ZF DFE TRANSCEIVER DESIGN FOR MIMO RELAY SYSTEMS WITH DIRECT SOURCE-DESTINATION LINK

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ABSTRACT

In this paper we consider a non-linear transceiver design for non-regenerative multiple-input multiple-output (MIMO) relay networks where a direct link exists between the source and destination. Our system utilises linear processors at the source and relay as well as a zero-forcing (ZF) decision feedback equaliser (DFE) at the receiver. Under the assumption that full channel state information (CSI) is available the precoding and equaliser matrices are designed to minimise the arithmetic mean square error (MSE) whilst meeting transmit power constraints at the source and relay. The source, relay, and destination processors are provided in closed form solution. In the absence of the direct link our design particularises to a previous ZF DFE solution and as such can be viewed as a generalisation of an existing work. We demonstrate the effectiveness of the proposed solution through simulation and show that it outperforms existing techniques in terms of bit error ratio (BER).

1. INTRODUCTION

The use of relaying nodes to forward data from a source to destination offers spatial diversity in a communication system which can extend network coverage, increase channel capacity, and improve link reliability [1], [2], [3]. When each node in the network is equipped with multiple antennas the system is referred to as a MIMO relay system. MIMO relaying has gained significant attention from researchers lately and are considered an integral component in the design of next generation wireless networks.

MIMO relay transceivers are generally categorised as either decode forward (DF) or amplify forward (AF) [1], which are also commonly known as regenerative and non-regenerative respectively. In the case of regenerative relaying the relay node decodes the received signal, re-encodes the data bits, and then forwards to the next node in the network. In non-regenerative protocols the relay simply transmits an amplified version of the received signal to the subsequent node in the system. It is well known performing linear precoding at the relay can significantly enhance performance when compared to the conventional AF protocol [2], [4], [5].

Linear transceiver designs have been well studied for non-regenerative MIMO relaying. In [4] the optimal relay precoder that minimises the arithmetic MSE is derived under the assumption that the source precoder is a scaled identity matrix. A unified framework, based on majorisation theory, is presented in [5] for multi-carrier non-regenerative MIMO relays where the optimal source and relay precoders are derived for Schur convex and Schur concave objective functions. It is shown that for Schur concave objective functions the optimal source and relay processors diagonalise the MSE matrix and subsequently the overall communication system is converted to a set of single-input single-output (SISO) subsystems. For the case of Schur convex objective functions the optimal processors result in a MSE matrix with non-diagonal structure and the system is only diagonalised up to a specific rotation of the transmit and receive data symbols.

Non-linear techniques have also been considered for two-hop MIMO relay systems. The solution for the source and relay precoders when a DFE is utilised at the destination was investigated in [6]. In [7] minimum MSE processors were derived for a non-regenerative MIMO relay network with Tomlinson Harashima precoding employed at the source.

The works in [4], [5], [6], and [7] all assumed that the destination did not use any information received directly from the source. The direct source-destination link provides extra spatial diversity in the system and can lead to further benefits in performance. The authors in [8] investigate the joint design of linear precoders to minimise the arithmetic MSE when the direct link is included. It is shown that with the inclusion of the direct link the optimisation problem is very complicated and a direct minimisation of the MSE is difficult. As such the authors propose to minimise an upper bound on the MSE which, although leads to a suboptimal solution, significantly simplifies the design process.

In this work we focus on the design of linear processors when a ZF DFE is employed at the destination and a direct path exists between the source and destination antennas. We assume that each node in the network has full CSI and derive processors to minimise the arithmetic MSE. The remainder of the paper is organised as follows: In section 2 we introduce the signal model for the system under consideration. Section 3 presents the transceiver design and a numerical example is provided in section 4. Finaly conclusions are drawn in 5.

Notation: We conform to the standard notation where vectors and matrices are denoted by lower and upper case bold font respectively. The sets of real and complex numbers are $\mathbb{R}$ and $\mathbb{C}$, which in the case of vector and matrix quantities indicate dimensions by means of a superscript. The operators $\mathbb{E}\{\cdot\}$, $\mathrm{tr}\{\cdot\}$, $(\cdot)^{\mathrm{H}}$, $(\cdot)^{-1}$, $(\cdot)^{\dagger}$, and $|\cdot|$ denote the expectation, trace, hermitian transpose, inverse, pseudo-inverse, and determinant respectively. $\mathbf{I}_M$ is the $M \times M$ identity matrix and $\mathbf{0}_{N\times M}$ is a matrix of dimension $N \times M$ with zero entries. The element in the i_{th} row and j_{th} column of matrix $\mathbf{A}$ is denoted as a_{ij} and the i_{th} element of vector $\mathbf{a}$ is denoted as a_i. Matrix rank is noted by $rank(\cdot)$ and $diag\{a_1, a_2, ..., a_N\}$ denotes a diagonal matrix with diagonal entries $\{a_1, a_2, ..., a_N\}$. The operators $min(a,b)$ and $max(a,b)$ return the minimum and maximum values of a and b and we define $[x]^+ \triangleq max(x,0)$.

