Antenna Subset Selection in Distributed Multiple-Radar Architectures: A Knapsack Problem Formulation

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***Abstract*—In this paper, a performance driven resource allocation scheme for target localization in multiple radar systems is proposed and evaluated. An optimal subset of active antennas of predetermined size, K, is selected such that the localization mean-square error (MSE) is minimized. The problem is formulated in a combinatorial optimization framework as a knapsack problem (KP). The Cramer-Rao bound (CRB) is used as a performance metric. Cost parameters, representing operational cost or any other utilization constraints, are associated with each of the antennas. These are incorporated into the KP formulation, integrating decision making factors in the selection process. Antenna subset selection is implemented through an approximation algorithm, by successively selecting antennas so as to maximize the temporal Fisher information matrix (FIM) for a given subset size. The proposed approximation algorithm offers considerable reduction in computational complexity when compared with exhaustive search, supporting distributive processing and low performance loss.**

***Index Terms*—MIMO radar, Multistatic radar, Cramer-Rao bound, resource allocation, target localization.**

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I. Introduction

A. Background and Motivation

There are a growing number of systems that employ mobile stations or systems operating over prolonged time periods, in which the cost of operation becomes significant. One example of such a system is ground surveillance radars (GSRs). These are usually portable light-weight systems that may be carried by one person or mounted on a vehicle. GSR systems are used in a variety of applications, including urban warfare, counter terrorism, border patrol and security, airport security etc. The notion of resource-aware design is of critical importance when it comes to this type of system. Power management is one way to address resource-aware operation. In [1], we have proposed methods for allocating power among the transmit antennas such that localization performance is optimized while the total transmitted power is minimized.

Another aspect of resource-aware operation is the use of the available infrastructure. In a previous study [2] we have considered the problem of identifying the smallest subset of transmit and receive antennas, out of the available M transmit and N receive ones, that achieves a given target localization estimation mean-square error (MSE) threshold. In particular, we have shown that a given localization accuracy threshold may be met by using a proper subset of the available transmit and receive antennas. In addition to its operational savings, selecting a subset of active radars offers reduced communication needs and computational complexity. In this paper we address the case in which the active set must have a specific size. Thus, the goal is to select a subset of predetermined size K such that the estimation capability of the subset is maximized. The choice of the appropriate subset of antennas depends on the system parameters, such as the topology of the system with respect to the target, the signal-to-noise ratio (SNR) over the different propagation paths, the effective bandwidth, and the transmitted power.

The problem of selecting K sensors out of a given set of M possible sensors has been addressed in the literature for passive wireless sensor networks (WSNs) [3]- [5]. In [3], the problem is addressed through convex optimization tools, in which the discrete problem is relaxed to a continuous one. The estimation performance is evaluated using a scalar measure of the volume (mean radius) of the confidence ellipsoid. A different approach is proposed in [4], in which a set of sensors is selected such that the joint observations of the selected sensors with the prior target location distribution yields the greatest reduction in the entropy of the target location distribution. A geometry-based approach is proposed in [5] for the selection of sensor subsets in WSNs for estimating a target's bearing. Radar systems perform active sensing as they generate energy emissions through their transmit antennas for the purpose of observation. The *reflections* of this energy by targets of interest are then sensed at the receive antennas. In these systems two groups of sensors need to be selected: energy emitting sensors and data observing (sensing) sensors. The problem of selecting an optimal subset of K sensors translates to choosing K_0 transmit antennas out of the given M and $K - K_o$ receive antennas out of N. An active localization system is constrained to have at least one transmit antenna and one receive antenna. Overall, at least four antennas are required for target localization [1].

This subset selection problem may be formulated, in a combinatorial optimization framework, as a knapsack problem (KP) [6]. The KP determines the items to be included in the collection (knapsack), such that the total capacity is kept within a given limit (K) while minimizing the total value (MSE). The Cramer-Rao bound (CRB) [7] is used to evaluate the temporal performance level for a given set. This formulation facilitates a natural integration of decision factors into the optimization and the use of existing combinatorial optimization methods with its performance evaluation tools. An optimal brute-force search for this problem is simple to implement; however, its computational complexity is exponential in the number of elements and thus tends to grow very quickly as the size of the problem increases [6]. Fast approximation algorithms have been proposed for linear and quadratic KPs [8]- [10]. Common heuristic methods are *greedy* and *branch-and-bound* algorithms. Metaheuristics that use combinations of these heuristic tools have been proposed, among them is *multi-start local search* (MLS) [11], [12], and [13]. These methods offer polynomial complexity and their performance is evaluated through normalized approximation error analysis [11].

The combinatorial optimization problem of selecting a subset of K transmit and receive antennas such that the localization MSE is minimized is formulated in this paper as a KP. An approximation algorithm is proposed for the identification of such a subset. Cost parameters, associated with each of the antennas, are used to integrate decision factors in the selection process. The paper is organized as

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follows: The system model and the CRB metric are introduced in Section II. An approximation algorithm is proposed in Section III. Numerical analysis of the proposed algorithm is provided in Section IV. Finally, Section V concludes the paper.

