THE RATE MAXIMIZATION PROBLEM IN DSL WITH MIXED SPECTRUM AND SIGNAL COORDINATION

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ABSTRACT

Theoretical research has demonstrated that the gains in data rate achievable with spectrum coordination or signal coordination techniques are substantial for digital subscriber line (DSL) networks. Work on these two fronts has progressed steadily and usually independently. In this paper, we combine the two types of coordination for a mixed DSL scenario, one in which some of the infrastructure required for full-fledged signal coordination is available, but not all. This kind of scenario, which is referred to as the discrete multitone MIMO interference channel (DMT MIMO IC), can be an important stepping stone for the development of DSL towards a fully signal coordinated architecture. Our solution has characteristics of both signal and spectrum coordination and delivers good performance.

1. INTRODUCTION

Digital subscriber line (DSL) is today the most widespread technology for high speed data transmission. In the past ten years, theoretical research has shown that the improvements achievable with spectrum or signal coordination techniques (called dynamic spectrum management [DSM]) are significant. The main objective of these techniques is to avoid or cancel multi-user interference. i.e. *crosstalk*, the main source of performance degradation for DSL networks.

Spectrum coordination (also known as DSM levels 1 and 2) aims to allocate power in the available spectrum so that crosstalk is avoided and minimized. Examples of well-known solutions are [1, 9, 11, 14, 15]. Spectrum coordination algorithms do not deliver the same gains as signal coordination algorithms do, but they profit from simplified infrastructure requirements and smaller complexity. For spectrum coordination, users do not have to be physically close, and a number of solutions optimize a network in which little or no message exchanges between users take place.

Signal coordination (also known as DSM level 3 or vectoring) aims to cancel crosstalk. Well-known solutions include [4, 7]. With signal coordination, requirements on computational complexity and signal processing are considerably higher. For this kind of techniques, users have to be physically co-located, and knowledge of all signals and all channel gains involved is usually required. On the plus side, signal coordination techniques are able to deliver substantial gains in comparison with only spectrum coordination, eliminating most or all crosstalk.

Work on these two fronts has progressed steadily and, more often than not, independently. Recently, attention was given to mixed scenarios, in which some of the infrastructure for signal coordination is at hand, but not all [2, 6, 8]. These mixed scenarios could turn out to be an important stepping stone for full-fledged signal coordinated DSL networks with promises of gigabit per second data rates. One approach enables signal coordination on costumer premises equipment initially not designed for this purpose as long as the lines are terminated at the same access node. A second approach, targeted in this paper, considers a DSL scenario where, for each tone, one user with A transceivers can coordinate its signals, but where inter-user signal coordination is not possible.[1] In this scenario, inter-user coordination has to be done also on the spectrum level. Examples that seem specially relevant are the cases when the number of users is too large or when the lines are not terminated at the same access node (e.g. when local loop unbundling is regulatory required).

For this scenario, every tone is an multi-input, multi-output (MIMO) interference channel (IC) channel, and, because of discrete multitone (DMT) modulation, tones are coupled through a per-user power constraint. Thus we refer to this scenario as the DMT MIMO IC. An optimal solution for such a scenario is one in which elements of both spectrum and signal coordination are present, i.e. crosstalk that cannot be canceled in the signal level should be avoided at the spectrum level.

In this paper, we profit from previous results in the literature to propose an algorithm for the mixed scenario. Our algorithm basically does MIMO IC processing and power loading in different and independent steps.

This paper is organized as follows. In Section 2, we present the problem of interest. In Section 3, we present our proposed solution. Section 4 contains numerical experiments, and a conclusion is presented in Section 5.

This paper uses standard notation. We use lower-case boldface letters to denote vectors, while upper-case boldface is used for matrices. We represent the identity matrix of size A by $\mathbf{I}_A$. We also use $(\cdot)^{\mathrm{H}}$ as the Hermitian transpose, $\mathrm{E}[\cdot]$ as the expectation operator, $\mathrm{tr}\{\cdot\}$ as trace, $|\cdot|$ as determinant and $\mathrm{diag}\{\mathbf{a}\}$ as the matrix with $\mathbf{a}$ in the main diagonal.