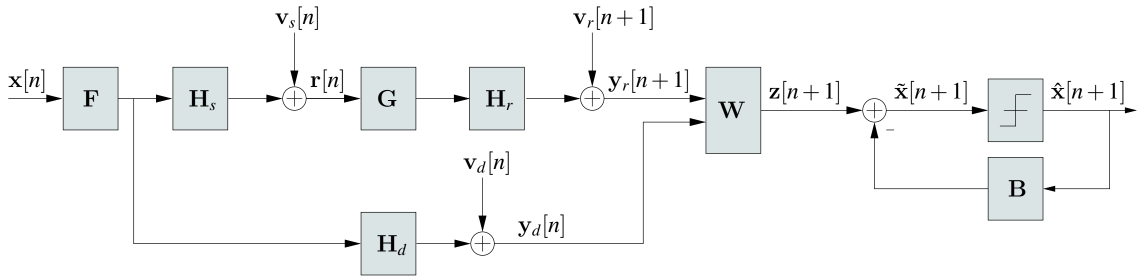

Figure 1: DFE signal model for a two-hop MIMO relay system with direct source-destination link.

2. SYSTEM MODEL

We consider transmission of N_s data streams over a two-hop MIMO relay system where the source, relay, and destination have N_s, N_r, and N_d antennas respectively and a direct link exists between the source and destination. To ensure N_s data streams can be transmitted across the network we assume that $N_s \leq min(N_r, N_d)$. For the purposes of interference cancellation we admit linear processors in each stage of the network and employ a DFE at the receiver. This configuration is shown in Figure 1.

Due to half-duplex relaying the transmission of data from source to destination is carried out over two seperate time-slots. In the first phase the data symbols $\mathbf{x}[n] \in \mathbb{C}^{N_s}$, which are assumed to be uncorrelated with covariance $\mathbb{E}\{\mathbf{x}[n]\mathbf{x}[n]^{\mathrm{H}}\} = \mathbf{I}_{N_s}$, are transmitted to both the relay and destination. The relay receives $\mathbf{r}[n] \in \mathbb{C}^{N_r}$ given by

$$\mathbf{r}[n] = \mathbf{H}_s\mathbf{F}\mathbf{x}[n] + \mathbf{v}_s[n] \quad (1)$$

where $\mathbf{H}_s \in \mathbb{C}^{N_r \times N_s}$ is the source-relay channel, $\mathbf{F} \in \mathbb{C}^{N_s \times N_s}$ is the linear source precoder, and $\mathbf{v}_s[n] \in \mathbb{C}^{N_r}$ is an additive white Gaussian noise (AWGN) vector with covariance $\mathbf{R}_{v_s v_s} = \mathbb{E}\{\mathbf{v}_s[n]\mathbf{v}_s^{\mathrm{H}}[n]\} = \sigma_{v_s}^2\mathbf{I}_{N_r}$. The signal $\mathbf{y}_d[n] \in \mathbb{C}^{N_d}$ received at the destination in the first time-slot is

$$\mathbf{y}_d[n] = \mathbf{H}_d\mathbf{F}\mathbf{x}[n] + \mathbf{v}_d[n] \quad (2)$$

where $\mathbf{H}_d \in \mathbb{C}^{N_d \times N_s}$ is the direct source-destination channel and $\mathbf{v}_d[n] \in \mathbb{C}^{N_d}$ is an AWGN vector with covariance matrix $\mathbf{R}_{v_d v_d} = \mathbb{E}\{\mathbf{v}_d[n]\mathbf{v}_d^{\mathrm{H}}[n]\} = \sigma_{v_d}^2\mathbf{I}_{N_d}$.

In the second phase of transmission the relay processes the signal $\mathbf{r}[n]$ and transmits across the relay-destination channel $\mathbf{H}_r \in \mathbb{C}^{N_d \times N_r}$ resulting in the data received at the destination in the second phase $\mathbf{y}_r[n+1] \in \mathbb{C}^{N_d}$ being

$$\mathbf{y}_r[n+1] = \mathbf{H}_r\mathbf{G}\mathbf{r}[n] + \mathbf{v}_r[n+1] \quad (3)$$

where $\mathbf{G} \in \mathbb{C}^{N_r \times N_r}$ and $\mathbf{H}_r \in \mathbb{C}^{N_d \times N_r}$ are the relay precoder and relay-destination channel respectively and the noise vector $\mathbf{v}_r[n+1] \in \mathbb{C}^{N_r \times N_d}$ contains AWGN samples with covariance $\mathbf{R}_{v_r v_r} = \mathbb{E}\{\mathbf{v}_r[n+1]\mathbf{v}_r^{\mathrm{H}}[n+1]\} = \sigma_{v_r}^2\mathbf{I}_{N_d}$.

