BEZOUT-BASED ROBUST PRECODING FOR MIMO FREQUENCY SELECTIVE CHANNEL USING IMPERFECT CHANNEL KNOWLEDGE

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ABSTRACT

In this paper, the problem of MIMO precoding is addressed for the case of single user, frequency selective wireless channel. Based on Bezout identity, two robust precoder designs are proposed that aim to inverse the effects of channel Intersymbol Interference (ISI) under two different scenarios of imperfect channel knowledge at the transmitter side. In the first one, specific statistical quantities of the channel are known at the transmitter while in the second one, a worst case scenario is adopted assuming that channel uncertainty falls within a given bounded set. Using standard optimization tools, both precoders are expressed in closed forms which are applicable for a wide range of channel conditions w.r.t. correlation in space and/or time, antenna numbers etc. Simulation results have shown that the proposed precoders perform comparably to the perfect channel knowledge case, offering at the same time some benefits in required transmitted power.

1. INTRODUCTION

In the last fifteen years MIMO communications have attracted a great research attention mainly because of the important performance improvement that can be achieved in the presence of multipath fading environments [1]. Typically, the performance of a MIMO system is evaluated under one of two different perspectives, either analytically or through simulations. In the first one, some kind of system capacity measure is analyzed and the fundamental limits of the system are explored. In the second one, the performance is investigated under a practical viewpoint in terms of complexity and BER of the employed processing techniques [2]. In [3], [4] some interesting conclusions are summarized for a variety of different assumptions regarding wireless channel conditions and characteristics, *e.g.* fades dependencies in time and space, time horizon analysis, amount of Intersymbol Interference (ISI) etc.

ISI is a form of signal distortion that is mainly caused by multipath transmission, limited available bandwidth and user mobility. In the presence of ISI, the quality of the transmission can be heavily degraded since the currently transmitted symbol may interfere with several subsequent transmitted symbols. In downlink transmission systems, ISI can be effectively equalized at receiver side. However, notable receiver simplification and power consuming benefits result when ISI is handled at transmitter side. Two main approaches are widely employed to combat ISI at transmitter side. In the first one, multicarrier transmission is performed, *e.g.*, by employing OFDM modulation while in the second one, the channel effects are pre-equalized (precoded) before single-carrier transmission is carried out [5], [6]. Especially in the latter case, the available Channel State Information (CSI) in any of the two communication ends plays a key role [7].

Channel State Information at the Receiver (CSIR) is usually obtained via training sequences and can be considered sufficiently accurate in most cases. However, this is not the case with Channel State Information at the Transmitter side (CSIT). Usually, perfect CSIT is not a trivial task in practice. Hence, precoder design has to be performed under imperfect or partial CSIT in a way that it remains robust under some kind of channel state uncertainties. Typically, robust precoding is classified in two main categories. In the first one, uncertainty is modeled as a random process and the whole design aims to guarantee a certain system performance on average. A plenty of relevant precoding designs have been proposed in literature particularly when Mean Square Error (MSE) criterion is used as a cost function [8], [9], [10]. In the second category, it is assumed that channel uncertainty at the transmitter is bounded within a range and no further restrictions are made for noise behavior [11]. Precoders of this category tend to be somewhat conservative and possible power consuming. However, a certain performance level is guaranteed for any possible channel uncertainty. In this work, two novel MIMO precoding schemes are proposed that aim to eliminate ISI at transmitter side. The optimization criterion appeared in [12] is adopted where the Bezout identity is used for channel inversion but the precoding problem is investigated for the two representative cases of imperfect CSIT described above. Using standard tools of optimization theory, the proposed precoders are expressed in closed forms which are applicable for a wide range of channel conditions w.r.t. time and space correlation with relatively low power consumption.

The rest of the paper is organized as follows: The system model and the addressed problem are described in Section 2 and emphasis is given on Bezout-based precoding and the assumed imperfect CSIT case. The proposed FIR precoders are presented in Section 3 and their performance is shown in Section 4. Finally, conclusions are drawn in Section 5.