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[1]We consider a transceiver to be connected to a physical communications channel, which can be a differential or a common mode of a twisted wire pair, or a phantom mode of two or more twisted wire pairs.

2. PROBLEM STATEMENT

We consider DSL with discrete multitone (DMT) modulation throughout this work. Consider an N user DMT system with K tones. Denote the set of users by $\mathcal{N} = \{1, \ldots, N\}$ and the set of tones by $\mathcal{K} = \{1, \ldots, K\}$. Let p_n^k be the transmit power of user n on tone k. We organize these values in the matrix $\mathbf{P} \in \mathbb{R}^{K\times N}$. The nth column of $\mathbf{P}$, denoted by $\mathbf{p}_n = \begin{bmatrix} p_n^1 & \cdots & p_n^K \end{bmatrix}^{\mathrm{T}}$, contains the power allocation of user n in all tones. The kth row of $\mathbf{P}$, $\mathbf{p}^k = \begin{bmatrix} p_1^K & \cdots & p_N^k \end{bmatrix}$, represents the power allocation of all users in tone k. We will focus on a situation where user n has A_n transceivers. For every tone, each user can coordinate the transmission of its own A_n transceivers. For $n \in \mathcal{N}$ and $k \in \mathcal{K}$, we obtain the received signal as

$$\mathbf{y}_n^k = \mathbf{H}_{n,n}^k \mathbf{T}_n^k \mathbf{x}_n^k + \sum_{j\neq n} \mathbf{H}_{n,j}^k \mathbf{T}_j^k \mathbf{x}_j^k + \mathbf{z}_n^k. \tag{1}$$

Here $\mathbf{y}_n^k,\ \mathbf{x}_n^k \in \mathbb{C}^{A_n}\ \forall n$ are, respectively, the received and transmitted signal vector for user n on tone k; $\mathbf{H}_{n,j}^k \in \mathbb{C}^{A_n \times A_j}$, $\mathbf{T}_n^k \in \mathbb{C}^{A_n\times A_n}$ are, respectively, the channel matrix from user j to user n on tone k and the transmit matrix for user n on tone k. In (1), we have $\mathrm{E}\left[\mathbf{x}_n^k (\mathbf{x}_n^k)^{\mathrm{H}}\right] = \mathbf{I}_{A_n}$. Without loss of generality, the noise vector $\mathbf{z}_n^k$ is assumed to be is spatially white with covariance matrix $\mathrm{E}\left[\mathbf{z}_n^k(\mathbf{z}_n^k)^{\mathrm{H}}\right] = \mathbf{I}_{A_n}$. Also, we have $\mathrm{tr}\{(\mathbf{T}_n^k)^{\mathrm{H}}\mathbf{T}_n^k\} = p_n^k$. The estimated signal vector for user n on tone k is given by

$$\hat{\mathbf{x}}_n^k = \mathbf{R}_n^k \mathbf{y}_n^k, \tag{2}$$

where $\mathbf{R}_n^k$ is the receive matrix for user n on tone k.

Assuming Gaussian signaling, the achievable bit loading for user n on tone k is given by

$$b_n^k = \log_2\left|\mathbf{I}_{A_n} + (\mathbf{M}_n^k)^{-1}\mathbf{H}_{n,n}^k\mathbf{T}_n^k(\mathbf{T}_n^k)^{\mathrm{H}}(\mathbf{H}_{n,n}^k)^{\mathrm{H}}\right|, \tag{3}$$

where

$$\mathbf{M}_n^k = \mathbf{I}_{A_n} + \sum_{j\neq n}\mathbf{H}_{n,j}^k\mathbf{T}_j^k(\mathbf{T}_j^k)^{\mathrm{H}}(\mathbf{H}_{n,j}^k)^{\mathrm{H}}$$

is the noise plus interference covariance matrix.