The data received at the destination over two consecutive time-slots, given in (2) and (3), is processed by the linear equaliser $\mathbf{W} \in \mathbb{C}^{N_s \times 2N_d}$ resulting in

$$\mathbf{z}[n+1] = \mathbf{W}\mathbf{H}\mathbf{F}\mathbf{x}[n] + \mathbf{W}\mathbf{v}[n+1] \quad (4)$$

where for notational convenience we define

$$\mathbf{H} \triangleq \begin{bmatrix} \mathbf{H}_d \\ \mathbf{H}_r\mathbf{G}\mathbf{H}_s \end{bmatrix}, \ \mathbf{v}[n+1] \triangleq \begin{bmatrix} \mathbf{v}_d[n] \\ \mathbf{H}_r\mathbf{G}\mathbf{v}_s[n] + \mathbf{v}_r[n+1] \end{bmatrix}. \quad (5)$$

Here $\mathbf{H} \in \mathbb{C}^{2N_d \times N_s}$ is the effective MIMO channel between the source and destination antennas and $\mathbf{v}[n+1] \in \mathbb{C}^{2N_d}$ is the equivalent noise vector at the equaliser input with covariance matrix $\mathbf{R}_{vv} = \mathbb{E}\{\mathbf{v}[n+1]\mathbf{v}^{\mathrm{H}}[n+1]\}$ given by

$$\mathbf{R}_{vv} = \begin{bmatrix} \sigma_{v_d}^2\mathbf{I}_{N_d} & \mathbf{0}_{N_d \times N_d} \\ \mathbf{0}_{N_d \times N_d} & \mathbf{H}_r\mathbf{G}\mathbf{G}^{\mathrm{H}}\mathbf{H}_r^{\mathrm{H}}\sigma_{v_s}^2 + \sigma_{v_r}^2\mathbf{I}_{N_d} \end{bmatrix}. \quad (6)$$

After processing by $\mathbf{W}$, successive interference cancellation is performed by a strictly upper right triangular matrix $\mathbf{B} \in \mathbb{C}^{N_s \times N_s}$. The data estimates in $\hat{\mathbf{x}}[n+1] \in \mathbb{C}^{N_s}$ are recursively computed according to [9]

$$\hat{x}_i[n+1] = C\left[z_i[n+1] - \sum_{j=i+1}^{N_s} b_{ij}\hat{x}_j[n+1]\right] \quad (7)$$

for $j = N_s, N_s - 1, \ldots, 1$. The operator $C[.]$ signifies a mapping to the nearest constellation point of the transmitted symbols. The operation in (7) is equivalent to successively making decisions [10] on

$$\tilde{\mathbf{x}}[n+1] = \mathbf{W}\mathbf{H}\mathbf{F}\mathbf{x}[n] + \mathbf{W}\mathbf{v}[n+1] - \mathbf{B}\hat{\mathbf{x}}[n+1]. \quad (8)$$

The error between the input to the decision device and the transmitted symbols is defined as $\mathbf{e}[n+1] \triangleq \tilde{\mathbf{x}}[n+1] - \mathbf{x}[n]$ which using (8) results in

$$\mathbf{e}[n+1] = (\mathbf{W}\mathbf{H}\mathbf{F} - \mathbf{U})\,\mathbf{x}[n] + \mathbf{W}\mathbf{v}[n+1] \quad (9)$$

where we have used the standard assumption of correct past decisions [9], [10], and we define $\mathbf{U} \triangleq \mathbf{B} + \mathbf{I}_{N_s}$ as a unit diagonal upper right triangular matrix. Using the error signal in (9) the error covariance matrix $\mathbf{R}_{ee} = \mathbb{E}\{\mathbf{e}[n+1]\mathbf{e}^{\mathrm{H}}[n+1]\}$ can be computed as

$$\mathbf{R}_{ee} = (\mathbf{W}\mathbf{H}\mathbf{F} - \mathbf{U})(\mathbf{W}\mathbf{H}\mathbf{F} - \mathbf{U})^{\mathrm{H}} + \mathbf{W}\mathbf{R}_{vv}\mathbf{W}^{\mathrm{H}}. \quad (10)$$

The transceiver in this paper aims to minimise the arithmetic MSE, which is simply given by $\mathrm{tr}\{\mathbf{R}_{ee}\}/N_s$, whilst meeting certain system constraints.

