{\pi_a}}\right) \quad (11)$$

with $\mathbf{e}_i = [e_{i,1}, e_{i,2}]^T$ referring to the signal of interferer i, $Q(u) = (\int_u^{+\infty} e^{-v^2/2} dv)/\sqrt{2\pi}$ and $k_l = 2(2L-1)/(2L)^{2P+1}$,

Table 1: SNR, INR$_i$ and $\mathbf{u}_i$ definitions

	P-WL-MMSE receiver	F-WL-MMSE receiver
SNR	$\frac{2\pi_s(\mathbf{g}_1^H \mathbf{R}_{\tilde{b}}^{-1}\mathbf{g}_1)^2}{\sigma^2 \mathbf{g}_1^H \mathbf{R}_{\tilde{b}}^{-2}\mathbf{g}_1}$	$\frac{\pi_s(\tilde{\mathbf{f}}_1^H \mathbf{R}_{\tilde{b}}^{-1}\tilde{\mathbf{f}}_1)^2}{\sigma^2 \tilde{\mathbf{f}}_1^H \mathbf{R}_{\tilde{b}}^{-2}\tilde{\mathbf{f}}_1}$
INR$_i$	$\frac{2\pi_i\|\mathrm{Re}\{\tilde{\mathbf{f}}_1^H \mathbf{R}_{\tilde{b}}^{-1}\mathbf{G}_i\}\|^2}{\sigma^2 \mathbf{g}_1^H \mathbf{R}_{\tilde{b}}^{-2}\mathbf{g}_1}$	$\frac{\pi_i\|\tilde{\mathbf{f}}_1^H \mathbf{R}_{\tilde{b}}^{-1}\tilde{\mathbf{F}}_i\|^2}{\sigma^2 \tilde{\mathbf{f}}_1^H \mathbf{R}_{\tilde{b}}^{-2}\tilde{\mathbf{f}}_1}$
$\mathbf{u}_i^H$	$\frac{\mathrm{Re}\{\mathbf{g}_1^H \mathbf{R}_{\tilde{b}}^{-1}\mathbf{G}_i\}}{\|\mathrm{Re}\{\mathbf{g}_1^H \mathbf{R}_{\tilde{b}}^{-1}\mathbf{G}_i\}\|}$	$\frac{\tilde{\mathbf{f}}_1^H \mathbf{R}_{\tilde{b}}^{-1}\tilde{\mathbf{F}}_i}{\|\tilde{\mathbf{f}}_1^H \mathbf{R}_{\tilde{b}}^{-1}\tilde{\mathbf{F}}_i\|}$

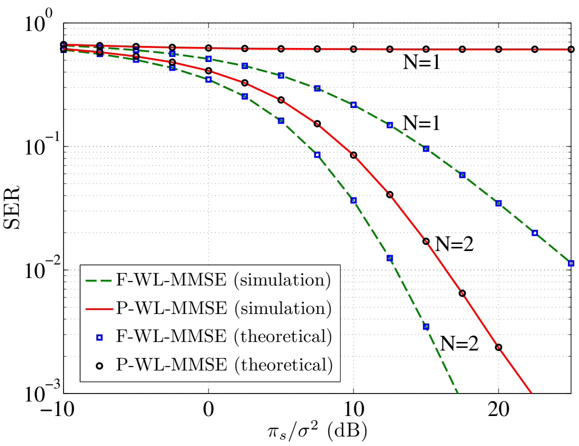


Figure 3: Theoretical and simulated SER for $N = 1, 2$

and where SNR is the Signal to Noise Ratio at the output of the receiver, INR$_i$ the Interference to Noise Ratio induced by interferer i at the output of the receiver and $\mathbf{u}_i$ the unitary vector induced by interferer i at the output of the receiver. The SNR, the INR$_i$ and the $\mathbf{u}_i$ are defined in Table 1, where we denoted Re$\{\mathbf{v}\}$ the vector whose components are the real part of the components of vector $\mathbf{v}$. Equation (11) extends the SER expression for one BPSK interference derived in [6].

6.2 Simulation and results

We consider a useful ASK signal with $2L = 4$ states $(\pm 1, \pm 3)$ corrupted by $P = 1$ synchronous internal interference with an Interference to Signal Ratio equal to 10 dB. The channel vectors of all the signals are assumed to be constant over a burst composed of 56 blocks of couples of information symbols. The channels vectors are zero-mean i.i.d Gaussian from a burst to another with independent components. The number of bursts used for the simulations is 10^5.

Under these assumptions Fig. 3 shows the variations of SER at the output of both introduced receivers as a function of π_s/σ^2, for $N = 1$ and $N = 2$ receiving antennas. Our analytical results are perfectly in line with the simulation. As expected from the SINR comparison, we can see that the F-WL-MMSE receiver performs better than the P-WL-MMSE receiver in terms of SER. For $N = 1$, the P-WL-MMSE receiver does not handle the interference while the F-WL-MMSE performs SAIC, as stated in section 3.2. For $N = 2$, both receivers handle the interference, but the F-WL-MMSE requires a lower π_s/σ^2 than the P-WL-MMSE for a given SER; e.g. the F-WL-MMSE has a 3 dB gain over the P-WL-MMSE at a SER of 10^{-2}.

7. CONCLUSION

In this paper we have highlighted the behavior of the F-WL-MMSE introduced in [11, 12] in the multiuser context: for $N = 1$ it performs two rotations and a homothety of the constellations to cancel the interferences, hence the SAIC of one Alamouti interference. We also derived the SINR and SER performance of these filters corrupted by P internal interferences. The results show the relevance in the multiuser context of the F-WL-MMSE receiver compared to the P-WL-MMSE receiver usually used in the literature, both for $N = 1$ and $N > 1$. The F-WL-MMSE receiver fully exploits the phase diversity and, for $N > 1$, the space diversity. A more detailed discussion about the performance of this receiver is presented in [12].

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