Now denote the set of all matrices $\mathbf{T}_n^k$ as $\mathbb{T} = \{\mathbf{T}_n^k | n \in \mathcal{N},\ k \in \mathcal{K}\}$. The problem we would like to solve is

$$\begin{aligned} \{\mathbf{P}^\star,\ \mathbb{T}^\star\} &= \arg\max_{\{\mathbf{P},\ \mathbb{T}\}} \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} w_n b_n^k \\ &\text{subject to}\ \ \mathrm{tr}\{(\mathbf{T}_n^k)^{\mathrm{H}}\mathbf{T}_n^k\} = p_n^k\ \forall k,\ n \\ &\qquad \sum_k p_n^k \le P_n^{\max}\ \forall n \end{aligned} \tag{4}$$

Here, $P_n^{\max}$ is the power budget for user n and w_n is the weight for user n. Notice that in this paper we use a per-user power constraint, not per-transceiver. We also do not use a per-tone power constraint, i.e. we do not consider a spectral mask.

We remark that the design of the receive matrices in (2) is the easy part of the problem. It has been shown that in a MIMO IC scenario the linear MMSE (LMMSE) receiver provides an optimal linear receiver given a set of linear transmit matrices [3, 10]. Given a set of transmit matrices, the LMMSE filter is given by

$$\mathbf{R}_n^k = (\mathbf{H}_{n,n}^k\mathbf{T}_n^k)^{\mathrm{H}}\left(\mathbf{M}_n^k + \mathbf{H}_{n,n}^k\mathbf{T}_n^k(\mathbf{H}_{n,n}^k\mathbf{T}_n^k)^{\mathrm{H}}\right)^{-1}. \tag{5}$$

This fact makes the optimization variables in (4) restricted only to $\mathbb{T}$ and $\mathbf{P}$.

The optimization in (4) comprises K distinct N-user MIMO ICs, in which, for all tones, user n has A_n transceivers. The challenge in (4) is twofold: first, we should design the matrices $\mathbf{T}_n^k$ for all users and tones given a power budget p_n^k; second, we should appropriately choose $\mathbf{P}$, i.e. allocate power for each user and tone. The design of the matrices $\mathbf{T}_n^k$ corresponds to the signal coordination part of the problem. The design of $\mathbf{P}$, i.e. the power allocation, is the spectrum coordination part. Notice that the per-user power constraint couples the optimization through tones, which complicates the problem significantly. We refer to (4) as the DMT MIMO IC problem.

We illustrate the problem in Fig. 1 for a system with three users and three tones.

We remark that special cases of (4) are well-known in the literature. The special case when $A_n = 1, \forall n$ is the pure spectrum coordination problem (DSM levels 1 or 2). The optimal solution for this case is known [1], and several other papers have worked on practical and low complexity solutions for an efficient implementation, e.g. [9,11,14,15]. For the special case when $N = 1$, the optimal solution is also known [7]. For this case, the optimal solution comprises two steps: first, set $\mathbf{R}^k = (\mathbf{U}^k)^{\mathrm{H}}$ and $\mathbf{T}^k = 1/\sqrt{A}\mathbf{V}^k$ for all tones, where $\mathbf{U}^k$ and $\mathbf{V}^k$ are, respectively, the matrices of left and right singular vectors of the singular value decomposition (SVD) of $\mathbf{H}^k$, i.e. $\mathbf{H}^k = \mathbf{U}\mathrm{diag}\left\{\begin{bmatrix}\tau^k(1) & \ldots & \tau^k(r)\end{bmatrix}\right\}(\mathbf{V}^k)^{\mathrm{H}}$—here r is the rank of $\mathbf{H}^k$; and second, consider the noise to channel ratio to be $1/\tau^k(i)^2$ and allocate power with a waterfilling algorithm . For the special case when $K = 1$, several solutions are also available, e.g. [5,10], but they are at best guaranteed to converge to a local optimum—i.e. it is not know how to solve this case optimally.