3. TRANSCEIVER DESIGN

In this section we derive the processors that minimise the system arithmetic MSE whilst abiding by the ZF condition as well as power constraints at both the source and relay terminals.

3.1 Optimal ZF Equaliser

The ZF condition ensures a perfect reconstruction of the transmit symbols in the absence of noise and imposes the following constraint on the equaliser matrix $\mathbf{W}$

$$\mathbf{WHF} = \mathbf{U} \tag{11}$$

which upon substituting in (10) results in the error covariance matrix $\mathbf{R}_{ee}$ reducing to

$$\mathbf{R}_{ee} = \mathbf{W}\mathbf{R}_{vv}\mathbf{W}^{\mathrm{H}}. \tag{12}$$

It is well known [10] that, for a given $\mathbf{U}$, $\mathbf{H}$, $\mathbf{F}$, and $\mathbf{R}_{vv}$, the optimal solution for $\mathbf{W}$ that minimises (12) and satisfies the condition in (11) is provided by

$$\mathbf{W} = \mathbf{U}\left(\mathbf{R}_{vv}^{-1/2}\mathbf{HF}\right)^{\dagger}\mathbf{R}_{vv}^{-1/2}. \tag{13}$$

Since $N_s \leq N_d$ the product $\mathbf{R}_{vv}^{-1/2}\mathbf{HF} \in \mathbb{C}^{2N_d \times N_s}$ has more rows than columns and as such the pseudo-inverse [11] is given by $(\mathbf{R}_{vv}^{-1/2}\mathbf{HF})^{\dagger} = (\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{HF})^{-1}\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1/2}$. Substituting (13) in (12) we arrive at the MSE matrix

$$\mathbf{R}_{ee} = \mathbf{U}\left(\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{HF}\right)^{-1}\mathbf{U}^{\mathrm{H}} \tag{14}$$

where we note that the error covariance is now no longer a function of the equaliser matrix $\mathbf{W}$.

3.2 Constrained Optimisation Problem

As previously mentioned the transceiver design in this paper aims to minimise the arithmetic MSE. However, as in [6], [7], and [10], we propose to minimise the geometric MSE, which lower bounds the arithmetic MSE, and then suitably contruct processors such that the arithmetic MSE achieves the minimised lower bound.

The relationship between the arithmetic and geometric MSE is a simple consequence of the arithmetic-geometric mean inequality [11] which states that for a positive semi-definite matrix $\mathbf{A} \in \mathbb{C}^{N \times N}$ we have the inequality $|\mathbf{A}|^{1/N} \leq \mathrm{tr}\{\mathbf{A}\}/N$ where equality is achieved if, and only if, $\mathbf{A} = \alpha\mathbf{I}_N$ for any $\alpha \geq 0$. Applying the arithmetic-geometric mean inequality to (14) we obtain the following bounds

$$\begin{aligned} &|\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{HF})|^{-1/N_s} \\ &\leq \mathrm{tr}\left\{\mathbf{U}(\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{HF})^{-1}\mathbf{U}^{\mathrm{H}}\right\}/N_s \end{aligned} \tag{15}$$

where we have used the facts that $|\mathbf{A}^{-1}| = |\mathbf{A}|^{-1}$, $|\mathbf{AB}| = |\mathbf{A}||\mathbf{B}|$ for square matrices $\mathbf{A}$ and $\mathbf{B}$, and $|\mathbf{U}| = |\mathbf{U}^{\mathrm{H}}| = 1$ since $\mathbf{U}$ is a unit diagonal triangular matrix. Using the lower bound in (15) as the objective function and considering the source and relay power constraints we arrive at

$$\min_{\mathbf{F},\mathbf{G}} \quad |(\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{HF})|^{-1/N_s} \tag{16}$$

$$\text{s.t.} \quad \mathrm{tr}\left\{\mathbf{FF}^{\mathrm{H}}\right\} = P_s \tag{17}$$

$$\mathrm{tr}\left\{\mathbf{G}(\mathbf{H}_s\mathbf{FF}^{\mathrm{H}}\mathbf{H}_s^{\mathrm{H}} + \sigma_{v_s}^2\mathbf{I}_{N_r})\mathbf{G}^{\mathrm{H}}\right\} = P_r \tag{18}$$

Here (17) and (18) are the source and relay power constraints and P_s and P_r are the available power budgets. We note that the optimisation problem in (16), (17), and (18) is the same as that for the ZF DFE design in [6]. However, as will be seen, the solution differs significantly due to the inclusion of the direct source-destination link.

