GRASSMANNIAN DIFFERENTIAL LIMITED FEEDBACK FOR INTERFERENCE ALIGNMENT

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ABSTRACT

Interference alignment (IA) can use channel state information (CSI) to precode, align, and reduce the dimension of interference at each of the receivers, enabling systems to achieve their maximum multiplexing gain. CSI, estimated at the receivers, can be shared with the transmitters by limited feedback. The number of channels to be shared grows with the square of the number of users creating too much overhead in conventional feedback methods. This paper proposes Grassmannian differential feedback to take advantage of temporal correlation in the channel and reduce overhead. Grassmannian differential feedback uses two manifold tools, tangent spaces and geodesic paths, to track the evolution of CSI on the manifold. Simulation results show that the proposed feedback strategy allows IA to perform well over a wide range of Doppler spreads, and to approach perfect CSI performance in slowly varying channels.

1. INTRODUCTION

Interference alignment (IA) is a linear precoding technique for the interference channel [1]. IA reduces the dimension of interference at each of the receivers, allowing systems to achieve maximum multiplexing gain. IA precoders can be found using various algorithms [2, 3], at the expense of requiring channel state information (CSI) for all interference channels in the network. Feedback is a flexible method to obtain the CSI required for IA. Limited feedback, where CSI is quantized, is a reasonable approach for low overhead feedback [4].

Prior work has applied both limited and analog feedback to IA [5, 6]. Unfortunately, the required CSI scales with the square of the number of users incurring a high overhead. Partitioning users into small interference alignment groups is one approach to reduce overhead [7]. Partitioning reduces the number of channels that must be fed back, but does not directly address the shortcomings of existing feedback strategies. The codebooks required for limited feedback, for example, still become prohibitively large at high SNR [5]. While analog feedback avoids codebook scaling, it suffers from the fact that extra feedback symbols provide only marginal improvements to CSI accuracy [6]. Both [5, 6] neglect temporal correlation which can be exploited by differential feedback [8] and predictive vector quantization [9].

In this paper we propose a limited feedback strategy for IA based on Grassmannian differential feedback. The proposed strategy tracks the slow evolution of the normalized and rotationally invariant channel impulse responses on the Grassmannian manifold. At each feedback update, a quantized tangent vector relating consecutive channel realizations is used at the transmitted reconstruct the channel. Grassmannian manifold tools, such as the tangent vector, were previously used in a predictive quantization strategy for the multiple-input single-output (MISO) broadcast channel [9]. In this paper, no prediction is performed and differential feedback is used in the new context of single-input single-output (SISO) interference channels. In a SISO IA system, alignment can be done by coding over frequency extensions in which case the wideband channel responses are the CSI to be quantized. New adaptive tangent codebooks are constructed to make use of the geometry of the CSI and dynamics of the system. The proposed algorithm outperforms memoryless quantization and competitive feedback strategies for temporally correlated channels [9]. Simulation results show that IA with the proposed feedback strategy performs well for practical SNRs with a limited number of feedback bits.

Throughout this paper we use the following notation: $\mathbf{A}$ is a matrix; $\mathbf{a}$ is a vector; and a is a scalar; $\mathbf{A}^*$ and $\mathbf{a}^*$ denote the conjugate transpose of $\mathbf{A}$ and $\mathbf{a}$ respectively; $\mathbf{A} \circ \mathbf{B}$ is the Hadamard product ; $\|\mathbf{A}\|_F$ is the Frobenius norm of $\mathbf{A}$ and $\mathrm{trace}(\mathbf{A})$ is its trace; $\|\mathbf{a}\|$ is the 2-norm of $\mathbf{a}$; $|a|$ is the absolute value of a; $\mathbf{I}_N$ is the $N \times N$ identity matrix; $\mathbf{0}_N$ is the N-dimensional zero vector; $\mathcal{F}_N$ is the N-point discrete Fourier transform (DFT); $\mathrm{diag}(\mathbf{a})$ is the diagonal matrix obtained by putting the elements of $\mathbf{a}$ on its diagonal; $\mathcal{CN}(\mathbf{a}, \mathbf{A})$ is a complex Gaussian random vector with mean $\mathbf{a}$ and covariance matrix $\mathbf{A}$. Expectation is denoted by $\mathbb{E}[.]$.