Notation: In the following, lowercase bold letters denote vectors and uppercase bold letters denote matrices. $(\cdot)^T$, $(\cdot)^H$, $tr(\cdot)$ and $\mathbf{A}^{\frac{1}{2}}$ denote transpose, Hermitian transpose, matrix trace and the square root of matrix $\mathbf{A}$, respectively. $\|.\|$ denotes the euclidean norm when it is applied on vectors and the associated induced norm when it is applied on matrices. $\star$ is the convolution operator, $\otimes$ is the Kronecker product operator, $\delta(.)$ is the Dirac function and $\mathbb{E}\{.\}$ is the expectation operator.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1 System Model and Bezout Precoding

A MIMO frequency selective system is considered with N_t transmit antennas and N_r receive antennas with $N_r < N_t$. A d-order quasi-static model is assumed for the wireless channel, whose discrete time impulse response is described by a set of $N_r \times N_t$ complex matrices $\mathbf{H}[0], \dots \mathbf{H}[d]$. When a FIR precoder $\mathbf{F}(n)$ of order r is employed at the transmitter side (w.l.o.g. r is equal to d), the input-output relation in time instant n is

$$\mathbf{y}(n) = \mathbf{H}(n) \star \underbrace{\mathbf{F}(n) \star \mathbf{x}(n)}_{\mathbf{s}(n)} + \mathbf{w}(n), \quad (1)$$

where $\mathbf{x}(n) \in \mathbb{C}^{N_r}$ denotes the vector of information symbols, $\mathbf{w}(n) \in \mathbb{C}^{N_r}$ is the Zero Mean Circularly Symmetric Complex Gaussian (ZMCSCG) vector of additive noise with i.i.d. elements of zero mean and variance σ_w^2 and $\mathbf{s}(n) = \mathbf{F}(n) \star \mathbf{x}(n) = \sum_{k=0}^{r} \mathbf{F}[k]\mathbf{x}[n-k]$ is the transmitted vector of symbols. Equation (1) can be expressed in $\mathscr{Z}$-plane as

$$\mathbf{y}(z) = \mathbf{H}(z)\mathbf{F}(z)\mathbf{x}(z) + \mathbf{w}(z),$$

where $\mathbf{F}(z) = \sum_{k=0}^{r} \mathbf{F}[k]z^{-k}$ and $\mathbf{H}(z) = \sum_{k=0}^{d} \mathbf{H}[k]z^{-k}$. Figure 1 illustrates the overall system model.

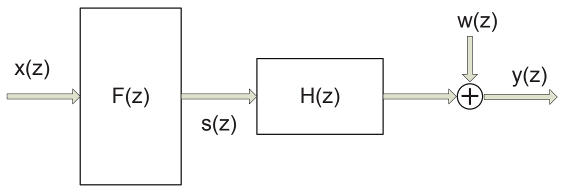

Figure 1: System Model.

When perfect CSI is available at transmitter side, a Zero Forcing (ZF) criterion can be adopted for complete ISI elimination at the receiver side. In such a case, each antenna's received symbol is a delayed version of the corresponding transmitted symbol [12], [13]

$$y_j(n) = x_j(n-k_j) + w_j(n), \ 1 \le j \le N_r. \quad (2)$$

Moreover, the detection at the receiver side can be based on the following criterion

$$\widehat{x}_j(n) = \arg\min_{k \in \mathscr{Q}} \|y_j(n) - s_k\|^2, 1 \le j \le N_r, \quad (3)$$

where $\mathscr{Q} = \{s_1 \dots s_M\}$ is the available transmission set of symbols. Nevertheless, providing transmitter with perfect CSI is a difficult task. In a more common approach, each channel tap is modeled as $\mathbf{H}[k] = \overline{\mathbf{H}}[k] + \widetilde{\mathbf{H}}[k]$, $k = 0 \dots d$, where $\overline{\mathbf{H}}[k]$ represents the long term channel evolution and $\widetilde{\mathbf{H}}[k]$ denotes the tap uncertainty. Generally, changes on $\overline{\mathbf{H}}[k]$ may be accurately tracked by the transmitter but regarding $\widetilde{\mathbf{H}}[k]$, only some kind of statistical knowledge is considered possible. Clearly, in such a case, complete ISI elimination is not possible. However, eq. (3) may be still used in the receiver's side for the detection of $\widehat{x}_j(n)$.