3.3 Precoder and Feedback Processors

We now focus on the design of the source and relay precoders, $\mathbf{F}$ and $\mathbf{G}$, as the solution to (16), (17), and (18) as well as the feedback matrix $\mathbf{B}$.

3.3.1 Source Precoder Structure

For a given relay precoder $\mathbf{G}$ that satisfies the relay power constraint in (18) we introduce the decomposition

$$\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{H} = \mathbf{V}_h\mathbf{\Lambda}_h\mathbf{V}_h^{\mathrm{H}} \tag{19}$$

where $\mathbf{V}_h \in \mathbb{C}^{N_s \times N_s}$ is a unitary matrix and the diagonal matrix $\mathbf{\Lambda}_h = diag\left\{\lambda_{h_1}, \lambda_{h_2}, \ldots, \lambda_{h_{N_s}}\right\}$ contains the non-zero singular values of $\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{H}$. We assume here that the matrix $\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{H}$ is full rank i.e. $rank\left\{\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{H}\right\} = N_s$.

Using the Hadamard determinant inequality [11] we can state from (16) and using the decomposition in (19) that

$$|\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{HF}|^{-1/N_s} \geq \prod_{i=1}^{N_s}\left(\gamma_i^2\lambda_{h_i}\right)^{-1/N_s} \tag{20}$$

where the lower bound is achieved with any processor $\mathbf{F}$ given by $\mathbf{F} = \mathbf{V}_h\mathbf{\Gamma\Psi}$ where $\mathbf{\Gamma} = diag\{\gamma_1, \gamma_2, \ldots, \gamma_{N_s}\}$ and $\mathbf{\Psi} \in \mathbb{C}^{N_s \times N_s}$ is an arbitrary unitary matrix. The elements of the power allocation matrix $\mathbf{\Gamma}$ can be found by substituting $\mathbf{F} = \mathbf{V}_h\mathbf{\Gamma\Psi}$ in (16) and (17) and solving

$$\min_{\gamma_i} \quad \prod_{i=1}^{N_s}\left(\gamma_i^2\lambda_{h_i}\right)^{-1/N_s} \tag{21}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_s}\gamma_i^2 = P_s, \quad \gamma_i^2 \geq 0, \ \forall i. \tag{22}$$

This is a standard convex optimisation problem and the solution can be found using the Karush-Kuhn-Tucker (KKT) method [12] and is given by $\gamma_i = \sqrt{P_s/N_s}$, $\forall i$. We can thus state that, for any relay precoder $\mathbf{G}$ that satisfies (18), the source precoder that minimises (16) and satisfies (17) is given by

$$\mathbf{F} = \sqrt{P_s/N_s}\mathbf{V}_h\mathbf{\Psi} \tag{23}$$

where $\mathbf{\Psi}$ is a unitary matrix yet to be determined.

3.3.2 Relay Precoder Structure

Having established the structure of the source precoder $\mathbf{F}$ we now focus on computing the relay processor $\mathbf{G}$. We firstly note that using the definitions for $\mathbf{H}$ and $\mathbf{R}_{vv}$ in (5) and (6) and the source precoder in (23) we can expand the determinant in (16) as

$$|\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{vv}^{-1}\mathbf{HF}|^{-1/N_s} = (N_s/P_s)|\mathbf{D} + \mathbf{XYZ}|^{-1/N_s} \tag{24}$$

where for convenience we define matrices $\mathbf{D} \triangleq \mathbf{H}_d^{\mathrm{H}}\mathbf{R}_{v_dv_d}^{-1}\mathbf{H}_d$, $\mathbf{X} \triangleq \mathbf{H}_s^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_r^{\mathrm{H}}$, $\mathbf{Y} \triangleq \left(\mathbf{H}_r\mathbf{GG}^{\mathrm{H}}\mathbf{H}_r^{\mathrm{H}}\sigma_{v_s}^2 + \sigma_{v_r}^2\mathbf{I}_{N_r}\right)^{-1}$, and $\mathbf{Z} \triangleq \mathbf{H}_r\mathbf{GH}_s$. We note here that when the direct link is negligible (24) particularises to the objective function for the ZF design in [6]. As such our design is a generalisation of the ZF DFE transceiver in [6] to include the direct source-destination link.