To the best of our knowledge, the problem in the more general form of (4) has not been analyzed in the literature.

3. PROPOSED SOLUTION

We first define the Lagrangian function related to (4) as

$$\begin{aligned} L(\mathbf{P},\ \mathbb{T},\ \boldsymbol{\mu},\ \boldsymbol{\lambda}) = &\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} w_n b_n^k + \sum_{n\in\mathcal{N}}\mu_n\left(P_n^{\max} - P_n^{\mathrm{tot}}\right) \\ &+ \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}}\lambda_n^k\left(\mathrm{tr}\{(\mathbf{T}_n^k)^{\mathrm{H}}\mathbf{T}_n^k\} - p_n^k\right), \end{aligned} \tag{6}$$

where b_n^k is given by (3), $P_n^{\mathrm{tot}} = \sum_{k\in\mathcal{K}} p_n^k$ and μ_n and λ_n^k are Lagrange multipliers. We now write the first order necessary condition for optimality of (4), the well-known Karush-Kuhn-Tucker (KKT) condition. The KKT condition states that, if $\{\mathbf{P},\ \mathbb{T}\}$ is a local optimizer of (4), there exist $\boldsymbol{\lambda} \in \mathbb{R}^{KN}$, $[\boldsymbol{\lambda}]_{k,n} = \lambda_n^k$ and $\boldsymbol{\mu} \in \mathbb{R}^N$, $[\boldsymbol{\mu}]_n = \mu_n$, such that

$$\nabla_{\mathbf{T}_n^k} L(\mathbf{P},\ \mathbb{T},\ \boldsymbol{\mu},\ \boldsymbol{\lambda}) = \mathbf{0}, \tag{7}$$

$$\nabla_{p_n^k} L(\mathbf{P},\ \mathbb{T},\ \boldsymbol{\mu},\ \boldsymbol{\lambda}) = 0, \tag{8}$$

$$\mathrm{tr}\{(\mathbf{T}_n^k)^{\mathrm{H}}\mathbf{T}_n^k\} - p_n^k = 0,$$

$$P_n^{\max} - \sum_{k\in\mathcal{K}} p_n^k \ge 0,$$

$$\mu_n \ge 0,\ \mu_n(P_n^{\max} - \sum_{k\in\mathcal{K}} p_n^k) = 0,$$

$n \in \mathcal{N},\ k \in \mathcal{K}$. The first two equations, (7) and (8), are know as the stationary conditions. Our approach is to solve (8) and (7) separately, first one then the other. We apply this process iteratively until convergence.

We now focus on how to separately solve each stationary equation.

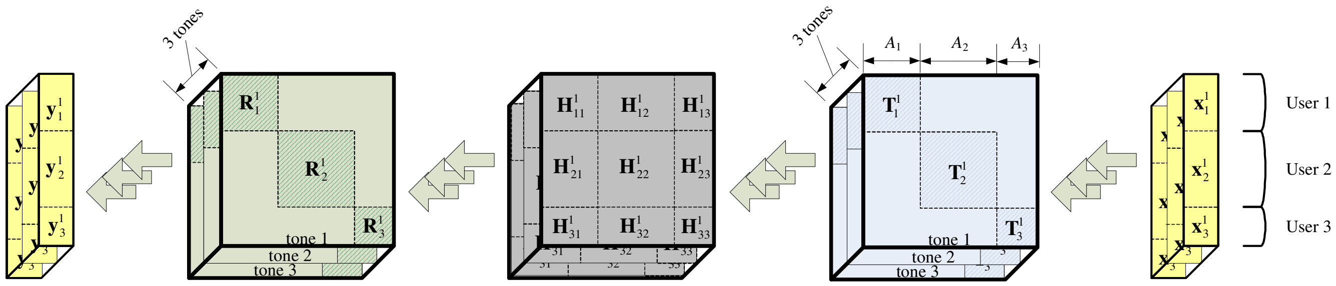

Figure 1: Illustration of the DMT MIMO IC problem for a scenario with 3 users and 3 tones. Each user n has A_n transceivers. Each user can coordinate its own transceivers, but inter-user coordination should be done on the spectral level.