II. System Model and Preliminaries

We consider a distributed multiple radar system with M transmit and N receive antennas, forming an $M \times N$ distributed multiple radar system. An extended target, with a center of mass located at position (x, y), is assumed. The system is tracking the target's location and has available estimates for unknown parameters, such as the target radar cross section (RCS) and propagation attenuation, from previous cycles. The search cell is confined to $(x_c \pm kc/\beta, y_c \pm kc/\beta)$, where k is an integer, c is the speed of light, and β is the waveform effective bandwidth. The transmit and receive radars are located in a two dimensional plane. The M transmit antennas are arbitrarily located at coordinates $(x_{m_{Tx}}, y_{m_{Tx}})$, $m = 1, \ldots, M$, and the N receiver antennas are arbitrarily located at coordinates $(x_{n_{Rx}}, y_{n_{Rx}})$, $n = 1, \ldots, N$. The transmit antenna set is defined as $\mathcal{S}_{Tx} := \{(x_{1_{Tx}}, y_{1_{Tx}}), (x_{2_{Tx}}, y_{2_{Tx}}), ..., (x_{M_{Tx}}, y_{M_{Tx}})\}$ and the receive radar set as $\mathcal{S}_{Rx} := \{(x_{1_{Rx}}, y_{1_{Rx}}), (x_{2_{Rx}}, y_{2_{Rx}}), ..., (x_{N_{Rx}}, y_{N_{Rx}})\}$. A set of orthonormal waveforms is transmitted, each with a lowpass equivalent $s_m(t)$. The waveforms' transmitted powers $p_{m_{tx}}$ are constrained by $\mathbf{p}_{tx} = [p_{1_{tx}}, p_{2_{tx}}, ..., p_{M_{tx}}]^T$.

The baseband representation for the signal transmitted from radar m received at radar n is

$$r_{m,n}(t) = \sqrt{\alpha_{m,n} p_{m_{tx}}} h_{m,n} s_m(t - \tau_{m,n}) + w_{m,n}(t), \quad (1)$$

where $\tau_{m,n}$ denotes the propagation time of a signal transmitted by radar m, reflected by the target, and received by radar n. The term $\alpha_{m,n}$ represents the variation in the signal strength due to path loss effects. The term $h_{m,n}$ incorporates the effect of phase offsets between the transmit and receive antennas and the target radar RCS impact on the phase and amplitude. This quantity is modeled as being deterministic and complex. The term $w_{m,n}(t)$ represents circularly symmetric, zero-mean, complex Gaussian noise, spatially and temporally white with autocorrelation function $\sigma_w^2 \delta(\tau)$. The propagation path from transmitter m to the target and from the target to receiver n is defined as channel (m, n).

We define a vector of unknown parameters as $\mathbf{u} = \left[x, y, \mathbf{h}^T\right]^T$, where $\mathbf{h} = [h_{1,1}, h_{1,2}, ..., h_{M,N}]^T$.

For an unbiased (or asymptotically unbiased) estimator, the maximum likelihood estimator (MLE) MSE asymptotically approaches the CRB as the SNR becomes large [7]. In [14] it was demonstrated that the MLE is asymptotically tight to the CRB at high SNR (over 10dB). Thus, the CRB is used here to represent the localization MSE as a function of the power allocation. Herein, the CRB expression, $\mathbf{C}_{x,y}(\mathbf{u})$, derived in [1], is used, resulting in the following expression:

$$\mathbf{C}_{x,y}(\mathbf{u}) = \left\{ \sum_{m=1}^{M} \sum_{n=1}^{N} \mathbf{J}_{m,n} \right\}^{-1}, \quad (2)$$

where the submatrix $\mathbf{J}_{m,n}$ is the Fisher information matrix (FIM) defined as

$$\mathbf{J}_{m,n} = \begin{bmatrix} u_{a_{m,n}} & u_{c_{m.n}} \\ u_{c_{m,n}} & u_{b_{m,n}} \end{bmatrix}, \quad (3)$$

and the elements $u_{a_{m,n}}$, $u_{b_{m,n}}$, and $u_{c_{m,n}}$, are defined in [2]. Introducing a set of binary variables

$$q_{tx_m} = \begin{cases} 1 & \text{if transmit radar } m \text{ is selected;} \\ 0 & \text{otherwise} \end{cases}, \quad m = 1, ..., M, \quad (4)$$

and

$$q_{rx_n} = \begin{cases} 1 & \text{if receive radar } n \text{ is selected;} \\ 0 & \text{otherwise} \end{cases}, \quad n = 1, ..., N, \quad (5)$$

the CRB for a set of radars, $\mathcal{S}_A = \{\mathbf{x}_{m_{Tx}} \in \mathcal{S}_{tx}, \mathbf{x}_{n_{Rx}} \in \mathcal{S}_{Tx} \mid q_{tx_m} = 1,\ q_{rx_n} = 1\}$, may be expressed as

