ROBUST PARETO OPTIMAL BEAMFORMING IN TWO-USER MULTIPLE-INPUT SINGLE-OUTPUT INTERFERENCE CHANNEL

Rami Mochaourab and Eduard Jorswieck

Communications Theory, Communications Laboratory
Dresden University of Technology, Dresden, Germany
e-mail: {Rami.Mochaourab, Eduard.Jorswieck}@tu-dresden.de

ABSTRACT

We consider a two-user multiple-input single-output interference channel in which the receivers treat interference as additive noise. The transmitters are assumed to have imperfect channel state information to the receivers. The transmitters choose their beamforming vectors considering worst case power gains at the receivers. We provide a real-valued parametrization of the beamforming vectors that achieve the Pareto boundary of the rate region with channel uncertainties. Simulation results and high SNR analysis show that the gain in spectrum sharing under imperfect channel state information converges to the setting of time division multiple access. Moreover, we provide analytical results for the maximum sum rate in asymptotic cases of low and high signal to noise ratios.

1. INTRODUCTION

We consider two transmitter-receiver pairs sharing the same spectral band. Each transmitter is equipped with multiple antennas while each receiver uses a single antenna. This setting corresponds to the multiple-input single-output (MISO) interference channel (IFC) [1]. We assume that the receivers treat the interference from unintended transmitters as additive Gaussian noise. The achievable rate region, of this setting is in general not a convex set. The outermost boundary of this region is called the Pareto boundary and consists of Pareto optimal points. At these points, it is impossible to increase the rate of one user without reducing the rate of the other.

In the case of perfect channel state information (CSI), the beamforming vectors that are relevant for Pareto optimal operation are proven in [2] to be a linear combination of maximum ratio transmission (MRT) and zero forcing transmission (ZF). For the general K-user case, real-valued parametrization of the efficient beamforming vectors is provided recently in [3, 4]. In [5], Pareto optimal beamforming for partial CSI at the transmitters is considered in the two-user MISO IFC.

The parametrization of the Pareto optimal beamforming vectors requires that the transmitters know the channels to all receivers. We consider the case where the transmitters have imperfect CSI and the uncertainty of the channel information is bounded by a spherical region. In [6], channel mismatches are modeled by spherical uncertainty, while in [7] by ellipsoidal uncertainty. The latter work encompasses the spherical uncertainty model as a special case. Different robust adaptive beamforming techniques are found in [8] and robust beamforming using convex optimization is discussed in [9].

Part of this work has been performed in the framework of the European research project SAPHYRE, which is partly funded by the European Union under its FP7 ICT Objective 1.1 - The Network of the Future. This work is also supported in part by the Deutsche Forschungsgemeinschaft (DFG) under grant Jo 801/4-1.

We investigate robust Pareto optimal transmission in which the transmitters have imperfect CSI. We adopt the spherical uncertainty model from [6]. Considering the worst case achievable rate of the links, we characterize the beamforming vectors that achieve points on the Pareto boundary of the robust rate region. In addition, we investigate the gain in spectrum sharing under channel information uncertainty compared to time division multiple access (TDMA). The spectral efficiency gain with spectrum sharing is larger in the mid signal to noise ratio (SNR) regime. We provide analytical results for asymptotic cases on the SNR for optimal sum rate transmission.

The paper is organized as follows. In Section 2, we describe the system model and channel uncertainty model. In Section 3, we provide the parametrization of the beamforming vectors that achieve the Pareto boundary of the Rate region with imperfect CSI. In Section 4, we study the optimal transmission in the low and high signal to noise ratio regimes. Section 6 gives conclusions and future work.

Notations

Column vectors and matrices are given in lowercase and uppercase boldface letters, respectively. $\|a\|$ is the Euclidean norm of $a, a \in \mathbb{C}^N$. $|b|$ is the absolute value of $b, b \in \mathbb{C}$. $(\cdot)^H$ denotes the Hermitian transpose. span$\{a_1,...,a_K\}$ denotes the space spanned by the vectors $a_1,...,a_K$. The orthogonal projector onto the column space of Z is $\Pi_Z := Z(Z^H Z)^{-1} Z^H$. The orthogonal projector onto the orthogonal complement of the column space of Z is $\Pi_Z^{\perp} := I - \Pi_Z$, where I is an identity matrix. Throughout the paper, the subscripts k,ℓ are from the set $\{1,2\}$.