A Bezout precoder is a ZF precoder that is based on the usage of Bezout identity [13]. According to Bezout identity, for every full rank $N_r \times N_t$ ($N_r < N_t$) rectangular FIR matrix $\mathbf{H}(z)$, there is a $N_t \times N_r$ FIR matrix $\mathbf{F}(z)$ such that

$$\mathbf{H}(z)\mathbf{F}(z) = \mathbf{diag}(z^{-k_i}), \ 1 \le i \le N_r, \quad (4)$$

where z^{-k_i} are integer delays. The only condition on $\mathbf{H}(z)$ imposed by Bezout identity is that it should be left coprime which is valid when $k_i \ge 0$ [14]. In the following subsection, Bezout identity is used to form an objective function that quantifies ISI elimination by the proposed precoder designs.

2.2 Objective Function

The mismatch between the ideal ZF impulse responses $\mathbf{E}(n) = diag\{\delta(n-k_1) \dots \delta(n-k_{N_r})\}$ and the performance of a precoder $\mathbf{F}(n)$ can be minimized by optimizing the following objective function [12]

$$J(\mathbf{F}) = tr([\Gamma(\mathbf{H})\mathbf{F} - \mathbf{E}]^H [\Gamma(\mathbf{H})\mathbf{F} - \mathbf{E}]), \quad (5)$$

where all the precoder taps have been stacked together in $(r+1)N_t \times N_r$ block matrix $\mathbf{F} = \left[[\mathbf{F}[0]]^T [\mathbf{F}[1]]^T \dots [\mathbf{F}[r]]^T\right]^T$, $\mathbf{E} = \left[[\mathbf{E}(0)]^T [\mathbf{E}(1)]^T \dots [\mathbf{E}(d+r)]^T\right]^T$ and

$$\Gamma(\mathbf{H}) = \begin{bmatrix} \mathbf{H}[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{H}[1] & \mathbf{H}[0] & \ddots & \vdots \\ \vdots & \mathbf{H}[1] & \ddots & \mathbf{0} \\ \mathbf{H}[d] & \ddots & \ddots & \mathbf{H}[0] \\ \mathbf{0} & \mathbf{H}[d] & \ddots & \mathbf{H}[1] \\ \vdots & \ddots & \ddots & \ddots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}[d] \end{bmatrix}$$

denotes the $(d+r+1)N_r \times (r+1)N_t$ block convolution matrix. $J(\mathbf{F})$ is a convex function since its Hessian $\nabla^2 J(\mathbf{F})$ is positive semidefinite [15]. Moreover, $J(\mathbf{F}) = \sum_{j=1}^{N_r} J_j(\mathbf{f}_j)$ holds, where each convex function $J_j(\mathbf{f}_j)$ is given by $J_j(\mathbf{f}_j) = \|\Gamma(\mathbf{H})\mathbf{f}_j - \mathbf{e}_j\|^2$ and vectors $\mathbf{f}_j$, $\mathbf{e}_j$ are the j-th column vectors of matrices $\mathbf{F}$ and $\mathbf{E}$, respectively. Hence, the precoder design may be based on separated minimization of each term $J_j(\mathbf{f}_j)$, $j = 1 \dots N_r$, instead of overall minimization over $J(\mathbf{F})$ and the following minimization problem is formed

$$\min_{\mathbf{f}_j} \|\Gamma(\mathbf{H})\mathbf{f}_j - \mathbf{e}_j\|^2, \ \forall j = 1, \dots, N_r. \quad (6)$$

Clearly, when the convolution matrix is perfectly known in the transmitter and Bezout precoding is applied, then $\Gamma(\mathbf{H})\mathbf{F} = \mathbf{E}$ holds and $J(\mathbf{F}) = 0$. In such a case, the resulting precoder is written as $\mathbf{F} = \mathbf{V}^{-1}\mathbf{U}^H\mathbf{E}$ by using SVD on $\Gamma(\mathbf{H}) = \mathbf{U}\Sigma\mathbf{V}^H$.