$$\mathbf{C}_{\mathcal{S}_A}(\mathbf{q}_{tx}, \mathbf{q}_{rx}, \tilde{\mathbf{u}}) = \left\{ \sum_{m=1}^{M} \sum_{n=1}^{N} q_{tx_m} q_{rx_n} \mathbf{J}_{m,n} \right\}^{-1}, \quad (6)$$

where $\mathbf{q}_{tx} = [q_{tx_1}, q_{tx_2}, ..., q_{tx_M}]^T_{M \times 1}$, $\mathbf{q}_{rx} = [q_{rx_1}, q_{rx_2}, ..., q_{rx_N}]^T_{N \times 1}$. We denote by $\tilde{\mathbf{u}} = \left[\tilde{x}, \tilde{y}, \tilde{\mathbf{h}}^T\right]^T$ a vector of prior estimates of the target location and RCS, obtained in previous cycles. Based on the FIM defined in (3), the trace of the CRB expression in (6), and therefore, the metric representing $\sigma_x^2(\mathbf{q}_{tx}, \mathbf{q}_{rx}) + \sigma_y^2(\mathbf{q}_{tx}, \mathbf{q}_{rx})$, may be formulated as

$$\text{tr}(\mathbf{C}_{\mathcal{S}_A}(\mathbf{q}_{tx}, \mathbf{q}_{rx}, \tilde{\mathbf{u}})) = \sum_{m=1}^{M} \sum_{n=1}^{N} U_{1_{m,n}} q_{tx_m} q_{rx_n} \quad (7)$$
$$\times \left[\sum_{m,m'=1}^{M} \sum_{n,n'=1}^{N} U_{2_{m,n,m',n'}} q_{tx_m} q_{rx_n} q_{tx_{m'}} q_{rx_{n'}} \right]^{-1},$$

where $U_{1_{m,n}} = u_{a_{m,n}} + u_{b_{m,n}}$ and $U_{2_{m,n,m',n'}} = u_{a_{m,n}} u_{b_{m',n'}} - u_{c_{m,n}} u_{c_{m',n'}}$. Locations in vector $\mathbf{q}_{tx}$ and vector $\mathbf{q}_{rx}$, corresponding to elements $\mathbf{x}_{m_{Tx}} \in \mathcal{S}_A$ and $\mathbf{x}_{n_{Rx}} \in \mathcal{S}_A$, respectively, are set to one; otherwise they are set to zero. The expression for the CRB, as given in (7), offers a metric that may be used to represent the MLE MSE in the KP formulation provided next.

III. Subset Selection

In the context of resource-aware operation of multiple active sensor systems, such as multiple radar systems, we developed tools for resource-aware operation in [1] and [2]. Herein, we address a scenario in which a required utilization factor is imposed on the system infrastructure operation. For example, under some emergency operation conditions, a multiple sensor system may be restricted to operate only a fraction of the system at any given time, denoted by $\rho_{\max}$. This means that if the system is equipped with $M + N$ sensors it will aim at working with a subset of $K = \lfloor \rho_{\max}(M + N) \rfloor$ sensors that present the best estimation performance attainable by a subset of this size. The number of possible combinations for choosing K antennas out of a set of $M + N$ is given by $\sum_{k_o=1; k_o \le M; K-k_o \le N}^{K-1} \binom{M}{k_o} \binom{N}{K-k_o}$. The objective is to choose a combination that offers the lowest MSE, i.e., minimal $\sigma_x^2 + \sigma_y^2 \ge \text{tr}(\mathbf{C}_{\mathcal{S}_A}(\mathbf{q}_{tx}, \mathbf{q}_{rx}))$.

1) Problem formulation: The problem of selecting K sensors, where K_o are transmit antennas and $K - K_o$ are receive antennas, that provide the best estimation performance in terms of localization MSE, may be formulated as

$$\begin{aligned} \underset{\mathbf{q}_{tx}, \mathbf{q}_{rx}}{\text{minimize}} \quad & \text{tr}(\mathbf{C}_{\mathcal{S}_A}(\mathbf{q}_{tx}, \mathbf{q}_{rx}, \tilde{\mathbf{u}})), \\ s.t. \quad & \sum_{m=1}^{M} q_{tx_m} + \sum_{n=1}^{N} q_{rx_n} = K, \\ & \sum_{m=1}^{M} v_{tx_m} q_{tx_m} + \sum_{n=1}^{N} v_{tx_m} q_{rx_n} \le K^v \\ & \sum_{m=1}^{M} q_{tx_m} \ge 1, \ \sum_{n=1}^{N} q_{rx_n} \ge 1, \\ & q_{tx_m} \in \{0, 1\}, \ q_{rx_n} \in \{0, 1\}, \end{aligned} \quad (8)$$

where K^v represents the total allowable cost. The matrix $\mathbf{C}_{\mathcal{S}_A}(\mathbf{q}_{tx}, \mathbf{q}_{rx})$ is given in (7). The search cell center coordinates, (x_c, y_c), may also be used instead of an estimated target location$(\tilde{x}, \tilde{y})$. The trace of the CRB represent the sum of the variances of the estimation error on the target's x and y location, $\sigma_x^2(\mathbf{q}_{tx}, \mathbf{q}_{rx}) + \sigma_y^2(\mathbf{q}_{tx}, \mathbf{q}_{rx})$.