2. PRELIMINARIES

2.1 System and Channel model

The quasi-static block flat-fading channel vector from transmitter k to receiver ℓ is denoted by $\hat{h}_{k\ell} \in \mathbb{C}^N$. We assume that transmission consists of scalar coding followed by beamforming. The beamforming vector used by transmitter k is $w_k \in \mathbb{C}^N$. The matched-filtered, symbol-sampled complex baseband data received at receiver k is

$$y_k = \hat{h}_{kk}^H w_k s_k + \hat{h}_{\ell k}^H w_\ell s_\ell + n_k, \quad k \neq \ell, \tag{1}$$

where s_k is the symbol transmitted by transmitter k. The random variables n_k are noise terms which are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and variance σ^2. Each transmitter has a total power constraint of $P := 1$ such that $\|w_k\|^2 \leq 1$. Define the signal to noise ratio (SNR) as $\rho = 1/\sigma^2$.

2.2 Spherical Uncertainty model

We assume a transmitter k does not know the channels $\hat{h}_{kk}$ and $\hat{h}_{k\ell}$ perfectly. In the spherical uncertainty model, the channel estimation errors are defined as

$$\delta_{k\ell} = \hat{h}_{k\ell} - h_{k\ell}, \quad \delta_{k\ell} \in D(\varepsilon_{k\ell}), \tag{2}$$

where $h_{k\ell}$ is the estimate of $\hat{h}_{k\ell}$ and

$$D(\varepsilon) = \{\delta : \|\delta\| \leq \varepsilon_{kk}\}. \tag{3}$$

The worst case power gain at intended receiver k is the squared of

$$x_{kk}(w_k) = \min_{\delta_{kk} \in D(\varepsilon_{kk})} |w_k^H \hat{h}_{kk}|. \tag{4}$$

The error vector as a function of the used beamforming vector which realizes the worst case intended power gain in (4) is determined in [6], and the worst case intended power gain can be written as [6, 10].

$$x_{kk}(w_k)^2 = \left((|w_k^H h_{kk}| - \|w_k\|\varepsilon_{kk})^+\right)^2, \tag{5}$$

where $(x)^+ := \max(0,x)$. Define the worst case interference gain as the squared of

$$x_{k\ell}(w_k) = \max_{\delta_{k\ell} \in D(\varepsilon_{k\ell})} |w_k^H \hat{h}_{k\ell}|. \tag{6}$$

Similarly, the error vector as a function of the used beamforming vector which realizes the worst case interference power gain can be calculated and the interference power gain at receiver ℓ is

$$x_{k\ell}(w_k)^2 = \left(|w_k^H h_{k\ell}| + \|w_k\|\varepsilon_{k\ell}\right)^2. \tag{7}$$

Thus, the worst case SINR at receiver k can be written as

$$\phi_k(w_1,w_2) = \frac{\left((|h_{kk}^H w_k| - \varepsilon_{kk}\|w_k\|)^+\right)^2}{\left(|h_{\ell k}^H w_\ell| + \varepsilon_{\ell k}\|w_\ell\|\right)^2 + \sigma^2}, \quad k \neq \ell. \tag{8}$$

2.3 Robust Rate Region

The achievable rate for link k is

$$r_k(w_1,w_2) = \log_2(1 + \phi_k(w_1,w_2)), \tag{9}$$

where single-user decoding is performed at the receiver. The *rate region* is the set of all achievable rate tuples:

$$R := \left\{(r_1(w_1,w_2), r_2(w_1,w_2)) : \|w_k\|^2 \leq 1\right\}. \tag{10}$$

Definition 1 *An operating point $(r_1,r_2) \in R$ is* Pareto optimal *if there is no other operating point $(r_1',r_2') \in R$ such that $(r_1',r_2') \geq (r_1,r_2)$, where the inequality is componentwise and strict for at least one component.* ■

The set of all Pareto optimal operating points makes up the *Pareto boundary* of the rate region. We are interested in operating points which are on the Pareto boundary of the rate region.

3. PARETO OPTIMAL BEAMFORMING

In this section, we will first review the results for Pareto optimal beamforming for perfect CSI. Then we provide the parametrization for the imperfect CSI case.