The relay precoder $\mathbf{G}$ must be designed such that $|\mathbf{D} + \mathbf{XYZ}|^{-1/N_s}$ is minimised and the power constraint in

$$(N_s/P_s)|\mathbf{D}|^{-1/N_s}|\mathbf{I}_{N_s}+\mathbf{D}^{-1/2}\mathbf{H}_s^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_r^{\mathrm{H}}\left(\mathbf{H}_r\mathbf{G}\mathbf{G}^{\mathrm{H}}\mathbf{H}_r^{\mathrm{H}}\sigma_{v_s}^2+\sigma_{v_r}^2\mathbf{I}_{N_d}\right)^{-1}\mathbf{H}_r\mathbf{G}\mathbf{H}_s\mathbf{D}^{-1/2}|^{-1/N_s}$$
$$\geq (N_s/P_s)\prod_{i=1}^{N_s}\lambda_{d_i}^{-1/N_s}\prod_{i=1}^{N_s}\left(1+\lambda_{t_i}^2\phi_i^2\lambda_{r_i}^2\left(\phi_i^2\lambda_{r_i}^2\sigma_{v_s}^2+\sigma_{v_r}^2\right)^{-1}\right)^{-1/N_s} \quad (28)$$

(18) is satisfied. From the Hadamard determinant inequality we can state that the determinant is minimised if the matrix $\mathbf{D}+\mathbf{XYZ}$ is diagonalised by the precoder $\mathbf{G}$. It is clear that this diagonalisation cannot be directly conducted since $\mathbf{D}$ is not a function of $\mathbf{G}$. However we note that by using $\mathbf{D}+\mathbf{XYZ}=\mathbf{D}^{1/2}(\mathbf{I}_{N_s}+\mathbf{D}^{-1/2}\mathbf{XYZD}^{-1/2})\mathbf{D}^{1/2}$ we can write the optimisation for the relay precoder in (16)-(18) as

$$\min_{\mathbf{G}} \quad (N_s/P_s)|\mathbf{D}|^{-1/N_s}|\mathbf{I}_{N_s}+\mathbf{D}^{-1/2}\mathbf{XYZD}^{-1/2}|^{-1/N_s} \quad (25)$$

$$\text{s.t.} \quad \mathrm{tr}\{\mathbf{G}(\mathbf{H}_s\mathbf{F}\mathbf{F}^{\mathrm{H}}\mathbf{H}_s^{\mathrm{H}}+\sigma_{v_s}^2\mathbf{I}_{N_r})\mathbf{G}^{\mathrm{H}}\}=P_r \quad (26)$$

where we note that the source power constraint has been omitted from the optimisation problem because it is guaranteed to hold with the precoder $\mathbf{F}$ given in (23).

Before calculating the relay precoder $\mathbf{G}$ as the solution to (25) and (26) we firstly introduce the following singular value decompositions

$$\mathbf{D}=\mathbf{U}_d\mathbf{\Lambda}_d\mathbf{V}_d^{\mathrm{H}}, \quad \mathbf{H}_s\mathbf{D}^{-1/2}=\mathbf{U}_t\mathbf{\Lambda}_t\mathbf{V}_t^{\mathrm{H}}, \quad \mathbf{H}_r=\mathbf{U}_r\mathbf{\Lambda}_r\mathbf{V}_r^{\mathrm{H}} \quad (27)$$

where $\mathbf{U}_d\in\mathbb{C}^{N_s\times N_s}$, $\mathbf{V}_d\in\mathbb{C}^{N_s\times N_s}$, $\mathbf{U}_t\in\mathbb{C}^{N_r\times N_s}$, $\mathbf{V}_t\in\mathbb{C}^{N_s\times N_s}$, $\mathbf{U}_r\in\mathbb{C}^{N_d\times N_r}$, and $\mathbf{V}_r\in\mathbb{C}^{N_r\times N_r}$ are all unitary matrices. The matrices $\mathbf{\Lambda}_d=diag\{\lambda_{d_1},\lambda_{d_2},\ldots,\lambda_{d_{N_s}}\}$, $\mathbf{\Lambda}_t=diag\{\lambda_{t_1},\lambda_{t_2},\ldots,\lambda_{t_{N_s}}\}$ and $\mathbf{\Lambda}_r=diag\{\lambda_{r_1},\lambda_{r_2},\ldots,\lambda_{r_{N_r}}\}$ are diagonal matrices containing the singular values of $\mathbf{D}$, $\mathbf{H}_s\mathbf{D}^{-1/2}$, and $\mathbf{H}_r$ respectively.