3.1 Solving for $\mathbf{T}_n^k$

When solving for the transmit matrices, first notice that the problem is independent for every tone. Thus, we can write (6) as

$$L(\mathbb{T},\ \boldsymbol{\lambda}) = \sum_{k\in\mathcal{K}} L^k(\mathbb{T}^k,\ \boldsymbol{\lambda}^k),$$

where

$$L^k(\mathbb{T}^k,\ \boldsymbol{\lambda}^k) = \sum_{n\in\mathcal{N}} w_n b_n^k - \sum_{n\in\mathcal{N}} \lambda_n^k \left(\mathrm{tr}\{(\mathbf{T}_n^k)^{\mathrm{H}}\mathbf{T}_n^k\} - p_n^k\right).$$

Here $\mathbb{T}^k$ denotes the set of transmit matrices for all users in one given tone k, i.e $\mathbb{T}^k = \{\mathbf{T}_n^k | n \in \mathcal{N}\}$ and $\boldsymbol{\lambda}^k = [\lambda_1^k \ \cdots \ \lambda_N^k]^{\mathrm{T}}$. This implies that we have to solve K different and independent MIMO IC problems. Each MIMO IC problem has N interfering users, each user with A_n transceivers. Previous work has dealt with this problem many times. Here, we take the same approach as [3] (also see [10]). In these references, the authors explore an equivalence between the weighted rate maximization problem of (4) and the weighted MMSE minimization problem. Consider the MMSE matrix for user n on tone k, i.e.

$$\begin{aligned}\mathbf{E}_n^k &= \mathrm{E}\left[\left(\hat{\mathbf{x}}_n^k - \mathbf{x}_n^k\right)\left(\hat{\mathbf{x}}_n^k - \mathbf{x}_n^k\right)^{\mathrm{H}}\right] \\ &= \left(\mathbf{I}_{A_n} + (\mathbf{M}_n^k)^{-1}\mathbf{H}_{n,n}^k\mathbf{T}_n^k(\mathbf{T}_n^k)^{\mathrm{H}}(\mathbf{H}_{n,n}^k)^{\mathrm{H}}\right)^{-1}\end{aligned}$$

Here, $\mathbf{E}_n^k \in \mathbb{C}^{A_n\times A_n}$ is calculated using the optimal LMMSE in (5) and considering a given set $\mathbb{T}$. Now consider the weighted MMSE (WMMSE) minimization problem,

$$\begin{aligned}(\mathbb{T}^k)^\star &= \arg\min_{\mathbb{T}^k} \sum_{n\in\mathcal{N}} \mathrm{tr}\{\mathbf{W}_n^k\mathbf{E}_n^k\} \\ &\text{subject to } \ \mathrm{tr}\{(\mathbf{T}_n^k)^{\mathrm{H}}\mathbf{T}_n^k\} = p_n^k \ \forall n,\end{aligned} \tag{9}$$

where $\mathbf{W}_n^k \in \mathbb{C}^{A_n\times A_n}$, $\forall n,k$ is a given weighting matrix. It is shown in [3, 10] that (4) and (9) have the same local optimizers if we set

$$\mathbf{W}_n^k = w_n(\mathbf{E}_n^k)^{-1}\ \forall n. \tag{10}$$

The crucial point is that solving the WMMSE minimization problem is easier. By solving the equation with the stationary condition related to (9), we get

$$\mathbf{T}_n^k = \left(\sum_{j\in\mathcal{N}}(\mathbf{R}_j^k\mathbf{H}_{j,n}^k)^{\mathrm{H}}\mathbf{W}_n^k\mathbf{R}_j^k\mathbf{H}_{j,n}^k - \tau_n^k\mathbf{I}\right)^{-1}(\mathbf{R}_n^k\mathbf{H}_{n,n}^k)^{\mathrm{H}}\mathbf{W}_n^k. \tag{11}$$