We replace the optimization problem with an approximate KP representation by replacing $\min\{\mathrm{tr}(\mathbf{C}_{\mathcal{S}_A}(\mathbf{q}_{tx}, \mathbf{q}_{rx}, \tilde{\mathbf{u}}))\}$ in (8) with $\max\left\{\mathrm{tr}\left(\mathbf{C}_{\mathcal{S}_A}^{-1}(\mathbf{q}_{tx}, \mathbf{q}_{rx}, \tilde{\mathbf{u}})\right)\right\}$. Using the relation in (6), we replace $\max\left\{\mathrm{tr}\left(\mathbf{C}_{\mathcal{S}_A}^{-1}(\mathbf{q}_{tx}, \mathbf{q}_{rx}, \tilde{\mathbf{u}})\right)\right\}$ by $\max\left\{\sum_{m=1}^{M}\sum_{n=1}^{N} q_{tx_m} q_{rx_n}\,\mathrm{tr}(\mathbf{J}_{m,n})\right\}$. Using the definition of $U_{1_{m,n}}$ following (7), the resulting approximate KP optimization is then

$$\begin{aligned} \underset{\mathbf{q}_{tx}, \mathbf{q}_{rx}}{\text{maximize}} \quad & \sum_{m=1}^{M}\sum_{n=1}^{N} U_{1_{m,n}} q_{tx_m} q_{rx_n}, \\ s.t. \quad & \sum_{m=1}^{M} q_{tx_m} + \sum_{n=1}^{N} q_{rx_n} = K, \\ & \sum_{m=1}^{M} v_{tx_m} q_{tx_m} + \sum_{n=1}^{N} v_{tx_m} q_{rx_n} \le K^v \\ & \sum_{m=1}^{M} q_{tx_m} \ge 1, \ \sum_{n=1}^{N} q_{rx_n} \ge 1, \\ & q_{tx_m} \in \{0,1\}, \ q_{rx_n} \in \{0,1\}. \end{aligned} \tag{9}$$

This is a quadratic knapsack problem (QKP). For later use, we define a cost objective function, $Q_K(\mathbf{q}_{tx}, \mathbf{q}_{rx})$, as

$$Q_{K-set}(\mathbf{q}_{tx}, \mathbf{q}_{rx}) = \sum_{m=1}^{M}\sum_{n=1}^{N} q_{tx_m} q_{rx_n} U_{1_{m,n}} \times \left(\sum_{m,m'=1}^{M}\sum_{n,n'=1}^{N} U_{2_{m,n,m',n'}} q_{tx_m} q_{rx_n} q_{tx_{m'}} q_{rx_{n'}}\right)^{-1}, \tag{10}$$

and a total utilization weight function

$$W_{K-set}(\mathbf{q}_{tx}, \mathbf{q}_{rx}) = \sum_{m=1}^{M} v_{tx_m} q_{tx_m} + \sum_{n=1}^{N} v_{tx_m} q_{rx_n}, \tag{11}$$

where K^v represents the total allowable cost.

An optimal solution to the KP in (9) may be obtained through an exhaustive search of all possible combinations of transmit and receive antenna sets of size K, which has exponential complexity as it requires $\sum_{k_o=1; k_o \le M; K-k_o \le N}^{K-1} \binom{M}{k_o}\binom{N}{K-k_o}$ iterations. Hereafter, we propose an approximate algorithm with lower computational cost.