3.1 Perfect CSI case

For the two-user MISO IFC with perfect CSI at the transmitters and receivers, the set of beamforming vectors for each transmitter that are relevant for Pareto optimal operation are parameterized by a single real-valued parameter $\lambda_k \in [0,1]$ as [2, Corollary 1]

$$w_k(\lambda_k) = \sqrt{\lambda_k}\frac{\Pi_{\hat{h}_{k\ell}}\hat{h}_{kk}}{\|\Pi_{\hat{h}_{k\ell}}\hat{h}_{kk}\|} + \sqrt{1-\lambda_k}\frac{\Pi^{\perp}_{\hat{h}_{k\ell}}\hat{h}_{kk}}{\|\Pi^{\perp}_{\hat{h}_{k\ell}}\hat{h}_{kk}\|}, \tag{11}$$

where $k \neq \ell$. This parametrization is valuable for designing efficient low complexity distributed resource allocation schemes [11]. The set of beamforming vector in (11) includes maximum ratio transmission (MRT) ($\lambda_k^{\text{MRT}} = \|\Pi_{\hat{h}_{k\ell}}\hat{h}_{kk}\|^2/\|\hat{h}_{kk}\|^2$) and zero forcing transmission (ZF) ($\lambda_k^{\text{ZF}} = 0$). According to [2, Corollary 2], it suffices that the parameters λ_k only be from the set $[0,\lambda_k^{\text{MRT}}]$ for Pareto optimal operation.

3.2 Imperfect CSI case

In case the channel knowledge at the transmitters is not perfect but lies within the uncertainty region, robust transmission can be modeled with worst case analysis. According to the SINR expression in (8), worst case signal power and interference powers include additive terms influenced only by the norm of the beamforming vectors. It is thus expected that robust Pareto optimal beamforming includes additionally varying transmission power.

Proposition 1 *The Pareto boundary of the rate region R in* (10) *is achieved by the beamforming vectors*

$$w_k(p_k,\lambda_k) = p_k\sqrt{\lambda_k}\frac{\Pi_{h_{k\ell}}h_{kk}}{\|\Pi_{h_{k\ell}}h_{kk}\|} + p_k\sqrt{1-\lambda_k}\frac{\Pi^{\perp}_{h_{k\ell}}h_{kk}}{\|\Pi^{\perp}_{h_{k\ell}}h_{kk}\|}, \tag{12}$$

where $k \neq \ell, p_k \in [0,1]$, and $\lambda_k \in [0,\lambda_k^{MRT}]$, with $\lambda_k^{MRT} = \|\Pi_{h_{k\ell}}h_{kk}\|^2/\|h_{kk}\|^2$.

Proof: The proof is provided in Appendix 7.1. ■

The result in Proposition 1 differs to the case with perfect CSI in (11) in the additional parameter which controls the power allocation. Otherwise, the estimated channels $\hat{h}_{kk}$ and $\hat{h}_{k\ell}$ replace the true channels from (11).

The beamforming vectors corresponding to MRT and ZF represent extreme strategies which have the objective of either maximizing the power at the intended receiver or minimizing the interference power gain. Robust MRT is the solution of the following problem

$$\max_{\|w_k\| \leq 1}. \quad |\hat{h}_{kk}^H w_k| = |h_{kk}^H w_k| - \|w_k\|\varepsilon_{kk}, \tag{13}$$

which is $w_k^{\text{R-MRT}} = h_{kk}/\|h_{kk}\|$. In other words, to maximize the power gain at the intended receiver in the worst case of spherical uncertainty, the transmitter chooses full power transmission in the direction of the estimated channel. Robust ZF transmission is achieved only by allocating zero power. This is observed in the expression in (7) which can only be zero for $\|w_k\| = 0$.

4. EFFICIENCY IN HIGH AND LOW SNR

4.1 Efficiency at High SNR

The quantitative performance is analyzed using the high-SNR offset concept in [12, Section II]. Denote as $C(\rho)$ the sum rate as a function of the SNR. The high-SNR slope is

$$S_\infty = \lim_{\rho\to\infty} \frac{C(\rho)}{\log_2(\rho)} \quad \text{and} \tag{14}$$

which corresponds to the multiplexing gain, i.e. the slope of the sum rate curve at high SNR. The maximum sum rate for the case of perfect CSI is

$$\begin{aligned}\hat{C}(\rho) = &\max_{\lambda_1,\lambda_2} \log_2\left(1 + \frac{\rho|\hat{h}_{11}^H w_1(\lambda_1)|^2}{1+\rho|\hat{h}_{21}^H w_2(\lambda_2)|^2}\right) \\ &+ \log_2\left(1 + \frac{\rho|\hat{h}_{22}^H w_2(\lambda_2)|^2}{1+\rho|\hat{h}_{12}^H w_1(\lambda_1)|^2}\right).\end{aligned} \tag{15}$$