The full solution should, for each tone, iteratively adjust $\mathbf{R}_n^k$ with (5), $\mathbf{W}_n^k$ in (10) and $\mathbf{T}_n^k$ with (11). In (11), the variable τ_n^k can be found with a simple bisection search until the total power is met, i.e. we adjust τ_n^k with bisection until $\mathrm{tr}\{(\mathbf{T}_n^k)^{\mathrm{H}}\mathbf{T}_n^k\} = p_n^k$. After convergence, we should reach a local optimum for every MIMO IC. We explicitly write all the steps in section 3.3.

3.2 Solving for P

For the spectrum coordination part of the problem, we can also decompose the problem in tones. We again can write (6) as

$$L(\mathbf{P},\ \boldsymbol{\mu}) = \sum_{n\in\mathcal{N}} \mu_n P_n^{\max} + \sum_{k\in\mathcal{K}} L^k(\mathbf{p}^k,\ \boldsymbol{\mu}),$$

where

$$L^k(\mathbf{p}^k,\ \boldsymbol{\mu}) = \sum_{n\in\mathcal{N}} w_n b_n^k - \sum_{n\in\mathcal{N}} \mu_n p_n^k.$$

Notice that here, unlike the case for the solution of $\mathbf{T}_n^k$, the problem is not independent through tones. The power budget couples the optimization in each tone.

The next step is to calculate $\nabla_{p_n^k} L^k(\mathbf{P}^k,\ \boldsymbol{\mu})$, set the resulting equation to zero and solve for p_n^k. We skip the calculations due to space limitations. The resulting power allocation should respect

$$\mathrm{tr}\left\{\left(\mathbf{I}_{A_n} + p_n^k(\mathbf{M}_n^k)^{-1}\mathbf{S}_n^k\right)^{-1}(\mathbf{M}_n^k)^{-1}\mathbf{S}_n^k\right\} + t_n^k - \mu_n = 0. \tag{12}$$

Here $\mathbf{S}_n^k = \mathbf{H}_{n,n}^k\overline{\mathbf{T}}_n^k(\overline{\mathbf{T}}_n^k)^{\mathrm{H}}(\mathbf{H}_{n,n}^k)^{\mathrm{H}}$ and $\overline{\mathbf{T}}_n^k = {}^1\!/\!\sqrt{p_n^k}\,\mathbf{T}_n^k$. The solution of (12) has to be non-negative, i.e. $p_n^k \geq 0$. In (12),

$$\begin{aligned} t_n^k &= \sum_{j\neq n} w_j \mathrm{tr}\{(\mathbf{D}_j^k)^{-1}\mathbf{F}_j^k\mathbf{H}_{j,j}^k\mathbf{T}_j^k(\mathbf{H}_{j,j}^k\mathbf{T}_j^k)^{\mathrm{H}}\}; \qquad (13)\\ \mathbf{D}_j^k &= \mathbf{I}_{A_j} + (\mathbf{M}_j^k)^{-1}\mathbf{H}_{j,j}^k\mathbf{T}_j^k(\mathbf{H}_{j,j}^k\mathbf{T}_j^k)^{\mathrm{H}};\\ \mathbf{F}_j^k &= (\mathbf{M}_j^k)^{-1}\mathbf{H}_{j,n}^k\overline{\mathbf{T}}_n^k(\mathbf{H}_{j,n}^k\overline{\mathbf{T}}_n^k)^{\mathrm{H}}(\mathbf{M}_j^k)^{-1}.\end{aligned}$$

In our proposed solution, we apply (12) iteratively for each user. We cannot write p_n^k in closed form, but (12) can nonetheless be solved. Eq. (12) is a type of waterfilling formula with some frequency selectivity. The frequency selectivity is due to the term t_n^k, which represents how much damage is inflicted to other users if user n allocates power on tone k. This variable should be large if there is potential for large interference to other users. We remark that similar power allocation formulae in [14, 15] are special cases of (12) when all users have only one transceiver.