2. SYSTEM MODEL

Consider a frequency selective SISO interference channel with K communicating user pairs, as shown in Fig. 1. Each user k communicates desired data to its paired receiver k and also interferes with all other receivers $\ell \neq k$. The wideband channel between transmitter ℓ and receiver k is modeled by the L-tap channel impulse response vector $\mathbf{h}_{k,\ell} = [h_{k,\ell}[0], h_{k,\ell}[1], \ldots, h_{k,\ell}[L-1]]$ $\forall k, \ell \in \{1, \ldots, K\}$, which is assumed to be known perfectly to the receiver k. The elements of the channel impulse response are drawn independently across k and ℓ from a continuous distribution and have covariance matrix $\mathbb{E}\left[\mathbf{h}_{k,\ell}\mathbf{h}^*_{k,\ell}\right] = \mathbf{R}_{\mathbf{h}_{\mathbf{k},\ell}}$, $\forall k, \ell \in \{1, \ldots, K\}$, such that $\mathrm{trace}\left(\mathbf{R}_{\mathbf{h}_{\mathbf{k},\ell}}\right) = 1$.

Using orthogonal frequency division multiplexing (OFDM), the observed wideband channels are transformed into a set of N_{sc} non-interfering narrowband subcarriers. Stacking each received OFDM symbol in a vector, the

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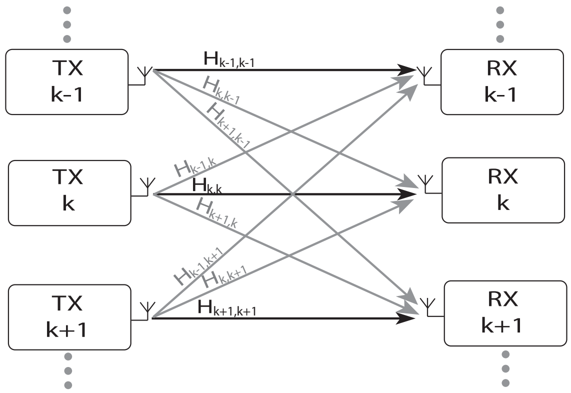

Figure 1: K-User SISO interference channel model

matrix input-output relationship is

$$\mathbf{y}_k[t] = \mathbf{H}_{k,k}[t]\mathbf{x}_k[t] + \sum_{\ell \neq k} \mathbf{H}_{k,\ell}[t]\mathbf{x}_\ell[t] + \mathbf{v}_k[t]. \quad (1)$$

where $\mathbf{x}_k[t]$ is the OFDM symbol sent by user k at time t with the average power constraint $\mathbb{E}\left[\|\mathbf{x}_k[t]\|^2\right] = N_{sc}P$, the $N_{sc} \times N_{sc}$ matrix $\mathbf{H}_{k,\ell}[t] = \operatorname{diag}\left(\mathcal{F}_{N_{sc}}\left[\mathbf{h}^*_{k,\ell}[t],\ \mathbf{0}_{N_{sc}-L}\right]^*\right)$ represents the channel frequency response between transmitter ℓ and receiver k at time t, and $\mathbf{v}_k[t]$ is the i.i.d. $\mathcal{CN}(0, \sigma_n^2 \mathbf{I}_{N_{sc}})$ thermal noise observed by user k. The system model assumes perfect time and frequency synchronization, and a cyclic prefix longer than all channel responses.

The channels seen by the t-th OFDM symbol are assumed to remain constant. The channels over consecutive OFDM symbols, however, are temporally correlated such that $\mathbb{E}\left[|\mathbf{h}_{k,\ell}[t+1]\mathbf{h}_{k,\ell}[t]|\right] = \eta \leq 1$.

3. INTERFERENCE ALIGNMENT IN FREQUENCY

In this section we review the concept of IA over frequency extensions and summarize the effect of imperfect CSI on the performance of IA.