In the high SNR regime, the maximum sum rate is achieved with ZF transmissions:

$$\begin{aligned}\hat{C}(\rho) = &\log_2\left(1+\rho|\hat{h}_{11}^H w_1(\lambda_1^{\mathrm{ZF}})|^2\right) \\ &+ \log_2\left(1+\rho|\hat{h}_{22}^H w_2(\lambda_2^{\mathrm{ZF}})|^2\right),\end{aligned} \tag{16}$$

which gives the maximum high-SNR slope of

$$\hat{S}_\infty = \lim_{\rho\to\infty} \frac{\hat{C}(\rho)}{\log_2(\rho)} = 2. \tag{17}$$

The maximum sum rate for the case of imperfect CSI is

$$\begin{aligned}C(\rho) = &\max_{p_1,\lambda_1,p_2,\lambda_2} \log_2\left(1 + \frac{\rho\left((|h_{11}^H w_1(p_1,\lambda_1)| - \varepsilon_{11}p_1)^+\right)^2}{\rho\left(|h_{21}^H w_2(p_2,\lambda_2)| + \varepsilon_{21}p_2\right)^2 + 1}\right) \\ &+ \log_2\left(1 + \frac{\rho\left((|h_{22}^H w_2(p_2,\lambda_2)| - \varepsilon_{22}p_2)^+\right)^2}{\rho\left(|h_{12}^H w_1(p_1,\lambda_1)| + \varepsilon_{12}p_1\right)^2 + 1}\right),\end{aligned} \tag{18}$$

where $\varepsilon_{12} > 0, \varepsilon_{21} > 0$ and $0 \leq \varepsilon_{11} \leq \|h_{11}\|$ and $0 \leq \varepsilon_{22} \leq \|h_{22}\|$. In the high-SNR regime single-user transmission is optimal achieving the largest high-SNR slope of $S_\infty = 1$. The maximum sum rate is

$$C(\rho) = \log_2\left(1 + \rho \max_{k=1,2}\left((\|h_{kk}\|^2 - \varepsilon_{kk})^+\right)\right), \tag{19}$$

where only one user operates using MRT and full power transmission. Note that the condition that determines the dominant user does not only depend on the channel gains but also on the amount of uncertainty present at the transmitter.

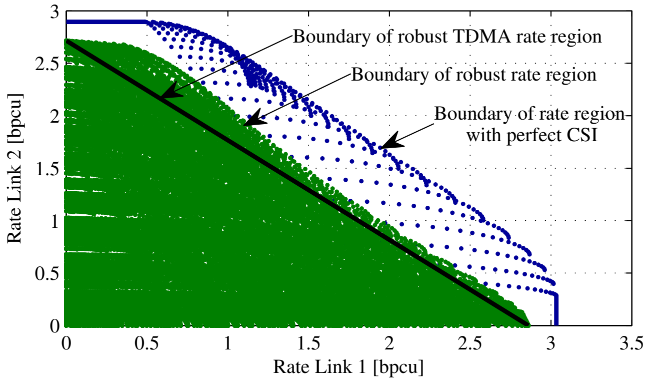

Figure 1: Comparison of rate regions for perfect CSI, imperfect CSI and TDMA with imperfect CSI. Two antennas are used at the transmitters, with 5 dB SNR and $\varepsilon_{kk} = \varepsilon_{k\ell} = 0.1$

4.2 Efficiency at Low SNR

In [13], the low-SNR regime has been analyzed and two performance measures namely the $(E_b/N_0)_{min}$ and the wideband slope S_0 were introduced. The system parameters bandwidth B, transmission rate R, transmit power P and spectral efficiency $C(E_b/N_0)$ satisfy the fundamental limit $\frac{R}{B} \leq C\left(\frac{E_b}{N_0}\right)$. The function $C(E_b/N_0)$ is directly related to the common capacity expression $C(\mathrm{SNR})$, i.e. $C(E_b/N_0) = C(\mathrm{SNR})$ for the SNR which solves $(E_b/N_0)C(\mathrm{SNR}) = \mathrm{SNR}$. At low SNR, the function $C(E_b/N_0)$ can be expressed as [13]

$$C\left(\frac{E_b}{N_0}\right) \approx \frac{S_0}{3dB}\left(\left(\frac{E_b}{N_0}\right)\Big|_{dB} - \left(\frac{E_b}{N_0}\right)_{min}\Big|_{dB}\right), \tag{20}$$

with $(E_b/N_0)_{min} = \frac{\log_e 2}{\dot{C}(0)}$ and $S_0 = \frac{2[\dot{C}(0)]^2}{-\ddot{C}(0)}$. The closer (E_b/N_0) gets to $(E_b/N_0)_{min}$ the better is the approximation in (20).