Applying the Hadamard determinant inequality to (25), and using the definitions for $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$, as well as the singular value decompositions in (27), we can state the inequality shown in (28) at the top of the page. The lower bound in (28) is achieved when the relay precoder is of the form

$$\mathbf{G}=\bar{\mathbf{V}}_r\mathbf{\Phi}\mathbf{U}_t^{\mathrm{H}} \quad (29)$$

where $\bar{\mathbf{V}}_r\in\mathbb{C}^{N_r\times N_s}$ contains the leftmost N_s columns of $\mathbf{V}_r$, $\mathbf{\Phi}=diag\{\phi_1,\phi_2,\ldots,\phi_{N_s}\}$, and $\mathbf{U}_t$ was defined in (27). The elements of the diagonal matrix $\mathbf{\Phi}$ can be found by minimising the objective function in (28) subject to the power constraint (26). The problem can be stated as

$$\min_{\phi_i} \quad \prod_{i=1}^{N_s}\left(1+\lambda_{t_i}^2\phi_i^2\lambda_{r_i}^2\left(\phi_i^2\lambda_{r_i}^2\sigma_{v_s}^2+\sigma_{v_r}^2\right)^{-1}\right)^{-1/N_s} \quad (30)$$

$$\text{s.t.} \quad \sum_{i=1}^{N_s}\phi_i^2 p_{ii}=P_r, \quad \phi_i^2\geq 0, \ \forall i. \quad (31)$$

where p_{ii} is the element in the i_{th} row and i_{th} column of the matrix $\mathbf{P}\triangleq\mathbf{U}_t^{\mathrm{H}}\mathbf{U}_s\left((P_s/N_s)\mathbf{\Lambda}_s^2+\sigma_{v_s}^2\mathbf{I}_{N_s}\right)\mathbf{U}_s^{\mathrm{H}}\mathbf{U}_t$. Here $\mathbf{U}_s\in\mathbb{C}^{N_r\times N_s}$ and $\mathbf{\Lambda}_s=diag\{\lambda_{s_1},\lambda_{s_2},\ldots,\lambda_{s_{N_s}}\}$ result from the singular value decomposition $\mathbf{H}_s=\mathbf{U}_s\mathbf{\Lambda}_s\mathbf{V}_s^{\mathrm{H}}$. We also note that the term $(N_s/P_s)\prod_{i=1}^{N_s}\lambda_{d_i}^{-1/N_s}$ has been omitted from (30) since it is a constant and has no effect on optimisation of the variables ϕ_i. The solution to the optimisation problem can be found using the KKT conditions of optimality to be

$$\phi_i^2=\left[\frac{-b_i+\sqrt{b_i^2-4a_ic_i}}{2a_i}\right]^+ \quad (32)$$

where we define the variables

$$a_i\triangleq\lambda_{r_i}^4\sigma_{v_s}^2(\lambda_{t_i}^2+\sigma_{v_s}^2), \qquad b_i\triangleq\lambda_{r_i}^2\sigma_{v_r}^2(\lambda_{t_i}^2+2\sigma_{v_s}^2),$$
$$c_i\triangleq\left(\sigma_{v_r}^4-\frac{\mu\lambda_{t_i}^2\lambda_{r_i}^2\sigma_{v_r}^2}{p_{ii}}\right). \quad (33)$$

Here the parameter μ must be calculated to satisfy (31).

The remaining task is to calculate the feedback matrix $\mathbf{B}$ and the unitary source matrix $\mathbf{\Psi}$ such that (15) holds with equality and the arithmetic MSE achieves the minimised lower bound in (28). With the decomposition in (19) and the source precoder in (23) we calculate $\mathbf{B}$ and $\mathbf{\Psi}$ similar to [10] by performing the following geometric mean decomposition

$$\sqrt{P_s/N_s}\mathbf{\Lambda}_h^{1/2}=\mathbf{Q}\bar{\mathbf{U}}\mathbf{\Psi}^{\mathrm{H}} \quad (34)$$

where $\mathbf{Q}\in\mathbb{C}^{N_s\times N_s}$ is an orthogonal matrix and $\bar{\mathbf{U}}\triangleq\beta^{-1/2}\mathbf{U}$ is an upper right triangular matrix with diagonal elements $\beta^{-1/2}$. Here we define β to be the lower MSE bound in (28). The feedback matrix $\mathbf{B}$ is now given by $\mathbf{B}=\beta^{1/2}\bar{\mathbf{U}}-\mathbf{I}_{N_s}$.

4. SIMULATION RESULTS

We assess the BER performance of the proposed system through simulations and compare it to existing linear techniques. We compare our design to the Naive AF (NAF), Pseudo-Matched Filter (PMF), optimal MMSE relay precoded design in [4], and the joint MMSE design in [8].