3.1 SISO IA via Frequency Extensions

IA for the SISO interference channel can achieve the maximum degrees of freedom when coding over infinite channel extensions [1]. Using IA over N_{sc} frequency extensions, each transmitter k sends $d_k < N_{sc}$ symbols along the precoding vectors $\mathbf{f}_k^m[t]$. As a result the transmitted symbol is

$$\mathbf{x}_k[t] = \sum_{m=1}^{d_k} \mathbf{f}_k^m[t] x_k^m[t], \quad (2)$$

where $x_k^m[t]$ are the symbols transmitted by user k at time t. To satisfy the power constraint, we set $\|\mathbf{f}_k^m[t]\|_2 = 1$, and $\mathbb{E}\left[x_k^m[t]^2\right] = N_{sc}P/d_k$, such that the total power in each N_{sc} subcarriers is $N_{sc}P$. The transmit directions $\mathbf{f}_k^m[t]$ are calculated such that the interference from $K-1$ users is aligned at all receivers, leaving interference free dimensions for the desired signal.

In this section, we restrict our attention to IA with a zero-forcing receiver. At the output of the linear receiver, the received signal is

$$\begin{aligned}\mathbf{w}_k^m[t]^*\mathbf{y}_k[t] =& \mathbf{w}_k^m[t]^*\mathbf{H}_{k,k}[t]\mathbf{f}_k^m[t]x_k^m[t] \\ &+ \sum_{(i,\ell)\neq(k,m)} \mathbf{w}_k^m[t]^*\mathbf{H}_{k,i}[t]\mathbf{f}_i^\ell[t]x_i^\ell[t] \\ &+ \mathbf{w}_k^m[t]^*\mathbf{v}_k[t],\end{aligned} \quad (3)$$

for $m \in \{1, \ldots, d_k\}$ and $k \in \{1, \ldots, K\}$, where $\|\mathbf{w}_k^m[t]\|^2 = 1$. With a linear receiver, the conditions for perfect IA can be restated as

$$\mathbf{w}_k^m[t]^*\mathbf{H}_{k,i}[t]\mathbf{f}_k^\ell[t] = 0, \qquad (i,\ell) \neq (k,m) \quad (4)$$

$$|\mathbf{w}_k^m[t]^*\mathbf{H}_{k,k}[t]\mathbf{f}_k^m[t]| \geq c > 0, \qquad \forall k, m \quad (5)$$

where alignment is achieved by (4), and (5) ensures the decodability of the d_k desired streams.

The achievability proof in [1] showed that if fading is independent on all subcarriers, then the vectors $\mathbf{f}_k^\ell[t]$ can be found to satisfy (4) and (5), if d_k's are chosen as in [1]. Fortunately, [5] has claimed that fading on each subcarrier need not be independent provided that the channel impulse response is long enough.

3.2 The Effect of Imperfect CSI Feedback

With imperfect or limited CSI feedback, condition (4) is not satisfied, resulting in residual interference. As the transmit power increases, so does the leakage interference power, which saturates the sum rate at high SNR.

In [6], it is shown that if imperfect CSI is used to calculate the IA precoders and combiners, $\widehat{\mathbf{f}}_k^m[t]$ and $\widehat{\mathbf{w}}_k^m[t]$, the mean loss in sum rate is upper bounded by

$$\Delta R_{sum} \leq \sum_{k,m} \frac{1}{N_{sc}} \log_2\left(1 + \frac{\mathbb{E}_{\mathbf{H}}\left[\mathcal{I}_{k,m}^1 + \mathcal{I}_{k,m}^2\right]}{\sigma^2}\right), \quad (6)$$

where $\mathcal{I}_{i,m}^1[t] = \sum_{\ell\neq m} \frac{N_{sc}P}{d_k}\left|\widehat{\mathbf{w}}_k^m[t]^*\mathbf{H}_{k,k}[t]\widehat{\mathbf{f}}_k^\ell[t]\right|^2$, and $\mathcal{I}_{i,m}^2[t] = \sum_{i\neq k}\sum_{\ell=1}^{d_i} \frac{N_{sc}P}{d_i}\left|\widehat{\mathbf{w}}_k^m[t]^*\mathbf{H}_{k,i}[t]\widehat{\mathbf{f}}_i^\ell[t]\right|^2$ are the inter-stream and inter-user interference respectively. The objective of the feedback algorithm then becomes minimizing the total leakage interference by improving effective CSI accuracy. Using the result from [5], the individual interference terms can be upper bounded as