For the sum spectral efficiency for perfect CSI we obtain

$$\left(\frac{E_b}{N_0}\right)_{\min}^{pCSI} = \frac{\log_e 2}{|\hat{h}_{11}^H w_1(\lambda_1)|^2 + |\hat{h}_{22}^H w_2(\lambda_2)|^2}, \tag{21}$$

where the minimum is achieved with $\frac{\log_e 2}{\|\hat{h}_{11}\|^2 + \|\hat{h}_{22}\|^2}$ corresponding to MRT transmission. For the case of imperfect CSI, we obtain

$$\left(\frac{E_b}{N_0}\right)_{\min}^{iCSI} = \frac{\log_e 2}{(\|\hat{h}_{11}\| - \varepsilon_{11})^2 + (\|\hat{h}_{22}\| - \varepsilon_{22})^2}, \tag{22}$$

which clearly shows the loss due to channel uncertainty.

5. ILLUSTRATIONS

In Figure 1, three rate regions are plotted. The largest region corresponds to the case of perfect CSI where the Pareto boundary is attained from (11). The second largest region corresponds to the case of imperfect CSI. For this case, we fix $\varepsilon_{kk} = \varepsilon_{k\ell} = 0.1$ and generate the beamforming vectors characterized in Proposition 1. We choose for the parameters p_k 10 samples uniformly in $[0,1]$. For the parameters λ_k

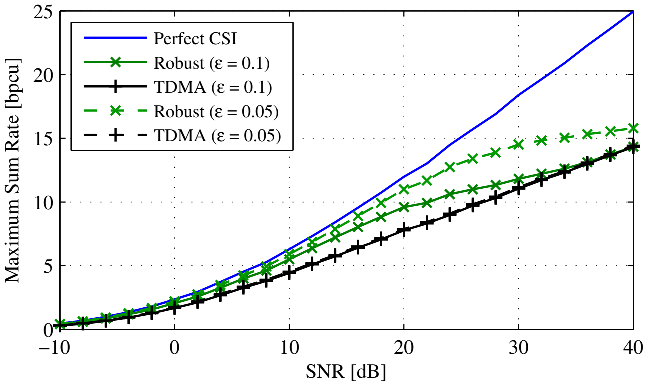

Figure 2: Comparison of maximum sum rates ($\varepsilon = \varepsilon_{kk} = \varepsilon_{k\ell}$). Two antennas are used at the transmitters.

we chose 20 samples uniformly in the range $[0, \lambda_k^{\mathrm{MRT}}]$. The smallest region corresponds to TDMA with imperfect CSI. The region is a triangle with two corners at the axes corresponding to the single-user points.

In Figure 2, maximum sum rate is plotted for different SNR levels. We compare the cases of perfect CSI, imperfect CSI, and TDMA with imperfect CSI. We generate 1000 random channel realizations and average over achievable sum rates. For each channel realization, the maximum sum rate point is found by a grid search over the rate samples corresponding to the parameterized beamforming vectors. In the case of perfect CSI, we used the parametrization in (11), and for the imperfect CSI case we used the parametrization in Proposition 1. For each parametrization, we generate 100 samples uniformly in the ranges of the parameters. The estimation error amplitude is equally chosen for all channels such that $\varepsilon = \varepsilon_{kk} = \varepsilon_{k\ell}$. The plots are generated for two values of ε, 0.1 and 0.5. The sum rate in TDMA is determined as the maximum of the single-user rates. The gain in spectrum sharing for the case of imperfect CSI to that of TDMA is observed as the distance between the two curves. This distance is highest in the mid SNR regime and increases for smaller ε, i.e., less channel uncertainty. The performance of spectrum sharing and TDMA is the same for low and high SNRs. The high-SNR slope for the imperfect CSI is 1 and for the perfect CSI case is 2 as calculated in Section 4. In the high SNR regime, ZF transmission is optimal to achieve maximum sum rate. In the low SNR regime, MRT is sum rate optimal and all curves in Figure 2 overlap.