3. PRECODER DESIGN USING IMPERFECT CHANNEL KNOWLEDGE

In the following two subsections, the problem of eq. (6) is solved using two different approaches, namely the Stochastic Robust Approximation (SRA) and the Worst-Case Robust Approximation (WCRA) [15]. In SRA, the minimization problem is solved over its expectation while in WCRA, a worst-case approach is followed. In both cases, transmitter possesses full knowledge of $\Gamma(\overline{\mathbf{H}})$ and only partial knowledge of $\Gamma(\widetilde{\mathbf{H}})$, as described in Section 2.

3.1 Precoder Design using SRA

When SRA is used, the problem of eq. (6) is solved over its expectation

$$\min_{\mathbf{f}_j} \ \mathbb{E}\left\{\left\|\Gamma(\mathbf{H})\mathbf{f}_j-\mathbf{e}_j\right\|^2\right\} \ \ \forall j=1,\ldots,N_r. \tag{7}$$

Given that $\Gamma(\mathbf{H})=\Gamma(\overline{\mathbf{H}})+\Gamma(\widetilde{\mathbf{H}})$ and $\mathbb{E}\left\{\Gamma(\widetilde{\mathbf{H}})\right\}=0$, the problem of eq. (7) can be written in the following Least-Square form

$$\min_{\{\mathbf{f}_j\}_{j=1}^{N_r}} \left(\|\Gamma(\overline{\mathbf{H}})\mathbf{f}_j-\mathbf{e}_j\|^2+\|\mathbb{E}\left\{\Gamma(\widetilde{\mathbf{H}})^H\Gamma(\widetilde{\mathbf{H}})\right\}^{\frac{1}{2}}\mathbf{f}_j\|^2\right) \tag{8}$$

Equation (8) can be solved by setting its gradient equal to zero. Thus, for $j=1\ldots N_r$, the analytical solution has the form

$$\mathbf{f}_j=\left[\Gamma(\overline{\mathbf{H}})^H\Gamma(\overline{\mathbf{H}})+\mathbb{E}\left\{\Gamma(\widetilde{\mathbf{H}})^H\Gamma(\widetilde{\mathbf{H}})\right\}\right]^{-1}\Gamma(\overline{\mathbf{H}})^H\mathbf{e}_j. \tag{9}$$

As it can be seen from eq. (9), only $\mathbb{E}\left\{\Gamma(\widetilde{\mathbf{H}})^H\Gamma(\widetilde{\mathbf{H}})\right\}$ is necessary for $\mathbf{f}_j$ computation ($\Gamma(\overline{\mathbf{H}})$ is commonly considered to be known). In most cases, the existence of such information at transmitter side is a reasonable assumption. For example, in case the entries of $\Gamma(\widetilde{\mathbf{H}})$ are considered i.i.d. random variables with $\mathcal{N}_{\mathbb{C}}(0,\sigma^2)$ distribution, then $\mathbb{E}[\Gamma(\widetilde{\mathbf{H}})^H\Gamma(\widetilde{\mathbf{H}})]$ will be a $(d+r+1)N_r\times(d+r+1)N_r$ block identity matrix multiplied by a factor $dN_r\sigma^2$. In Subsection 3.3 a similar conclusion is made for a more generic channel model w.r.t. tap and space correlation. In any case, the required knowledge by eq. (9) is certainly a lower burden than full CSIT which is required by Bezout precoder.

3.2 Precoder Design using WCRA

In WCRA design, the minimization of eq. (6) is performed under a worst-case approach. Let $\Phi\subseteq\mathbb{C}^{(d+r+1)N_r\times(r+1)N_t}$ denotes the non-empty and bounded set of every possible matrix $\Gamma(\mathbf{H})$. Given a feasible precoding vector $\mathbf{f}_j$, the worst-case error is defined as $e_{wc}(\mathbf{f}_j)=\sup\left\{\left\|\Gamma(\mathbf{H})\mathbf{f}_j-\mathbf{e}_j\right\| \mid \Gamma(\mathbf{H})\in\Phi\right\}$. Precoder design in WCRA aims to the following minimization

$$\min_{\{\mathbf{f}_j\}_{j=1}^{N_r}} \ \sup\{\|\Gamma(\mathbf{H})\mathbf{f}_j-\mathbf{e}_j\| \mid \Gamma(\mathbf{H})\in\Phi\}. \tag{10}$$