$$\begin{aligned}&\left|\widehat{\mathbf{w}}_k^m[t]^*\mathbf{H}_{k,i}[t]\widehat{\mathbf{f}}_i^\ell[t]\right|^2 \leq \\ &\|\widehat{\mathbf{w}}_k^m[t] \circ \widehat{\mathbf{f}}_i^\ell[t]\|^2\|\mathbf{h}_{k,i}[t]\|^2\left(1 - \left|\frac{\mathbf{h}_{k,i}[t]^*\widehat{\mathbf{h}}_{k,i}[t]}{\|\mathbf{h}_{k,i}[t]\|\|\widehat{\mathbf{h}}_{k,i}[t]\|}\right|^2\right).\end{aligned} \quad (7)$$

From (7) we see that leakage interference is directly related to the angle between the normalized channel impulse response, $\frac{\mathbf{h}_{k,i}[t]}{\|\mathbf{h}_{k,i}[t]\|}$, and its quantized version $\frac{\widehat{\mathbf{h}}_{k,i}[t]}{\|\widehat{\mathbf{h}}_{k,i}[t]\|}$. As a result, to limit performance degradation an efficient feedback strategy must attempt to minimize the angle between the actual and quantized channels.

4. GRASSMANNIAN DIFFERENTIAL FEEDBACK

In this section we show that the CSI required for IA evolves on the Grassmannian manifold and present the proposed differential feedback algorithm. We also summarize the main design parameters of our system.

4.1 Motivation and Setup

As shown in (7), the performance of IA with imperfect CSI is tightly related to the angle between the normalized channel and its quantized version. Defining $\mathbf{g}_{k,i}[t] \triangleq \frac{\mathbf{h}_{k,i}[t]}{\|\mathbf{h}_{k,i}[t]\|}$ and $\widehat{\mathbf{g}}_{k,i}[t] \triangleq \frac{\widehat{\mathbf{h}}_{k,i}[t]}{\|\widehat{\mathbf{h}}_{k,i}[t]\|}$, we see that only knowledge of the normalized channels $\mathbf{g}_{k,i}[t]$ is necessary for IA. Further, if a channel is scaled and rotated by a constant, $\alpha \in \mathbb{C}$, the subspaces spanned by $\mathbf{H}_{k,i}[t]\mathbf{f}_i^m[t]$ do not change since $\text{diag}\left(\mathcal{F}_{N_{sc}}\alpha\mathbf{h}_{k,i}[t]\right)\mathbf{f}_i^m[t] = \alpha\text{diag}\left(\mathcal{F}_{N_{sc}}\mathbf{h}_{k,i}[t]\right)\mathbf{f}_i^m[t]$. As a result, the CSI needed for IA with frequency extensions evolves on the manifold of L-dimensional unit norm, rotationally invariant vectors otherwise known as the Grassmannian manifold, $\mathcal{G}_{n,1}$. From (7), the interference power is directly related to the chordal distance between two points on the manifold [10]. Defining the chordal distance as $d(\mathbf{g}_{k,i}[t], \widehat{\mathbf{g}}_{k,i}[t]) = \sqrt{1 - |\mathbf{g}_{k,i}[t]^*\widehat{\mathbf{g}}_{k,i}[t]|^2}$ allows rewriting the right hand side of (7) as

$$\|\widehat{\mathbf{w}}_k^m[t] \circ \widehat{\mathbf{f}}_i^\ell[t]\|^2 \, \|\mathbf{h}_{k,i}[t]\|^2 d(\mathbf{g}_{k,i}[t],\, \widehat{\mathbf{g}}_{k,i}[t])^2. \tag{8}$$

The dependence of sum rate loss on chordal distance motivated the use of Grassmannian quantization [4, 5]. Memoryless quantization, however, is inefficient since it requires large codebook sizes, and thus high feedback rates, to achieve the required CSI resolution. We propose to exploit slow channel variations to reduce feedback and improve the accuracy of quantized CSI.