2) KNAPsetK Algorithm: The KP in (9) defines a search for a combination of K transmit and receive antennas that maximizes the FIM. The proposed approximate search algorithm in detailed in Table 1. An initial subset of the antennas is generated by selecting one transmitter and one receiver $\left\{\mathbf{x}_{i_{Tx}}^{\circ}, \mathbf{x}_{j_{Rx}}^{\circ}\right\}$. As in the aKNAPminset algorithm, all possible pairs of transmit and receive antennas are exhausted for the given available set of antennas. The algorithm repeats the search for all possible MN initial pairs $\left\{\mathbf{x}_{i_{Tx}}^{\circ}, \mathbf{x}_{j_{Rx}}^{\circ}\right\}$. The locations in vector $\mathbf{q}_{tx}$ and vector $\mathbf{q}_{rx}$, corresponding to the resulting $\mathbf{x}_{i_{Tx}}^{\circ}$ and $\mathbf{x}_{j_{Rx}}^{\circ}$, are set to one and the antennas are added to the active antenna set $\mathcal{S}_{\min} = \{\mathbf{x}_{i_{Tx}}, \mathbf{x}_{j_{Rx}}\}$. At the same time, these antennas are discarded from the remaining, inactive, transmit and receive antenna sets, $\mathcal{S}_{Tx}' = \mathcal{S}_{Tx} \backslash \mathbf{x}_{i_{Tx}}$ and $\mathcal{S}_{Rx}' = \mathcal{S}_{Rx} \backslash \mathbf{x}_{j_{Rx}}$, respectively. At each iteration step either one transmit antenna or one receive antenna is added to the active subset, such that the trace of the *temporal* FIM matrix is maximized, $\max_{\mathbf{x}_{i_{Tx}} \in \mathcal{S}_{Tx}'} \left\| \mathrm{tr}\left(\mathbf{J}_{\mathcal{S}_{\min} \cup \mathbf{x}_{i_{Tx}}}\right) \right\|$ for a transmit antenna or $\max_{\mathbf{x}_{j_{Rx}} \in \mathcal{S}_{Rx}'} \left\| \mathrm{tr}\left(\mathbf{J}_{\mathcal{S}_{\min} \cup \mathbf{x}_{j_{Rx}}}\right) \right\|$ for a receive antenna. The temporal FIM is defined as the FIM obtained for a specific subset of transmit and receive antennas at a given iteration step, where $q_{tx_m} = 1$ if $\mathbf{x}_{m_{Tx}} \in \mathcal{S}_{temp}$ and $q_{rx_n} = 1$ if $\mathbf{x}_{n_{Rx}} \in \mathcal{S}_{temp}$; otherwise, they are set to zero.

Table 1: K-Subset selection algorithm

1	init: $\mathbf{q}_{tx} = \mathbf{0}$, $\mathbf{q}_{rx} = \mathbf{0}$, $\mathcal{A}_{\min} = \varnothing$, $\mathcal{A}_{\min}^{*} = \varnothing$,
2	for $m = 1, ..., M$ and $n = 1, ..., N$
2.1	Select initial subset: $\mathcal{S}_{\min} = \{\mathbf{x}_{m_{Tx}}, \mathbf{x}_{n_{Rx}}\}$,
2.2	Update: $\left\{ \begin{array}{c} \mathcal{S}_{Tx}' = \mathcal{S}_{Tx} \backslash \mathbf{x}_{m_{Tx}},\ \mathcal{S}_{Rx}' = \mathcal{S}_{Rx} \backslash \mathbf{x}_{n_{Rx}} \\ q_{tx_m} = 1,\ q_{rx_n} = 1 \end{array} \right.$;
2.3	Set: $set = 0, k = 2$
2.4	while $k < K$
2.4.1	$\left\{ \begin{array}{c} \text{if } \mathcal{S}_{Tx}' \neq null \text{ than select } \mathbf{x}_{i_{Tx}}' \text{ s.t.} \\ \mathbf{x}_{i_{Tx}}' = \arg \max_{\mathbf{x}_{i_{Tx}} \in \mathcal{S}_{Tx}'} \mathrm{tr}\left(\mathbf{J}_{\mathcal{S}_{\min} \cup \mathbf{x}_{i_{Tx}}}\right) \\ \text{Update:} \left\{ \begin{array}{c} \mathbf{x}_{Tx_t} = \mathbf{x}_{i_{Tx}}' \\ J_{Tx_t} = \mathrm{tr}\left(\mathbf{J}_{\mathcal{S}_{\min} \cup \mathbf{x}_{i_{Tx}}'}\right) \end{array} \right. \end{array} \right.$
2.4.2	$\left\{ \begin{array}{c} \text{if } \mathcal{S}_{Rx}' \neq null \text{ than select } \mathbf{x}_{j_{Rx}}' \text{ s.t.} \\ \mathbf{x}_{j_{Rx}}' = \arg \max_{\mathbf{x}_{j_{Rx}} \in \mathcal{S}_{Rx}'} \mathrm{tr}\left(\mathbf{J}_{\mathcal{S}_{\min} \cup \mathbf{x}_{i_{Tx}}}\right) \\ \text{Update:} \begin{array}{c} \mathbf{x}_{Rx_t} = \mathbf{x}_{j_{Rx}}'; \\ J_{Rx_t} = \mathrm{tr}\left(\mathbf{J}_{\mathcal{S}_{\min} \cup \mathbf{x}_{j_{Rx}}'}\right) \end{array} \end{array} \right.$
	if $J_{Tx\_temp} > J_{Rx\_temp}$
	$\left\{ \begin{array}{c} \text{Update: } \mathcal{S}_{\min} = \mathcal{S}_{\min} \cup \{\mathbf{x}_{Tx_t}\}, \\ \mathcal{S}_{Tx}' = \mathcal{S}_{Tx}' \backslash \{\mathbf{x}_{Tx_t}\} \\ \text{Set: } k = k+1, q_{\mathbf{x}_{Tx_t}} = 1, \end{array} \right.$
	else
	$\left\{ \begin{array}{c} \text{Update: } \mathcal{S}_{\min} = \mathcal{S}_{\min} \cup \{\mathbf{x}_{Rx_t}\}, \\ \mathcal{S}_{Rx}' = \mathcal{S}_{Rx}' \backslash \{\mathbf{x}_{Rx_t}\} \\ \text{Set: } k = k+1, q_{\mathbf{x}_{Rx_t}} = 1 \end{array} \right.$
	go to (2.4)
2.5	$\mathcal{A}_{\min} = \mathcal{A}_{\min} \cup \{\mathbf{q}_{tx}, \mathbf{q}_{rx}\}$;
2.6	$\mathbf{q}_{tx} = \mathbf{0}$, $\mathbf{q}_{rx} = \mathbf{0}$, $set = set + 1$;
	end (2)
3	for $index = 1 : set$
3.1	select vectors $\left\{\mathbf{q}_{tx_k}^{*}, \mathbf{q}_{rx_k}^{*}\right\} \in \mathcal{A}_{\min}$ s.t. $\arg \min_{\mathbf{q}_{tx_k}, \mathbf{q}_{rx_k} \in \mathcal{A}_{\min}} \mathrm{tr}\left(\mathbf{C}_{\mathcal{S}_A}(\mathbf{q}_{tx_k}, \mathbf{q}_{rx_k})\right)$
	end (3)
	end $\left\{\mathbf{q}_{tx_k}^{*}, \mathbf{q}_{rx_k}^{*}\right\}$