6. CONCLUSIONS

In this work, we studied robust Pareto optimal beamforming in a two-user multiple-input single-output interference channel. It turns out that a similar beamforming parametrization can be used as in the perfect CSI case. However, additional power allocation is necessary. Since ZF beamforming is not possible due to uncertainty, the achievable high SNR sum rate converges to the TDMA case with slope one.

For future work, we plan to extend the setting to the K-user scenario and include also ellipsoidal uncertainty regions. Furthermore, an important question is related to strategic bargaining in interference channels with imperfect channel state information.

7. APPENDIX

7.1 Proof of Proposition 1

The proof follows the lines of proof of Proposition 1 and Corollary 1 in [2]. First we prove that any Pareto optimal operating point is achieved by beamforming vectors which lie in the span of h_{kk} and $h_{k\ell}$. The proof is by contradiction. Let $\{z_{k1}, \ldots, z_{k(N-2)}\}$ be an orthonormal basis for the null space of span$\{h_{kk}, h_{k\ell}\}$. Assume that the beamforming vector w_k is not in span$\{h_{kk}, h_{k\ell}\}$ and achieves a Pareto optimal point. The beamforming vector w_k can be written as

$$w_k = \alpha_k h_{kk} + \beta_k h_{k\ell} + \sum_{i=1}^{N-2} \gamma_i z_{ki} \tag{23}$$

where α_k, β_k, and γ_{ki} are complex-valued and $|\gamma_{ki}| > 0$ for at least one i. Since the vectors h_{kk} and $h_{k\ell}$ span the same space as $\frac{\Pi_{h_{k\ell}} h_{kk}}{\|\Pi_{h_{k\ell}} h_{kk}\|}$ and $\frac{\Pi^{\perp}_{h_{k\ell}} h_{kk}}{\|\Pi^{\perp}_{h_{k\ell}} h_{kk}\|}$ we can rewrite (23) as

$$w_k = \alpha_k \frac{\Pi_{h_{k\ell}} h_{kk}}{\|\Pi_{h_{k\ell}} h_{kk}\|} + \beta_k \frac{\Pi^{\perp}_{h_{k\ell}} h_{kk}}{\|\Pi^{\perp}_{h_{k\ell}} h_{kk}\|} + \sum_{i=1}^{N-2} \gamma_i z_{ki} \tag{24}$$

The power of the beamforming vector is

$$\|w_k\|^2 = |\alpha_k|^2 + |\beta_k|^2 + \sum_{i=1}^{N-2} |\gamma_i|^2 \leq 1 \tag{25}$$

because all the vectors are orthonormal. By definition of Pareto optimality, it is not possible to find another beamforming vector than w_k in (24) which increases the rate of one link without reducing the rate of the other link. Consider the SINR of link k, the intended power gain is

$$\left(\left(|h_{kk}^H w_k| - \varepsilon_{kk}\|w_k\|\right)^+\right)^2. \tag{26}$$

For link $\ell, \ell \neq k$, the interference power gain is

$$\left(|h_{k\ell}^H w_k| + \varepsilon_{k\ell}\|w_k\|\right)^2. \tag{27}$$

If for any γ_{ki} in (24) with $|\gamma_{ki}|^2 > 0$ we can construct a beamforming vector w_k' such that $\|w_k'\|^2 = \|w_k\|^2$ by modifying w_k in the following manner: the power $|\gamma_{ki}|^2$ which is assigned in w_k in the component z_i is traded to the component $\frac{\Pi^{\perp}_{h_{k\ell}} h_{kk}}{\|\Pi^{\perp}_{h_{k\ell}} h_{kk}\|}$. Increasing $|\beta_k|^2$ increases the intended power gain in (26) without affecting the interference power gain at the unintended receiver in (27). This is a contradiction to the assumption that w_k achieves a Pareto optimal point. Hence, the Pareto optimal beamforming vectors should have the form

$$w_k = \alpha_k \frac{\Pi_{h_{k\ell}} h_{kk}}{\|\Pi_{h_{k\ell}} h_{kk}\|} + \beta_k \frac{\Pi^{\perp}_{h_{k\ell}} h_{kk}}{\|\Pi^{\perp}_{h_{k\ell}} h_{kk}\|} \tag{28}$$