4.2 Differential Feedback Framework

The proposed algorithm encodes CSI increments using a tangent vector which defines the geodesic path between consecutive channel realizations. Our feedback framework uses tools presented in [10], which were simplified and first used for MISO broadcast channel feedback in [9].

We consider the slowly varying normalized channel vectors $\mathbf{g}_{k,i}[t]$ $\forall t \geq 0$ that are to be separately quantized. Though $\mathbf{g}_{k,i}$ $\forall k, i$ may be quantized jointly, in this paper we consider separate quantization. Therefore, we restrict our attention to one of the channels and drop the user subscripts. The smooth structure of the manifold allows us to relate consecutive channel realizations via a tangent vector defined as [10]

$$\mathbf{e}[t] = \tan^{-1}\left(\frac{d[t]}{\rho[t]}\right)\frac{\mathbf{g}[t]/\rho[t] - \mathbf{g}[t-1]}{d[t]/\rho[t]}, \tag{9}$$

where $\rho[t] = \mathbf{g}[t-1]^*\mathbf{g}[t]$ and $d[t] = \sqrt{1 - |\rho[t]|^2}$. The tangent can be viewed as a length preserving unwrapping of the shortest path between the two channel realizations onto the tangent space at $\mathbf{g}[t-1]$. This means $\|\mathbf{e}[t]\| = \tan^{-1}\left(\frac{d[t]}{\rho[t]}\right) = \theta[t]$ where $\theta[t]$ is the arc length between the two channel vectors. The shortest path between the two channels, known as the geodesic path on the manifold, is

$$\begin{aligned} G(\mathbf{g}[t-1], \mathbf{e}[t], \ell) =& \mathbf{g}[t-1]\cos\left(\|\mathbf{e}[t]\|\ell\right) \\ &+ \frac{\mathbf{e}[t]}{\|\mathbf{e}[t]\|}\sin\left(\|\mathbf{e}[t]\|\ell\right). \end{aligned} \tag{10}$$

It is possible to verify that $G(\mathbf{g}[t-1], \mathbf{e}[t], 0) = \mathbf{g}[t-1]$, $G(\mathbf{g}[t-1], \mathbf{e}[t], 1) = \mathbf{g}[t]$ and that $\|G(\mathbf{g}[t-1], \mathbf{e}[t], \ell)\| = 1$, $\forall \ell$ since $\mathbf{e}[t]$ is orthogonal to the base vector $\mathbf{g}[t-1]$. Therefore, the tangent vector and geodesic path can be used to build a feedback framework to track the evolution of CSI on the Grassmann manifold. Given the previous channel realization, the transmitter can reconstruct the current channel once the tangent vector is fedback by applying (10). Using predictive vector quantization, feeding back the tangent vector was shown in [9] to provide good sum-rate performance for the narrow band MISO broadcast channel.

Algorithm 1 Receiver

```
Input: ĝ[t − 1]
for all t = 1, 2, . . . do
    Estimate the channel g[t]
    Calculate tangent between ĝ[t − 1] and g[t] using (9)
    Quantize and feedback quantized tangent vector ê[t]
    Reconstruct the channel ĝ[t] = G(ĝ[t − 1], ê[t], 1)
end for
```

The pseudo code used to encode the channel evolution over time is given in Algorithm 1 and 2. At each new channel realization, the receiver estimates the normalized L-tap channel $\mathbf{g}[t]$. Perfect channel estimation is assumed here to decouple the quantization error from the estimation error. Given the current observation, and the output of the Grassmannian feedback algorithm in the previous channel realization, $\widehat{\mathbf{g}}[t-1]$, the receiver calculates the tangent vector, $\mathbf{e}[t]$, from $\widehat{\mathbf{g}}[t-1]$ to $\mathbf{g}[t]$. The receiver then quantizes the tangent vector to produce a quantized tangent, $\widehat{\mathbf{e}}[t]$. The details of the quantization method are given in depth in Section 4.3. This quantized tangent is then fedback to the transmitter over a delay and error free feedback link. Equipped with the previous quantized channel $\widehat{\mathbf{g}}[t-1]$ and the quantized error vector $\widehat{\mathbf{e}}[t]$, *both transmitter and receiver* reconstruct the quantized channel $\widehat{\mathbf{g}}[t]$, which will be used as the base point in the next iteration. The transmitter then uses the quantized CSI, $\widehat{\mathbf{g}}_{k,i}[t]$, $\forall k, i$ to do IA, i.e. this algorithm runs in parallel for all channels in the network.