All benchmark systems use the optimal MMSE receiver, that takes the direct source-destination link into consideration, given by $\mathbf{W}=\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}(\mathbf{H}\mathbf{F}\mathbf{F}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}+\mathbf{R}_{vv})^{-1}$. Both the NAF and PMF algorithms employ $\mathbf{F}=\sqrt{P_s/N_s}\mathbf{I}_{N_s}$ at the source. The relay precoders for the PMF and NAF designs are given by $\mathbf{G}=\sqrt{P_r/\mathrm{tr}\{\mathbf{H}_r^{\mathrm{H}}\mathbf{H}_s^{\mathrm{H}}(\mathbf{H}_s\mathbf{F}\mathbf{F}^{\mathrm{H}}\mathbf{H}_s^{\mathrm{H}}+\sigma_{v_s}^2\mathbf{I}_{N_r})\mathbf{H}_s\mathbf{H}_r\}}\mathbf{H}_r^{\mathrm{H}}\mathbf{H}_s^{\mathrm{H}}$ and $\mathbf{G}=\sqrt{P_r/\mathrm{tr}\{\mathbf{H}_s\mathbf{F}\mathbf{F}^{\mathrm{H}}\mathbf{H}_s^{\mathrm{H}}\sigma_s^2+\sigma_{v_s}^2\mathbf{I}_{N_r}\}}\mathbf{I}_{N_r}$ respectively. The precoders for the optimal MMSE relay precoded system and joint MMSE design are given in [4] and [8] respectively.

We assume the channel matrices contain complex Gaussian entries with zero mean and unit variance and the symbols from the source antennas are drawn from QPSK constellations. We define the signal-noise ratio (SNR) for the source-relay, relay-destination, and source-destination stages as $\mathrm{SNR}_s\triangleq P_s/\sigma_{v_s}^2$, $\mathrm{SNR}_r\triangleq P_r/\sigma_{v_r}^2$, and $\mathrm{SNR}_d\triangleq P_s/\sigma_{v_d}^2$.

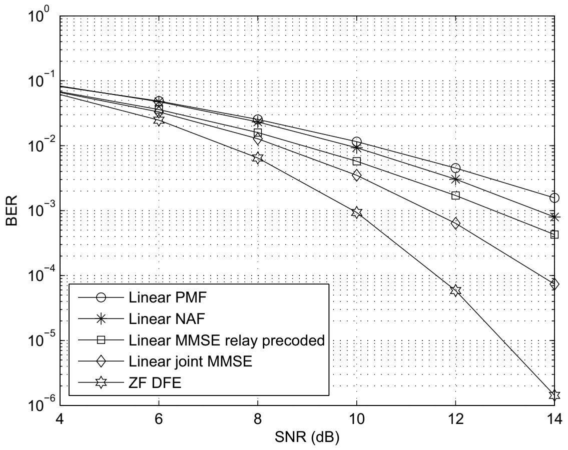

Figure 2: BER versus SNR for $N_s = N_r = N_d = 4$.

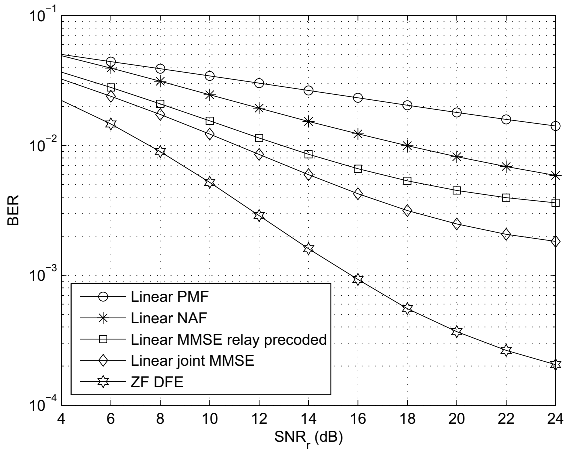

Figure 3: BER versus SNR$_r$ for $N_s = N_r = N_d = 3$, SNR$_s$ = 15 dB and SNR$_d$ = 5 dB

In the first simulation we consider transmission of data streams over a network with $N_s = N_r = N_d = 4$ antennas at the source, relay, and destination. We set SNR = SNR$_s$ = SNR$_r$ = SNR$_d$ and compare the BER performance of the proposed and benchmark designs for varying SNR shown in Figure 2. In the second scenario we consider $N_s = N_r = N_d = 3$ and set SNR$_s$ = 15 dB and SNR$_d$ = 5 dB. Figure 3 shows the BER performance of all designs for varying SNR$_r$. All simulation results obtained were averaged over 1000 channel realisations. It is clear that the proposed ZF DFE design offers a significant increase in performance in terms of BER compared to the linear benchmark systems.

5. CONCLUSIONS

In this paper we considered a non-linear transceiver design for AF MIMO relay networks where a direct link exists between the source and destination. Linear processors were utilised in each layer of the network and a decision feedback device at the receiver. The processors were designed to minimise the arithmetic MSE under the ZF condition and power constraints at the source and relay. Our design generalises an existing ZF DFE for the case of a direct source-destination link. Simulations demonstrate that the proposed design outperforms existing linear techniques in terms of BER.

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