Algorithm 1: Mixed signal and spectrum coord.

1 Initialize w_n, $\mathbf{P}$, $\mathbf{T}_n^k = 1/\alpha(\mathbf{H}_{n,n}^k)^{\mathrm{H}}$, $k \in \mathcal{K}$;
2 **repeat**
3   Calculate $\mathbf{R}_n^k$ with (5) $\forall n, k$;
4   Calculate $\mathbf{W}_n^k$ with (10) $\forall n, k$;
5   Calculate $\mathbf{T}_n^k$ with (11) $\forall n, k$;
6   Guess initial $\boldsymbol{\mu}$;
7   Calculate t_n^k with (13) $\forall n, k$;
8   **for** $n = 1, \ \ldots, \ N$ **do**
9   **repeat**
10   Calculate p_n^k with (12) $\forall k$;
11        $P_n^{\mathrm{tot}} = \sum_k p_n^k$;
12   **if** $P_n^{\mathrm{tot}} > P_n^{\mathrm{max}}$ **then**
13   increase μ_n;
14   **else**
15   decrease μ_n;
16   **until** $\left|P_n^{\mathrm{tot}} - P_n^{\mathrm{max}}\right| / P_n^{\mathrm{max}} < \epsilon_1$ *or* $\mu_n < \epsilon_2$
17 **until** *until convergence*

3.3 Algorithm

As already mentioned, our proposal is to solve (4) for the signal coordination part (i.e., for the $\mathbf{T}_n^k$'s) and the spectrum coordination part (i.e., $\mathbf{P}$) in separate steps and iteratively. The solution we propose is detailed in Algorithm 1.

The initialization of the algorithm is done in a simple way, as shown in line 1. We initialize the $\mathbf{T}_n^k$'s with the transmit matched filter. The constant α in line 1 is to make sure that $\mathrm{tr}\{(\mathbf{T}_n^k)^{\mathrm{H}}\mathbf{T}_n^k\} = p_n^k$. In the experiment section, the power matrix is initialized in a couple different ways. It is not know how to choose the initial point so that global optimality is achieved, so these choices were made because they provide good results.

The signal coordination part of the algorithm is contained in lines 3 to 5. Like [3], we first calculate $\mathbf{R}_n^k$, then $\mathbf{W}_n^k$ and then $\mathbf{T}_n^k$ for all users and tones. The spectrum coordination part of the algorithm is contained in lines 6 to 16. The main step for this part is the power allocation in line 10. The loop in lines 8 to 16 contains the adjustment of the Lagrangian multipliers μ_n. These should be adjusted so that the power constraint for each user is met. In line 16, the constants ϵ_1 and ϵ_2 are very small positive numbers. For the simulations to be presented in the next section, we set them to 10^{-6} and 10^{-10}, respectively.

4. NUMERICAL RESULTS

In this section, we present simulations results for the proposed algorithm. All simulations in this section consider upstream VDSL2. Cables AWG 26 are used and a SNR gap of 9.45 dB is considered. The tone spacing is denoted by Δ_f and the symbol rate by f_s. These values are set to 4.3125 kHz and 4 kHz, respectively. We calculate data rate with $R_n = f_s \sum_k b_n^k$. Each transceiver has at their disposal a maximum power of 14.5 dBm. For each line, noise model ETSI A is adopted [12] with a background noise level of -140 dBm/Hz. We use the FDD 998 frequency bandplan over POTS up to 12 MHz [13].