Algorithm 2 Transmitter

```
Input: ĝ[t − 1]
for all t = 1, 2, . . . do
    Receive the feedback tangent vector ê[t]
    Reconstruct the channel ĝ[t] = G(ĝ[t − 1], ê[t], 1)
end for
Output: ĝ[t]
```

4.3 Design Considerations

Having described the Grassmannian differential feedback framework, we present the adaptive quantization codebooks used by our algorithm and discuss its proper initialization.

Initialization: Synchronous operation of the Grassmannian differential feedback algorithm is ensured by the fact that at each iteration, both transmitter and receiver calculate a quantized channel vector based on the same commonly available knowledge. For this to hold, however, both transmitter and receiver need a common initial vector, $\widehat{\mathbf{g}}(0)$, as input to the algorithm, otherwise the time series observed by transmitter and receiver will not be coupled. This vector can be based on a memoryless quantization of the channel [4] or initialized with a common random vector.

Tangent Magnitude Quantization: The tangent vector calculated in (9) is decomposed naturally into a tangent magnitude and a unit norm tangent direction. In this paper, the magnitude and direction are quantized separately as it can be shown in simulation that there is little to gain from joint quantization.

The problem of quantizing the tangent magnitude is that of quantizing a positive scalar and is done as follows

$$\widehat{e}_{mag} = \underset{e_i \in \mathcal{C}_{mag}}{\arg\min} \; \left| \|\mathbf{e}[t]\| - e_i \right|, \tag{11}$$

where $\mathcal{C}_{mag}$ is the magnitude quantization codebook. The index of the minimizer is then sent to the transmitter via a delay and error free link which requires $\log_2(|\mathcal{C}_{mag}|)$ bits. Finding the exact probability density function of the magnitudes is intractable, and thus we do not seek to find an optimal quantization codebook. One solution is to uniformly quantize a range of magnitudes, $\|\mathbf{e}[t]\| \in [0, 1]$, which is suboptimal. For example, [9] has observed from simulations that in highly correlated channels, where such feedback strategies are most useful, quantization error in the magnitude dominates the error in tangent direction.

Motivated by the correlation between magnitudes in consecutive iterations, we propose to adapt the quantization range to the dynamics of the system. Given the magnitude of a tangent at time t, $\|\widehat{\mathbf{e}}[t]\|$, the codebook at time $t+1$ becomes a uniform quantization codebook in the range $[\alpha\|\widehat{\mathbf{e}}[t]\|, \min\{\beta\|\widehat{\mathbf{e}}[t]\|, \pi/2\}]$, where $0 < \alpha < 1 < \beta$ are fixed parameters of the codebook. This allows the feedback algorithm to accurately track the statistics of the magnitude and quantize the current range of magnitudes with higher resolution. In static channels, this allows our approach to converge to perfect CSI.

Tangent Direction Quantization: The problem of quantizing the tangent direction vector is that of quantizing a unit norm vector which lies in the tangent space orthogonal to the base vector $\widehat{\mathbf{g}}[t-1]$. General vector quantization codebooks, such as a random vector codebook, can not be used to quantize the tangent directly for several reasons. First, traditional codebooks quantize the full L dimensional space whereas the tangent vector is of lower dimension. Further, traditional codebooks do not enforce the structural constraint that requires the tangent direction codewords to be orthogonal to the base vector $\widehat{\mathbf{g}}[t]$. With such a non-orthogonal tangent vector, the geodesic path is undefined and the output of $G(\widehat{\mathbf{g}}[t-1], \widehat{\mathbf{e}}[t], \ell)$ does not lie on the manifold. Finally, note that the tangent space changes for each base vector, which necessitates an adaptive codebook.