The problem of eq. (10) can be solved by Norm Bound Error method [15]. In such a case, uncertainty on $\Gamma(\mathbf{H})$ is considered within a norm ball of radius α and set Φ is written as $\Phi=\{\Gamma(\overline{\mathbf{H}})+\Gamma(\widetilde{\mathbf{H}}) \mid \|\Gamma(\widetilde{\mathbf{H}})\|\leq\alpha\}$, $\alpha\in\mathbb{Z}_+$. Let $e_{wc}^{NBE}(\mathbf{f}_j)=\sup\{\|\Gamma(\overline{\mathbf{H}})\mathbf{f}_j-\mathbf{e}_j+\Gamma(\widetilde{\mathbf{H}})\mathbf{f}_j\| \mid \|\Gamma(\widetilde{\mathbf{H}})\|\leq\alpha\}$ be the the worst-case error given precoding vector $\mathbf{f}_j$. It can be easily shown that e_{wc}^{NBE} is equal to $e_{wc}^{NBE}(\mathbf{f}_j)=\|\Gamma(\overline{\mathbf{H}})\mathbf{f}_j-\mathbf{e}_j\|+\alpha\|\mathbf{f}_j\|$ and it is attained for $\Gamma(\widetilde{\mathbf{H}})=\alpha\mathbf{u}\mathbf{v}^H$ where $\mathbf{u}=\frac{\Gamma(\overline{\mathbf{H}})\mathbf{f}_j-\mathbf{e}_j}{\|\Gamma(\overline{\mathbf{H}})\mathbf{f}_j-\mathbf{e}_j\|}$ and $\mathbf{v}=\frac{\mathbf{f}_j}{\|\mathbf{f}_j\|}$, given that $\Gamma(\overline{\mathbf{H}})\mathbf{f}_j-\mathbf{e}_j\neq 0$ and $\mathbf{f}_j\neq 0$ [15]. Thus, the minimization problem of eq. (10) can be written in the following Tikhonov Regularization form with parameter α

$$\min_{\{\mathbf{f}_j\}_{j=1}^{N_r}} \ \|\Gamma(\overline{\mathbf{H}})\mathbf{f}_j-\mathbf{e}_j\|+\alpha\|\mathbf{f}_j\|. \tag{11}$$

Depending on the way the parameter α is specified by the design, more emphasis may be given either on power consumption or on channel effects mitigation. The analytical solution of eq.(11) is attained by setting $\nabla e_{wc}^{NBE}(\mathbf{f}_j)=0$

$$\mathbf{f}_j=\left[\Gamma(\overline{\mathbf{H}})^H\Gamma(\overline{\mathbf{H}})+\alpha\mathbf{I}\right]^{-1}\Gamma(\overline{\mathbf{H}})^H\mathbf{e}_j. \tag{12}$$

3.3 Precoding in the Presence of Tap and Space Correlation

Clearly, both problems of eq. (8) and eq. (10) are examples of Tikhonov Regularization problems. As it is well known in optimization theory, Tikhonov Regularization does not pose any restriction on the rank of the involved matrices, *e.g.* rank of $\Gamma(\widetilde{\mathbf{H}})$ and $\Gamma(\overline{\mathbf{H}})$ in eq. (9) and eq. (12), respectively [15]. Hence, a noticeable advantage of the analytical solutions of eq. (9) and eq. (12) is that the resulted precoders may be efficiently applied in a wide range of channel conditions w.r.t. spatial and time correlations. Under the common assumption of independent and separable tap and spatial correlation, a generic representation of the ISI channel $\mathbf{H}=[\mathbf{H}[0]\ldots\mathbf{H}[d]]$ in time domain is [16]