The simulation assesses how much we can gain with some signal coordination. The scenario of interest is given in Fig. 2. The scenario has two users, one with line length l_1 = 1.2 km and the other with l_2 = 0.9 km. Each user has two lines, hence two transceivers. In this upstream VDSL2 scenario, the user with the shorter lines has to avoid excessive crosstalk to the user with the longer lines. This kind of near-far scenario is perhaps the main testing ground for DSM algorithms. We remark that, for the signal plus spectrum coordination algorithm, the power budget applies to a user. Thus, a user with two lines has 17.5 dBm of power at its disposal.

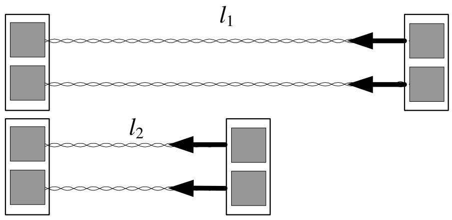

Figure 2: VDSL2 upstream scenario.

We will run two different algorithms: in the first one, each user can coordinate the transmission of its two lines on the signal level, but inter-user coordination has to be done on the spectrum level. In the second one, there is only spectrum coordination for all users and lines. In other words, the first experiment mixes signal and spectrum coordination, while the second experiment uses pure spectrum coordination. For the mixed signal and spectrum coordination algorithm, we use Algorithm 1. For the pure spectrum coordination, we use the distributed spectrum balancing (DSB) [14], which has been shown to be close to optimal. The rate regions for both experiments are depicted in Fig. 3. The increase in data rate is significant.

In Fig. 4, we plot the convergence behavior of the proposed algorithm. This curve corresponds to the convergence behavior of the point in the rate region marked with an arrow in Fig. 3. Such fast convergence was observed in all experiments.

5. CONCLUSION

In this paper, we have presented a new algorithm for a DSL scenario with mixed signal and spectrum coordination. We have called this scenario DMT MIMO IC. This type of mixed scenario could turn out to be an important stepping stone for the development of DSL networks towards full coordination on the signal level.

We have combined elements of previous solutions dealing separately with signal and spectrum coordination. The proposed algorithm has been observed to perform well in typical near-far DSL scenarios.

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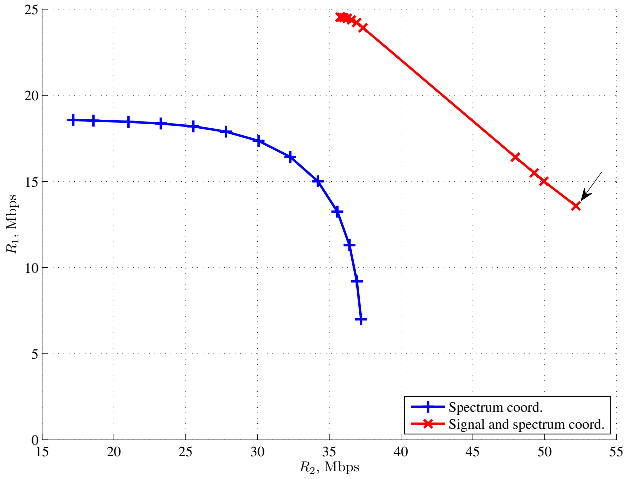

Figure 3: Rate region for VDSL2 scenario.

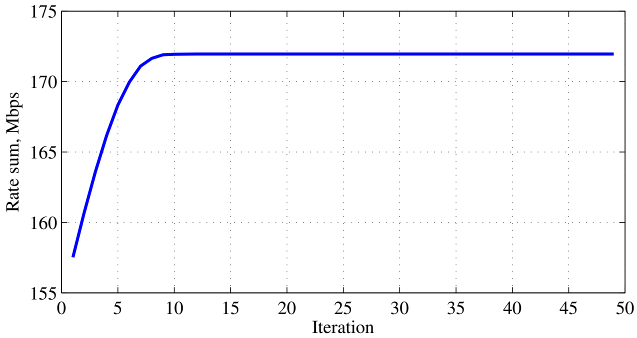

Figure 4: Iteration vs. rate sum.

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