with $|\alpha_k|^2 + |\beta_k|^2 \leq 1$. Next, we prove that the parameters α_k and β_k have to be nonnegative real-valued. We can write the intended power gain of link k as

$$|h_{kk}^H w_k| - \varepsilon\|w_k\| = \left| \alpha_k \frac{h_{kk}^H \Pi_{h_{k\ell}} h_{kk}}{\|\Pi_{h_{k\ell}} h_{kk}\|} + \beta_k \frac{h_{kk}^H \Pi_{h_{k\ell}}^{\perp} h_{kk}}{\|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|} \right| - \varepsilon\|w_k\|$$
$$\leq |\alpha_k| \frac{h_{kk}^H \Pi_{h_{k\ell}} h_{kk}}{\|\Pi_{h_{k\ell}} h_{kk}\|} + |\beta_k| \frac{h_{kk}^H \Pi_{h_{k\ell}}^{\perp} h_{kk}}{\|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|} - \varepsilon\|w_k\|, \tag{29}$$

where we used the triangle inequality. The upper bound in (29) is achieved for α_k and β_k having the same phase. Since w_k can be multiplied with any complex number without affecting the power gains, we can choose α_k and β_k to be real-valued. Thus, the condition for the parameters in (28) can be written as $\alpha_k^2 + \beta_k^2 \leq 1$.

Next, we show that the intended power gain $x_{kk}(w_k)$ is a concave function of λ_k and has its maximum at $\lambda_k = \lambda_k^{\mathrm{MRT}}$. Also, we show that the interference power gain $x_{k\ell}$ is monotonically increasing in λ_k. Hence, for Pareto optimal operation, λ_k need only be in the range $[0, \lambda_k^{\mathrm{MRT}}]$.

The square root of the intended signal power gain x_{kk}, given in (5), is

$$x_{kk}(w_k(\lambda_k)) = p_k \sqrt{\lambda_k} \frac{h_{kk}^H \Pi_{h_{k\ell}} h_{kk}}{\|\Pi_{h_{k\ell}} h_{kk}\|} + p_k \sqrt{1-\lambda_k} \frac{h_{kk}^H \Pi_{h_{k\ell}}^{\perp} h_{kk}}{\|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|} - p_k^2 \varepsilon_{kk}. \tag{30}$$

The first derivative of $x_{kk}(w_k(\lambda_k))$ w.r.t. λ_k is calculated as

$$\frac{\partial x_{kk}(w_k(\lambda_k))}{\partial \lambda_k} = \frac{p_k}{2\sqrt{\lambda_k}} \frac{h_{kk}^H \Pi_{h_{k\ell}} h_{kk}}{\|\Pi_{h_{k\ell}} h_{kk}\|} - \frac{p_k}{2\sqrt{1-\lambda_k}} \frac{h_{kk}^H \Pi_{h_{k\ell}}^{\perp} h_{kk}}{\|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|}$$
$$= \frac{p_k}{2} \left(\frac{\|\Pi_{h_{k\ell}} h_{kk}\|}{\sqrt{\lambda_k}} - \frac{\|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|}{\sqrt{1-\lambda_k}} \right) \tag{31}$$

The second derivative of $x_{kk}(w_k(\lambda_k))$ w.r.t. λ_k is

$$\frac{\partial^2 x_{kk}(w_k(\lambda_k))}{\partial \lambda_k^2} = -\frac{p_k}{4} \left(\frac{\|\Pi_{h_{k\ell}} h_{kk}\|}{\sqrt{(\lambda_k)^3}} + \frac{\|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|}{\sqrt{(1-\lambda_k)^3}} \right) \tag{32}$$

which is strictly less than zero for $p_k > 0$. Hence, $x_{kk}(w_k(\lambda_k))$ is strictly concave in λ_k. To find λ_k which achieves the maximum of $x_{kk}(w_k(\lambda_k))$ we equate the first derivative to zero and solve for λ_k:

$$\frac{p_k}{2} \left(\frac{\|\Pi_{h_{k\ell}} h_{kk}\|}{\sqrt{\lambda_k}} - \frac{\|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|}{\sqrt{1-\lambda_k}} \right) = 0 \tag{33}$$

$$\frac{\|\Pi_{h_{k\ell}} h_{kk}\|}{\sqrt{\lambda_k}} = \frac{\|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|}{\sqrt{1-\lambda_k}} \tag{34}$$