$$\mathbf{H}=\mathbf{H}_G\left(\mathbf{R}_d^{T/2}\otimes\mathbf{R}_{N_t}^{1/2}\right), \tag{13}$$

where elements of $\mathbf{H}_G$ follow i.i.d. $\mathcal{N}_{\mathbb{C}}(0,1)$ and matrices $\mathbf{R}_d$, $\mathbf{R}_{N_t}$ correspond to tap and transmit antenna correlation, respectively. If the model of imperfect channel knowledge that has been used so far is merged on eq. (13), $\mathbf{H}$ is described as

$$\mathbf{H}=\left(\overline{\mathbf{H}}+\widetilde{\mathbf{H}}\right)\left(\mathbf{R}_d^{T/2}\otimes\mathbf{R}_{N_t}^{1/2}\right)$$

where $\overline{\mathbf{H}}\left(\mathbf{R}_d^{T/2}\otimes\mathbf{R}_{N_t}^{1/2}\right)$ and $\widetilde{\mathbf{H}}\left(\mathbf{R}_d^{T/2}\otimes\mathbf{R}_{N_t}^{1/2}\right)$ correspond to the known and partially known channel parts, respectively. Thus, it can be easily shown that each (sub)diagonal of the block-Toeplitz matrix $\mathbb{E}\left\{\Gamma(\mathbf{H})^H\Gamma(\mathbf{H})\right\}$ equals to the sum of corresponding (sub)diagonal of the matrix $\mathbb{E}\left\{\mathbf{H}^H\mathbf{H}\right\}$. Hence, the only required knowledge by SRA precoder of eq. (9) is related to matrices $\mathbf{R}_d$ and $\mathbf{R}_{N_t}$.

4. SIMULATION RESULTS

In this section, simulation results are presented illustrating the performance of the two proposed precoders. A 8×2 MIMO structure is used as a baseline configuration. However, it should be noted that extensive experiments have been contacted for several antenna configurations leading to similar conclusions as the ones presented here. The order of both the precoder and the ISI channel is $d=r=4$. Each channel tap k, $k = 0\ldots d$, is generated as the sum of two ZMCSCG matrices $\overline{\mathbf{H}}[k]$ and $\widetilde{\mathbf{H}}[k]$ with elements that follow i.i.d. $\mathscr{N}_{\mathbb{C}}(0,1)$ and $\mathscr{N}_{\mathbb{C}}(0,\sigma^2_{\widetilde{\mathbf{H}}})$ distributions, respectively. The matrix $\widetilde{\mathbf{H}}$ is refreshed 1000 times faster than $\overline{\mathbf{H}}$. An exponential model is used for transmit antenna correlation, i.e., $\mathbf{R}_{N_t}(i,j) = \rho_{N_t}^{|i-j|}\ \forall i,j = 1\ldots N_t$ and $0 \leq |\rho_{N_t}| \leq 1$. Moreover, a Toeplitz tap correlation matrix $\mathbf{R}_d$ is used where the elements of the first column are specified by vector $\mathbf{p} = [p_0, p_1, \cdots, p_d]$. In all figures, QPSK modulation is used and SNR is defined as $SNR_{dB} = 10\log_{10}\frac{tr(\mathbf{F}\mathbf{F}^H)\sigma_x^2}{\sigma_w^2}$ where the variance of the transmitted symbols is equal $\sigma_x^2 = 2$.

In Fig. 2 and Fig. 3, the BER performance of SRA and WCRA is shown for $\sigma_{\widetilde{\mathbf{H}}} = 0.1$ and different cases of channel correlation, respectively. More explicitly, besides the case where elements of $\mathbf{H}$ are uncorrelated, three more cases are examined. In the first one, only space correlation is considered with $\rho_{N_t} = 0.7$. In the second one, low-correlation is considered where tap and space correlation are specified by $\rho_{N_t} = 0.35$ and $\mathbf{p} = [1, 0.35, 0.25, 0.2, 0.1]^T$, respectively. Finally, in the third one, high-correlation is considered where tap and space correlation are specified by $\rho_{N_t} = 0.7$ and $\mathbf{p} = [1, 0.7, 0.5, 0.4, 0.2]^T$, respectively.