The proposed heuristic algorithm offers a fast, polynomial-time approximation scheme that allows a multiple radar system to continuously adapt its operation to changing conditions. It has a reduced complexity of $\sim O(KMN(M+N))$, where K is the number of antennas in the final subset. Comparatively, an exhaustive search has a complexity of $\sim O(2^{M+N})$. For large numbers of radars, significant computational savings are obtained through the use of the proposed algorithm. For example, a 10×12 MIMO radar system with a subset size of $K = 6$ will required $2^{22} = 4{,}194{,}304$ iterations with an exhaustive search while the proposed heuristic algorithm requires only $10 \times 12 \times 22 \times 6 = 15{,}840$ iterations. The gap increases rapidly for larger numbers of antennas. The proposed solution presents a practical and useful way of approximating the exact solution. Another advantage of the proposed algorithm is that it may be solved using distributed processors by solving MN subproblems defined by step 2.4 in Table 1. An important issue that needs to be addressed is how closely the algorithm's solution approximates the global optimum. Additionally, since the selection of the active subset is reliant on estimated values of the channel, the robustness of the proposed algorithm to estimation errors should be evaluated. Next, a metric for algorithm performance evaluation and robustness is defined.

3) Normalized approximation error: The proposed algorithm will not necessarily provide a global optimum. Its performance may be evaluated based on the normalized approximation error ε_{K-set} given by

$$\varepsilon_{K-set} = \frac{Q_{K-set}\left(\mathbf{q}_{tx}^{*}, \mathbf{q}_{rx}^{*}\right) - Q_{K-set}\left(\mathbf{q}_{tx}^{opt}, \mathbf{q}_{rx}^{opt}\right)}{Q_{K-set}\left(\mathbf{q}_{tx}^{opt}, \mathbf{q}_{rx}^{opt}\right)}. \quad (12)$$

This performance error metric represents the effectiveness of the proposed algorithms. A value of $\varepsilon_K = 1$ means that the received suboptimal solution is twice the size of the optimal solution. The metric, ε_{K-set}, refers to the fraction of additional localization MSE incurred by the approximate approach when compared with the optimal one.

IV. Numerical Analysis

The spatially diverse multiple propagation paths between the transmit and receive radars have different error characteristics, depending on the specific path loss, target reflectivity, effective bandwidth, and transmitted power. In this section, numerical analysis is provided for the proposed subset selection algorithm. A 5×7 MIMO radar system ($M = 5$ and $N = 7$) is chosen for this analysis. To evaluate the effect of the radars' spread, four different angular spreads with respect to the target are chosen, as illustrated in Figure 1 in Case 1 through Case 4. The ranges from the transmit and receive radars to the target are set to be all equal, i.e., $R_{m_{Tx}} = R_{n_{Rx}} = 10^3$m, $\forall m, n$. This is equivalent to setting all $\alpha_{m,n}$'s to be equal. Radar layouts with different ranges with respect to the target position are generated by using the same angular spread as before and setting the transmit radars' ranges to $[5; 3; 2.4; 3.4; 5] \times 10^3$m and the receive radars' ranges to $[2.85; 2.85; 2.77; 2.97; 1.92; 1.82; 2.67] \times 10^3$m. Case 5 through Case 8 in Figure 2 demonstrate these spreads. The target location is set to the axis origin, $(x, y) = (0, 0)$. Two target RCS models are used; The first model is uniform reflectivity, denoted by $\mathbf{h}_1$, supporting the evaluation of system geometry, with the target RCS factored out. The second RCS model is $\mathbf{h}_2$, mimicking a scenario of two transmitters with high reflectivity conditions (transmitters 1 and 5), while one exhibits significant loss (transmitter 2).