$$\|\Pi_{h_{k\ell}} h_{kk}\| \sqrt{1-\lambda_k} = \|\Pi_{h_{k\ell}}^{\perp} h_{kk}\| \sqrt{\lambda_k} \tag{35}$$

$$\|\Pi_{h_{k\ell}} h_{kk}\|^2 - \lambda_k \|\Pi_{h_{k\ell}} h_{kk}\|^2 = \|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|^2 \lambda_k \tag{36}$$

$$\lambda_k (\|\Pi_{h_{k\ell}} h_{kk}\|^2 + \|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|^2) = \|\Pi_{h_{k\ell}} h_{kk}\|^2 \tag{37}$$

$$\lambda_k = \|\Pi_{h_{k\ell}} h_{kk}\|^2 / \|h_{kk}\|^2 \tag{38}$$

since $\|\Pi_{h_{k\ell}}^{\perp} h_{kk}\|^2 = \|h_{kk}\|^2 - \|\Pi_{h_{k\ell}} h_{kk}\|^2$.

The square root of the interference signal power gain $x_{k\ell}$ given in (7) is

$$x_{k\ell}(w_k(\lambda_k)) = p_k \sqrt{\lambda_k} \frac{|h_{k\ell}^H \Pi_{h_{k\ell}} h_{kk}|}{\|\Pi_{h_{k\ell}} h_{kk}\|} - p_k^2 \varepsilon_{k\ell} \tag{39}$$

The first derivative of $x_{k\ell}(w_k(\lambda_k))$ w.r.t. λ_k is

$$\frac{\partial x_{k\ell}(w_k(\lambda_k))}{\partial \lambda_k} = \frac{p_k}{2\sqrt{\lambda_k}} \frac{|h_{k\ell}^H \Pi_{h_{k\ell}} h_{kk}|}{\|\Pi_{h_{k\ell}} h_{kk}\|} \tag{40}$$

which is strictly larger than zero for $p_k > 0$. Thus, $x_{k\ell}(w_k(\lambda_k))$ is monotonically increasing with λ_k.

REFERENCES

[1] S. Vishwanath and S. Jafar, "On the capacity of vector Gaussian interference channels," in *Proc. ITW*, pp. 365–369, Oct. 2004.

[2] E. A. Jorswieck, E. G. Larsson, and D. Danev, "Complete characterization of the Pareto boundary for the MISO interference channel," *IEEE Trans. Signal Process.*, vol. 56, pp. 5292–5296, Oct. 2008.

[3] R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *IEEE Trans. Signal Process.*, vol. 58, pp. 5454–5462, Oct. 2010.

[4] R. Mochaourab and E. A. Jorswieck, "Optimal beamforming in interference networks with perfect local channel information," *IEEE Trans. Signal Process.*, vol. 59, pp. 1128–1141, Mar. 2011.

[5] J. Lindblom, E. G. Larsson, and E. A. Jorswieck, "Parameterization of the MISO IFC rate region: the case of partial channel state information," *IEEE Trans. Commun.*, vol. 9, pp. 500–504, Feb. 2010.

[6] S. Vorobyov, A. Gershman, and Z.-Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," *IEEE Trans. Signal Process.*, vol. 51, pp. 313–324, Feb. 2003.

[7] R. Lorenz and S. Boyd, "Robust minimum variance beamforming," *IEEE Trans. Signal Process.*, vol. 53, pp. 1684–1696, May 2005.

[8] J. Li and P. Stoica, eds., *Robust Adaptive Beamforming*. Hoboken, NJ: Wiley, 2006.

[9] A. Gershman, N. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *IEEE Signal Process. Mag.*, vol. 27, pp. 62–75, May 2010.

[10] A. Tajer, N. Prasad, and X. Wang, "Robust linear precoder design for multi-cell downlink transmission," *IEEE Trans. Signal Process.*, vol. 59, pp. 235–251, Jan. 2011.

[11] R. Mochaourab, E. A. Jorswieck, K. M. Z. Ho, and D. Gesbert, "Bargaining and beamforming in interference channels," in *Proc. Asilomar*, 2010.

[12] A. Lozano, A. Tulino, and S. Verdu, "High-SNR power offset in multiantenna communication," *IEEE Trans. Inf. Theory*, vol. 51, pp. 4134–4151, Dec. 2005.

[13] S. Verdu, "Spectral efficiency in the wideband regime," *IEEE Trans. Inf. Theory*, vol. 48, pp. 1319 –1343, Jun. 2002.