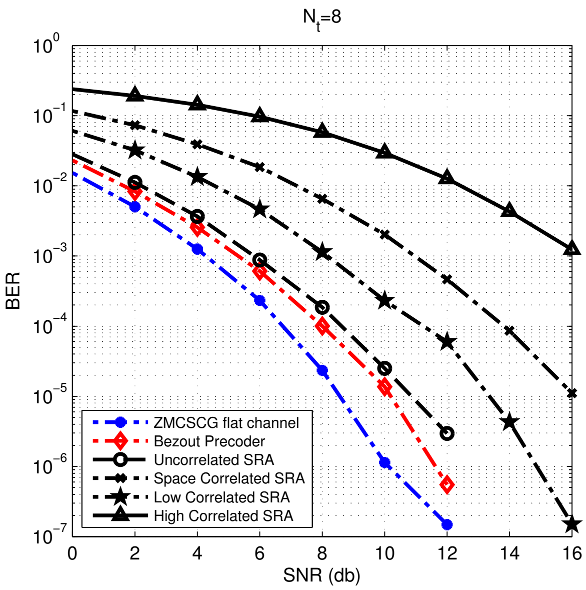

Figure 2: BER versus SNR for SRA precoder for $\sigma_{\widetilde{\mathbf{H}}} = 0.1$

As it can be seen from Fig. 2 and Fig. 3, both SRA and WCRA precoders perform closely to the perfect channel inversion by Bezout precoder [12], [13], especially in the area of low and medium SNRs. In the high SNRs area, the Bezout Precoder outperforms both SRA and WCRA since ISI is the main cause of degradation and lack of perfect CSIT affects notably the performance of the proposed schemes. As regards the role of correlation, it seems that correlation results in performance degradation of both SRA and WCRA precoders. As correlation becomes stronger, this degradation becomes higher, and hence higher SNR values are required for BERs with practical interest.

In Fig. 4 and Fig. 5, the SRA and WCRA transmitted powers versus $\sigma_{\widetilde{\mathbf{H}}}$ are shown for different values of N_t and

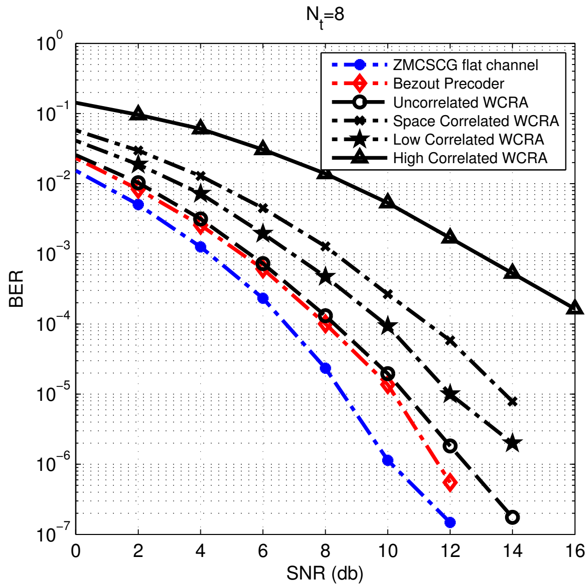

Figure 3: BER versus SNR for WCRA precoder for $\sigma_{\widetilde{\mathbf{H}}} = 0.1$

three different cases of channel correlation, namely the uncorrelated case, the case where only space correlation exists with $\rho_{N_t} = 0.7$ and the case where space and tap correlation exist with $\rho_{N_t} = 0.7$ and $\mathbf{p} = [1, 0.7, 0.5, 0.4, 0.2]^T$.