The subset size is set to $K = 6$. The subset selection algorithm, proposed in Section III, is applied to the system layouts given in Figure 1. At first, all cost factors $\mathbf{v}_{tx}$ and $\mathbf{v}_{rx}$ are set to be equal and $\mathbf{p}_{tx_{\max}} = 100 \times \mathbf{1}_{5\times1}$. The resulting subsets selected for Case 1 through Case 4, where $\mathbf{h}_2$ is used to model the target RCS, are given in Table 2.

	Table 2: RCS model $\mathbf{h}_2$.		
	$\mathbf{q}_{tx}^{*}$ Tx, $\mathbf{v}_{tx} = \mathbf{1}$	$\mathbf{q}_{rx}^{*}$ Rx, $\mathbf{v}_{rx} = \mathbf{1}$	Q_{K-set}
Case 1	$1, 0, 0, 0, 1$	$0, 1, 1, 0, 0, 1, 1$	3.88
Case 2	$1, 0, 0, 0, 1$	$1, 1, 0, 0, 0, 1, 1$	4.32
Case 3	$1, 0, 0, 0, 1$	$1, 1, 0, 0, 0, 1, 1$	7.4
Case 4	$1, 0, 0, 0, 1$	$0, 0, 1, 1, 1, 1, 0$	5.95

In all four cases, transmitter 1 and 5 are chosen as active radars in the subset, as they have the best channel conditions in the system. The receivers selected for the subset are the ones that provide the best angular spread, following a given selection of transmitters. Next, Case 5 through Case 8 are evaluated with $\mathbf{h}_1$ as the target RCS, eliminating the target reflectivity effect and concentrating on the geometric layout of the system. The resulting subset selections are as provided in Table 3. Transmitters 1, 2, and 3 are selected, as before, since they are closer to the target, compared with transmitter 1 or 5. On the receiver side, receiver 6 is selected in all four cases, as it is located closer to the target. As in the previous analysis, receivers that minimize the distance to the target and maximize angular spread are added to the subset. For that reason, receiver 1, 2, or 3 is added instead of receiver 5, in most cases. Although receiver 5 is located closer to the target, it does not always have a good angular position with respect to receiver 6. An exhaustive search will result with the same selections for the antennas subsets and the same MSE performance for all cases (Case 1 through Case 8), represented by $Q_{K-set}\left(\mathbf{q}_{tx}^{*}, \mathbf{q}_{rx}^{*}\right)$. As before, a decision vector with various cost factors for each antenna is used for Case 5 through Case 8. The transmit cost factors are chosen as $\mathbf{v}_{tx} = [1, 1, 7, 1, 5]$ and the receive cost factors are chosen as $\mathbf{v}_{rx} = [2, 3, 1, 1, 1, 5, 1]$. The resulting subset selections, with different cost factors, are as presented in Table 4.

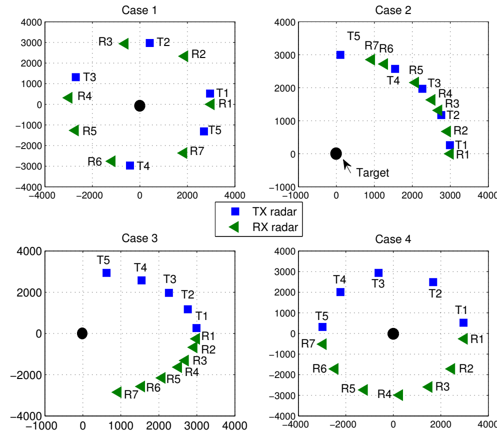

Fig. 1. Multiple radar layouts with equal ranges between transmit and receive radars and the target; Cases 1, 2, 3, and 4.

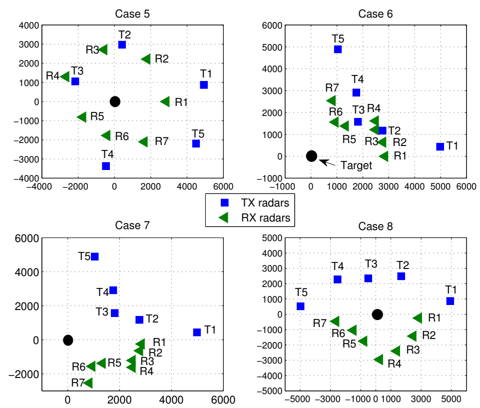

Fig. 2. Multiple radar layouts with different ranges between transmit and receive radars and the target; Cases 5, 6, 7, and 8.