To respect the varying tangent space geometry and orthogonality constraints, we propose to use a canonical generating codebook to be adapted at each iteration. The codebook design provided allows perfectly projecting a canonical codebook onto the tangent plane at each iteration. This ensures that the output of the Grassmannian differential feedback algorithm remains on the manifold. We define a canonical tangent codebook as $\mathcal{C}_{gen}$ which has $|\mathcal{C}_{gen}| = N$ unit norm vector entries $\mathbf{x}_1, \ldots, \mathbf{x}_N$. This vector codebook can be any L dimensional vector codebook whose entries span the full L-dimensional space, such as the random codebook. At each iteration we form a codebook, $\mathcal{C}_{dir}$, with entries orthogonal to the base vector by using a projection operation.

Definition 1 *The normalized projection matrix function*

$$\mathbf{P}(\mathbf{x}, \mathbf{x}_b) = \frac{\mathbf{I}_L - \mathbf{x}_b\mathbf{x}_b^*}{\sqrt{1 - (\mathbf{x}^*\mathbf{x}_b)^2}}\mathbf{x},$$

computes the closest unit vector to $\mathbf{x}$ *that is also orthogonal to the base vector* $\mathbf{x}_b$*.*

Definition 2 *The tangent direction codebook,* $\mathcal{C}_{dir}(\widehat{\mathbf{g}}[t-1])$*, for the base point* $\widehat{\mathbf{g}}[t-1]$ *is*

$$\mathcal{C}_{dir}(\widehat{\mathbf{g}}[t-1]) = \{\mathbf{P}(\mathbf{x}_1, \widehat{\mathbf{g}}[t-1]), \; \ldots, \; \mathbf{P}(\mathbf{x}_N, \widehat{\mathbf{g}}[t-1])\}.$$

To construct good tangent codebooks, note that if the change in the channel is assumed to be isotropic, then it can be shown that the tangent direction vector is also isotropically distributed in the tangent space. This motivates finding canonical codebooks that lead to an isotropic distribution in the tangent space. Further improving the direction codebook design, or constructing an optimal one, is left for future work.

To formalize the tangent direction quantization, recall that the quantized channel in the next time instant will be calculated as $G(\widehat{\mathbf{g}}[t-1], \widehat{\mathbf{e}}[t], 1)$. Given that the loss in sum rate is related to the chordal distance between the actual and quantized channel, the quantized tangent direction will be given as

$$\widehat{\mathbf{e}}_{dir} = \underset{\mathbf{x}_i \in \mathcal{C}_{dir}(\widehat{\mathbf{g}}[t-1])}{\arg\min} \; d(G(\widehat{\mathbf{g}}[t-1], \widehat{e}_{mag}\mathbf{x}_i, 1), \mathbf{g}[t]) \tag{12}$$

where the tangent magnitude, $\widehat{e}_{mag}$, is given by the output of the magnitude quantization step.

5. SIMULATION RESULTS

In this section we present simulation results to demonstrate the performance of IA when channel knowledge at the transmitter is obtained via the Grassmannian differential feedback strategy detailed in Section 4. To remove the limitation of a per-stream receiver, we calculate the sum rate of a decoder which considers all desired symbols jointly and treats leakage interference as colored Gaussian noise. Since the frequency extended system can be viewed as a virtual $N_{sc} \times N_{sc}$ MIMO system, the sum rate achieved is given by,

$$R_{sum} = \sum_{k=1}^{K} \frac{1}{N_{sc}} \log_2 \left| \mathbf{I} + \left(\sigma^2\mathbf{I} + \mathbf{R}_k\right)^{-1} \left(\mathbf{H}_{k,k}\mathbf{F}_k\mathbf{F}_k^*\mathbf{H}_{kk}^*\right) \right|,$$

where $\mathbf{R}_k = \sum_{i \neq k} \mathbf{H}_{k,i}\mathbf{F}_i\mathbf{F}_i^*\mathbf{H}_{k,i}^*$ is the interference covariance matrix and the precoders, $\mathbf{F}_k = \left[\mathbf{f}_k^1, \mathbf{f}_k^2, \ldots, \mathbf{f}_k^{d_k}\right]$, are calculated given ideal or quantized CSI. For the results in this section, we use the IA algorithm in [2]. Although a closed form solution for the IA precoders exists for the SISO frequency extended interference channel in [1], it can