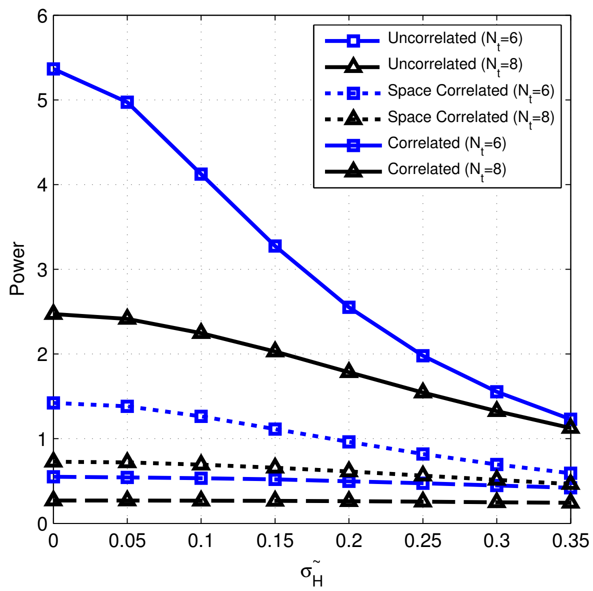

Figure 4: Precoder power versus $\sigma_{\widetilde{\mathbf{H}}}$ for SRA precoder

It can be seen that as the value of $\sigma_{\widetilde{\mathbf{H}}}$ increases, the power of SRA and WCRA precoders either decreases or remains almost the same. In other words, as the available CSIT gets more inaccurate, the proposed precoders avoid to increase the transmitted power in order to not amplify the residual ISI. Such a degradation has been verified experimentally (as the value of $\sigma_{\widetilde{\mathbf{H}}}$ increases). However the corresponding curves have not been included due to space limitations. Moreover, the same conclusions hold true in the presence of channel correlation with the only difference that a higher amount of power is required as correlation becomes stronger, especially in the area of $\sigma_{\tilde{\mathbf{H}}} \leq 0.2$ where the proposed precoders perform competitively. Clearly, as $\sigma_{\tilde{\mathbf{H}}} \to 0$, the SRA and WCRA reduce to Bezout precoder. Hence, both SRA and WCRA precoders offer a more balanced power management as compared to Bezout Precoder which is considerable in the case of correlated involved channels.

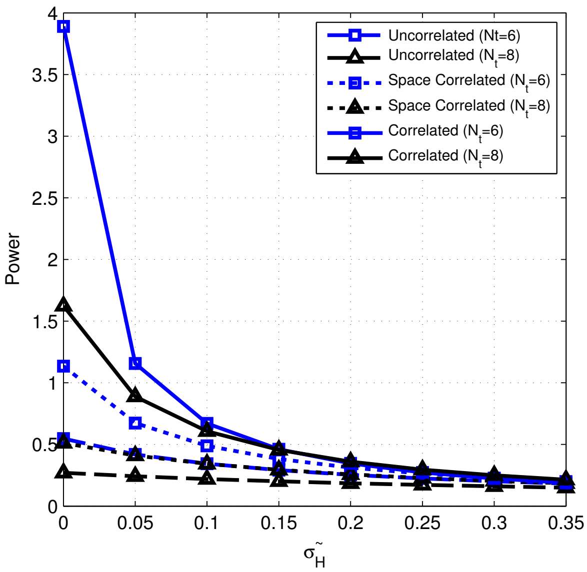

Figure 5: Precoder power versus $\sigma_{\tilde{\mathbf{H}}}$ for WCRA precoder

Finally, it should be noticed that $\|\mathbf{f}_j\|$ minimization was emphasized in the case of WCRA precoder. As it was mentioned in Section 3.2, parameter α is proportionally related to the value of $\sigma_{\tilde{\mathbf{H}}}$. Here, a relatively high value was selected for α, i.e., $\alpha = 5\sigma_{\tilde{\mathbf{H}}}$. The role of parameter α and the way it affects WCRA precoder performance is a subject of a relevant future work.

5. CONCLUSIONS

In this paper two novel precoding schemes have been presented for the case of frequency selective channels in single user transmission. In both schemes it has been assumed that the transmitter has only imperfect channel knowledge, which is in a statistical form for the first precoder and in a channel worst-case form for the second precoder. Using Tikhonov Regularization theory closed form expressions have been derived for both precoders. The new schemes turn out to have competitive performance even under correlated channels' conditions and, furthermore, they offer savings in required transmitted power. Finally, there have been presented simulation results which confirm the advantages of the proposed schemes.

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