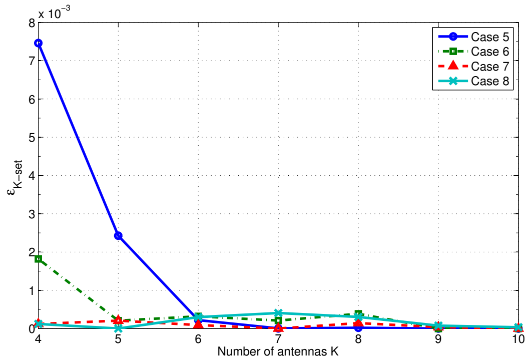

Fig. 3. Averaged normalized approximation error, $\bar{\varepsilon}_{K-set}$, and averaged normalized MSE error, $\bar{\varepsilon}_{MSE_{\min\, set}}$, for Cases 5, 6, 7, and 8 for a range of set size values $K = 4, 5, ..., 10$.

	Table 3: RCS model $\mathbf{h}_1$		
	$\mathbf{q}^*_{tx}$ Tx, $\mathbf{v}_{tx} = \mathbf{1}$	$\mathbf{q}^*_{rx}$ Rx, $\mathbf{v}_{tx} = \mathbf{1}$	Q_{K-set}
Case 5	$0, 1, 1, 1, 0$	$0, 0, 1, 0, 1, 1, 0$	2.26
Case 6	$0, 1, 1, 1, 0$	$1, 1, 0, 0, 0, 1, 0$	6.80
Case 7	$1, 1, 1, 1, 0$	$1, 0, 0, 0, 0, 1, 0$	11.12
Case 8	$0, 1, 1, 1, 0$	$1, 0, 0, 0, 0, 1, 1$	6.70

The integration of different cost values changed the final selection of antennas for one of the cases (Case 6), for which the constraint on the total cost, K^v, imposed an adaptation in the antennas selection to integrate one with lower cost. This results in a slight increase in the MSE performance.

	Table 4: RCS model $\mathbf{h}_1$ and $K^v = 17$			
	$\mathbf{q}^*_{tx}$ Tx	$\mathbf{q}^*_{rx}$ Rx	Q_{K-set}	K^v
Case 5	$0, 1, 1, 1, 0$	$0, 0, 1, 0, 1, 1, 0$	2.26	16
Case 6	$0, 1, 1, 1, 0$	$1, 0, 0, 0, 0, 1, 1$	6.90	17
Case 7	$1, 1, 1, 1, 0$	$1, 0, 0, 0, 0, 1, 0$	11.12	17
Case 8	$0, 1, 1, 1, 0$	$1, 0, 0, 0, 0, 1, 1$	6.70	17

In order to evaluate the influence of the fast approximation on solution quality, the normalized approximation error is evaluated numerically for the systems given in Figure 2, Case 5 through Case 8. The channel $\mathbf{h}$ is chosen randomly at each iteration, from a complex Gaussian distribution. Different target positions are selected at random in each iteration, uniformly distributed within a square of dimensions $\Delta x = \pm 100$ and $\Delta y = \pm 100$ around the axis origin $(x, y) = (0, 0)$. A set of 1000 simulations is performed, each with a different target location and channel matrix. At each simulation, the normalized approximation error is evaluated and an average normalized approximation error is generated.

The normalized approximation error is evaluated for a range of K values. The results are given in Figure 3. The approximate algorithm solution follows the optimal one in more than 99% of the cases. Case 3 and Case 4 have almost no errors when compared with an exhaustive search. The larger the size of the subset, the smaller the error is. Overall, all algorithms follow the optimal solution determined by exhaustive search closely.

The following may be deduced from the numerical analysis: The *channel* conditions affect the choice of antennas selected for a given subset. Antennas with better channel conditions and shorter distances to the targets are favorable over others. The use of the term *channel* in this context refers to the overall attenuation over a given propagation path between transmit radar m, the target, and receive radar n, defined as channel (m, n). The angular spread of the transmit and receive antennas with respect to the target impacts the selection of the active antennas. As higher angular spread results in lower MSE, the choice of a subset that also minimizes the localization MSE is reliant on the geometric layout of the system. Cost factors affect the manner in which radars are added to the subset, such that radars with good channel condition that are assigned a high cost factor are replaced by ones with lower operational cost. The fast approximate algorithm follows the optimal performance of an exhaustive search with high efficiency. A match of more than 99% is achieved, when compared with an exact algorithm.

V. Conclusions

An efficient method has been developed for the identification of a subset of K transmit and receive antennas that maximizes radar system performance in terms of target localization accuracy. The selection problem has been defined as a KP and a fast approximation algorithm, based on a greedy strategy with multi-start local search, has been proposed. The algorithm supports distributed processing of MN subproblems and has a reduced complexity of $\sim O(KMN(M+N))$. Comparatively, an exhaustive search has a complexity of $\sim O(2^{M+N})$. For large numbers of radars, significant computational savings are obtained through the use of the proposed algorithm. Cost parameters have been introduced to integrate decision factors in the selection process. The algorithm has been shown to perform very close to the optimum, with little or no penalty in terms of the final localization MSE.

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