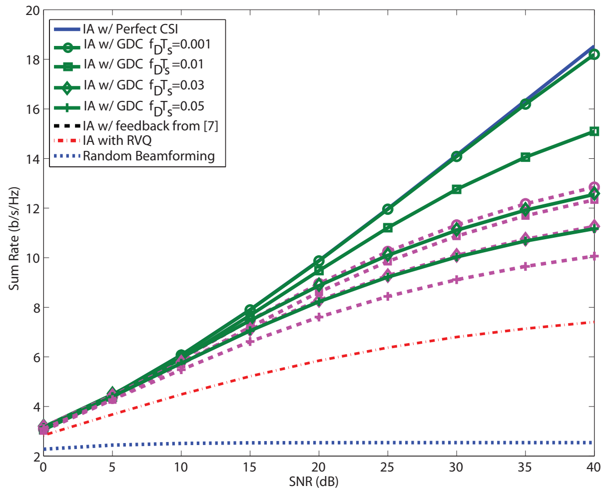

Figure 2: This figure shows the performance of IA. For slowly varying channels, the proposed algorithm allows IA networks to approach perfect CSI performance.

be shown that the proposed solution results in poor performance if not further improved [11].

We consider channels with $L = 3$ and a uniform fading profile, i.e. $\mathbf{R}_{\mathbf{h}_{k,i}} = 1/\sqrt{L}\mathbf{I}_L$, temporally correlated according to a first order autoregressive model. Each channel is given by $\mathbf{h}_{k,i}[t] = \eta\mathbf{h}_{k,i}[t-1] + \sqrt{1-\eta^2}\mathbf{z}_{k,i}[t]$ where $\mathbf{z}_{k,i} = \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\mathbf{h}_{k,i}})$ and η is a function of the normalized Doppler spread $f_D T_s$ such that $\eta = J_0(2\pi f_D T_s)$.

Fig. 2 shows the sum rate achieved by 3 users using IA over 16 channel extensions. CSI is obtained by our proposed GDC algorithm, the strategy in [9], and random vector quantization. For all the feedback strategies shown, each channel is quantized using 10 bits. For our GDC algorithm and the strategy in [9], 7 and 3 bits are used for the tangent direction and magnitude respectively. The magnitude codebook parameters are set to $\beta = \alpha^{-1} = 2$.

We see that no matter the quality of CSI, IA greatly outperforms random beamforming. In the case of perfect CSI, the rate of increase of sum-rate with SNR is 1.33, which approaches the 1.46 degrees of freedom predicted in theory when coding over 16 extensions.

As for the performance of IA with our GDC feedback framework, the sum rate achieved by IA has been greatly improved by exploiting temporal correlation in the channel. Although for constant feedback resources the multiplexing gain will never be preserved, the proposed algorithm exhibits close to perfect performance in correlated channels over the SNR range of interest. Since a system will almost never function in the asymptotically high SNR regime, the goal is to optimize for the medium to high SNR where IA is likely to be used. The proposed algorithm succeeds in providing negligible sum rate loss up to an SNR of 30dB, in channels with a normalized Doppler of up to 10^{-2}. In fact, this can be achieved with much less feedback bits as opposed to the 10 bits/channel shown in Fig. 2. The proposed algorithm continues to outperform memoryless quantization even at a significant Doppler of $f_D T_s = 0.05$. Compared to the strategy in [9], our algorithm consistently outperforms at all Dopplers, and does so increasingly for slower fading channels. This is due mainly to the proposed adaptive codebooks. A more detailed performance evaluation and comparison to other differential feedback strategies is omitted due to space constraints and will be included in future work.

6. CONCLUSION

We proposed a feedback strategy based on Grassmannian differential coding for SISO interference channels with IA over frequency extensions. Channel responses are tracked by moving along geodesic paths defined via quantized tangent feedback. Our approach exhibits significant improvements over earlier methods due to the optimized tangent and magnitude codebooks. The proposed algorithm can use the channel's temporal correlation to provide more accurate channel knowledge and better sum rate performance for a large range of SNR even with small quantization codebooks.

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